ON THE REGULARITY OF FLUCTUATIONS IN ANNUAL AND SECULAR POLAR MOTIONS

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Abstract. It may become possible to explain much of the behaviour and remarkable regularity of fluctuations in reported annual and secular polar motions if the potential that causes an immediate excitation is also a factor in that excitation's rate of change afterwards.

The Chandler frequency, for example, being a function of the deformation excitation, should then vary with the librational and nutational displacements of the pole of rotation. It is found that the frequency does in fact do so.

The annual excitation would be affected as well. The steady and seasonal primary excitations are known to cause free and forced nutations that are accompanied by periodic secondary excitations. These would arise partly at once and partly in the course of time; they would modulate the primary annual excitation and also one another, according to the period of the beats between the annual and Chandler nutations. It is found that the reported annual excitation shows phase and amplitude fluctuations of this kind. (The data also show another large excitation that occurred briefly on two occasions).

Finally, the amplitude and phase of the secular librations appear to have followed an expression that is obtained by integrating the rates of change of excitation. This expression is a function of the amplitude of the annual excitation and the period of the beats between the annual and Chandler nutations.

Attention is drawn to certain relationships in polar motions, and to possible creep excitations caused by flow, slippage or changes in torque. An excitation arising from the Chandler nutation is presumed to cause modulations of the annual excitation; the smoothed integrated effects are then found to resemble librations.

1. Deformation Excitations

m is the position of the rotation pole P with respect to the position to which P would revert if the disturbing potential ceased. In the notation of Munk and MacDonald (1960) $\psi_D = (k/k_s)m$. This indicates that the stress which produces a deformation excitation ψ_D is proportional to m, so the rate of possible creep due to this stress would also be expected to vary with m. Then after an interval τ the actual deformation excitation would become $f(m) + f(m^2)$.

The annual and Chandler frequencies are σ and σ_0 respectively, in radians per year. Then the Chandler component in $f(m)$ would introduce an excitation

$$w_0 \exp(i\sigma_0 t) = w_0 \exp\{i(\sigma - \mu)t\},$$

where $\mu = \sigma - \sigma_0$. Evidence of such a 'Chandler' component in the 'annual' excitation appears in the annual positions of the anticlockwise excitation pole shown in Figure 1, using data from Iijima (1965). The positions revolve clockwise with frequency about 1/7 cycle per year. The average value of w_0 is about 0.″004.

P. Melchior and S. Yumi (eds.), Rotation of the Earth, 61–67.

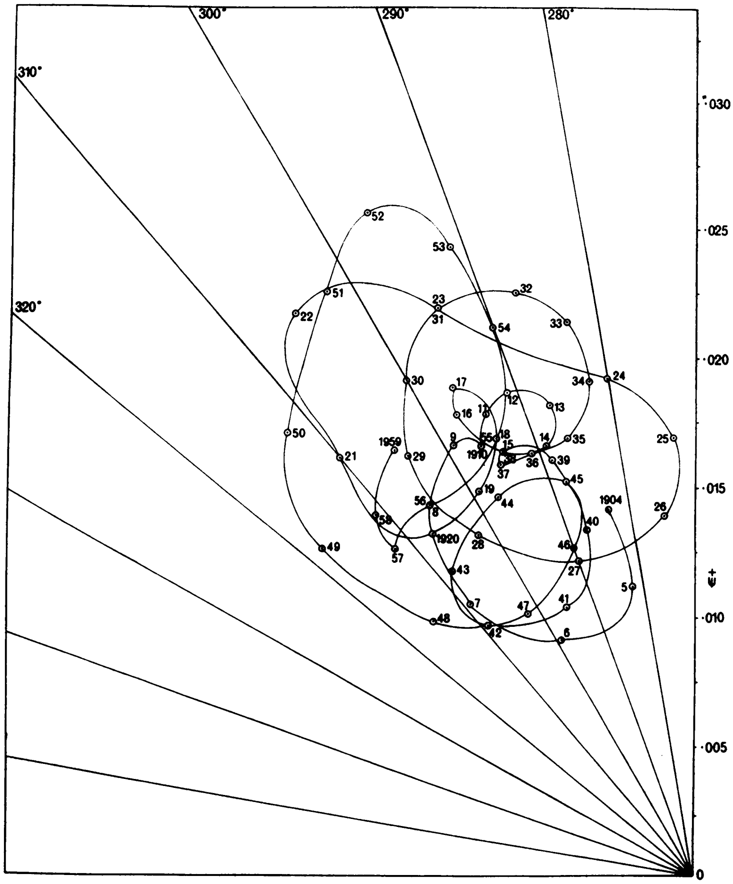

Fig. 1. Position of the anticlockwise excitation pole at the beginning of each year.

The term $f(m^2)$ implies that if creep exists the annual and Chandler nutations could introduce excitations with frequencies 2σ and $2\sigma_0$ respectively. For this or other reasons, both frequencies do in fact appear in the spectrum by Mandelbrot and McCamy (1970).

Another phenomenon consistent with creep is the lengthening of the Chandler period during greater nutational or librational stress. As pointed out by Abraham and Boots, under these conditions the creep deformation would be larger and the free period longer, (cf. the lengthening from 10 months to 14 months because of deformation). Calculated and observed values for σ_0 are compared in Figure 2.

2. Annual Excitations

Certain properties of annual excitations should be mentioned. Let the semi-major and semi-minor axes of the annual excitation ellipse be a_1 and b_1 respectively, and the

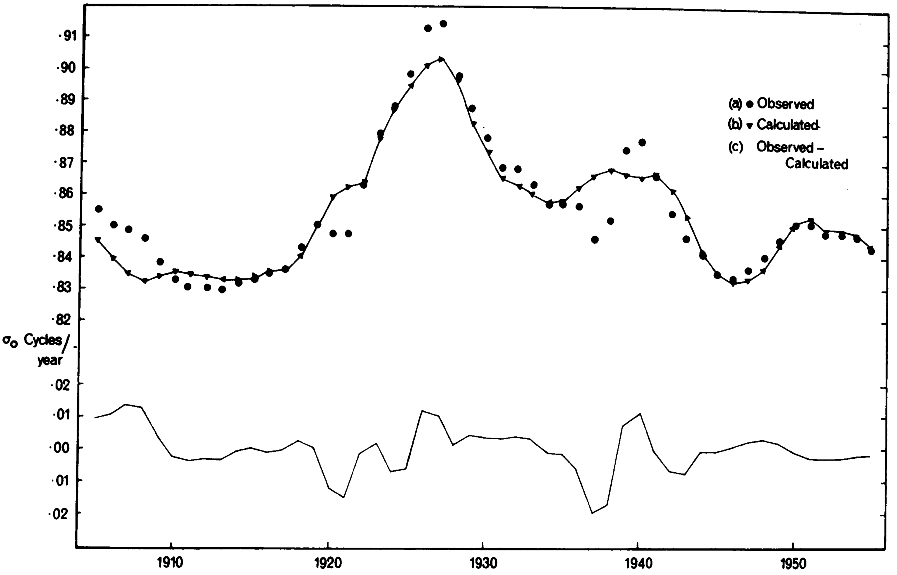

Fig. 2. Annual mean values of σ_0, (a) observed; (b) calculated; (c) observed-calculated.

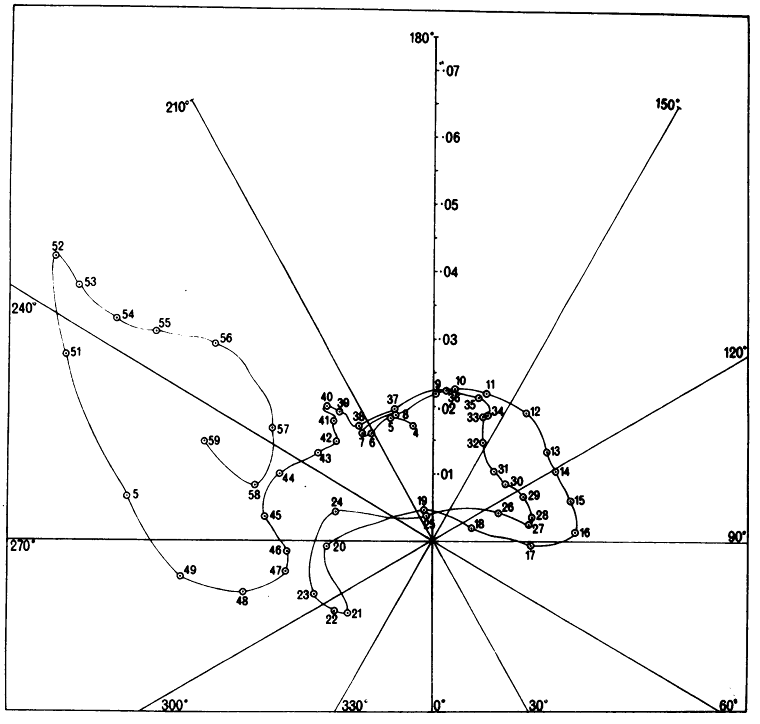

Fig. 3. Position of the clockwise excitation pole at the beginning of each year.

circular components be $\overset{+}{\psi} \exp(i\sigma t)$ and $\overset{-}{\psi} \exp(-i\sigma t)$. Values for a_1 and b_1 for 1904–59 are from data by Iijima (1965) with appropriate signs; values for later years are as calculated by J. N. Boots using data from Yumi (IPMS Reports).

Apart from the 'Chandler' component the anticlockwise excitation seems remarkably steady, for Figure 1 shows the centre of $w_0 \exp(i\sigma_0 t)$ as always near the same meridian and at about 0".018 ±0".004 from the annual mean position 0. The pole of the clockwise excitation, however, according to Figure 3, is very unstable in phase, and usually at about 0".020 to 0".025 from 0. Moreover, at about 1918–26 and 1948–56 there was a further strong excitation with amplitude about 0".04 and much the same phase on each occasion.

3. Modulated Annual Excitations

The combined 'Chandler' plus annual excitation in the meridian of a_1 is $w = w_0 \cos\sigma_0 t + a_1 \cos\sigma t$. It can be shown that w is successively equal to groups of excitations $w_{jk} = W_{jk}\, U_{jk} \cos(\sigma t - \mu_{jk} + L_{jk})$ as in Table I, in which $W' = |a_1 - w_0|$; $W'' = 2w_0 |\cos \mu t/_2|$; $W''' = |W' - W''|$; $U_1 = |\cos(\mu t/4 - A/2)|$;... (Figure 4), e.g. when $a_1 > w_0$, $w = = 2w_0 |\cos \mu t/2| \cos\{\sigma + \sigma_0) t/2 + A\} + (a_1 - w_0)\cos\sigma t$, where $A = 0$ when $-\pi < \mu t < \pi$, and $A = \pi$ when $\pi < \mu t < 3\pi$ (Figure 5). Similarly

$$w = \sum_{k=1}^{3} w_{jk}$$

where $j = \alpha, \beta, \gamma$ or δ.

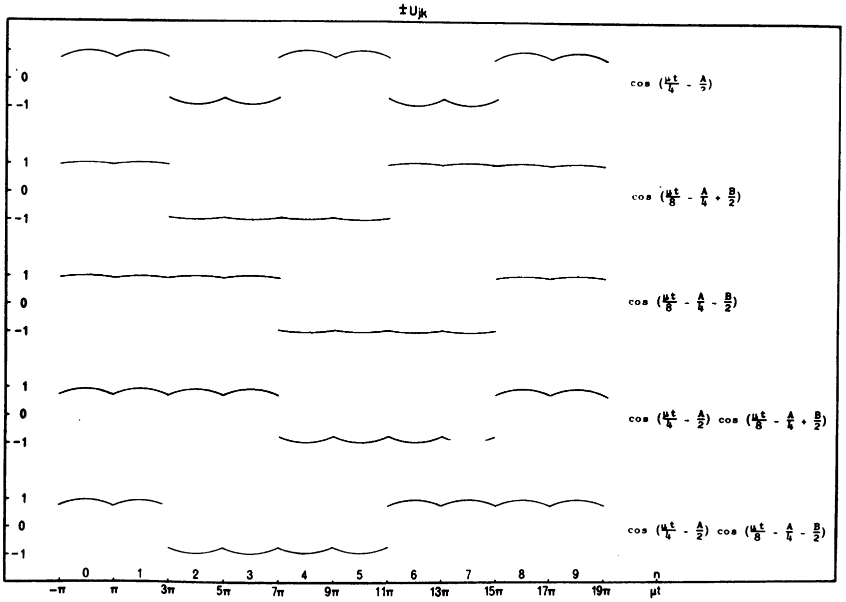

Fig. 4.

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TABLE I

			w_{j1}			w_{j2}			w_{j3}		
		j	W_{j1}	U_{j1}	L_{j1}	W_{j2}	U_{j2}	L_{j2}	W_{j3}	U_{j3}	L_{j3}
$W' < W''$	$W''' > 2W'U_1$	δ	W'''		$n\pi$	$-2W'$		$n\pi$	$4W'$	U_1U_2	$3n\pi/4$
	$W''' < 2W'U_1$	γ	$-W'''$	1	$n\pi/2$	$2W'$	U_1	$n\pi/2$	$2W'''$	U_2	
$W' > W''$	$W''' < 2W''U_1$	β				$2W''$				U_3	$n\pi/4$
	$W''' > 2W''U_1$	α	W'''		O	$-2W''$		O	$4W''$	U_1U_3	

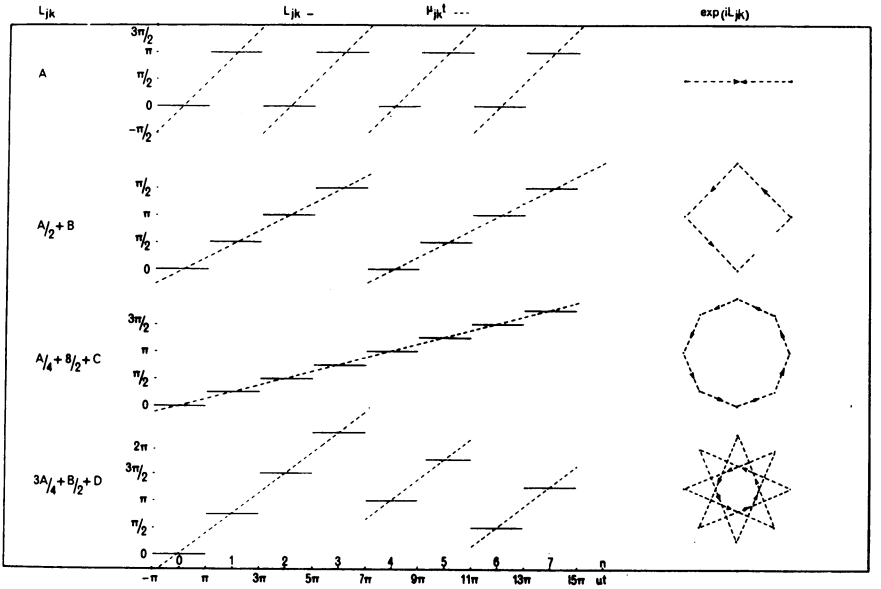

Fig. 5.

4. Librations

The phase angle L_{jk} advances after every annual-Chandler beat period ΔT; consequently the paths of nutations caused by w_{jk} keep changing their directions and mean points at intervals that vary greatly according to σ_0. The integrated path during each period is proportional to a constant $i\ W_{jk}\ U_{jk} \exp(iL_{jk})$.

Thus when $L_{jk} = n\pi$ the mean pole oscillates with period $2\Delta T$ (frequency $\mu/2$) in the meridian of b_1 (Figure 5), but is heavily smoothed. Similarly when $L_{jk} = n\pi/2$ the direction of the mean pole revolves with mean frequency $\mu/4$. After every odd period U_1 becomes zero, and then the displacement should be small and the orbit of the mean pole should be flattened, the major axis being 45° west of the meridian of a_1. Similarly frequencies $\mu/8$ and $3\mu/8$ should occur when L_{jk} is $n\pi/4$ or $3n\pi/4$. The positions of the

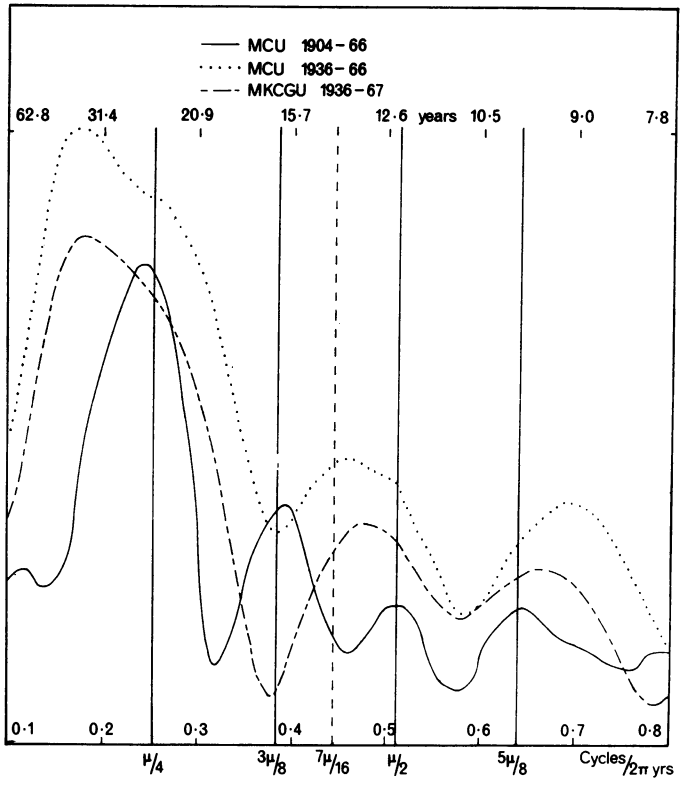

Fig. 6. Frequency response of the positions of the barycentre as given by Yumi and Wakō (1966).

barycentre given by Yumi and Wakō (1966) for 3 stations during 1904–66 have been analysed by J. N. Boots and do in fact show responses at $\mu/2$, $3\mu/8$ and $\mu/4$ (Figure 6).

5. Changes in Librations

Was this agreement accidental? The same frequency response is not always present. For example, the means of 3 stations and 5 stations agree that $\mu/2$ was more important than $3\mu/8$ during 1936–66. However, this was to be expected because the large Chandler nutations in 1936–66 should strengthen w_0 in $w_{\delta 1}+w_{\delta 2}$ (which causes $\mu/2$) at the expense of $w_{\delta 3}$ (which causes $3\mu/8$). The other period with large nutations was 1904–23 when again there was a strengthening at $\mu/2$.

In short, librations appear to stem from the annual excitation; and this aspect should be further examined.

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