GALAXY FORMATION: SOME COMPARISONS BETWEEN THEORY AND OBSERVATION

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I. INTRODUCTION

Before theoretical ideas in this subject can be compared with observational data, it is necessary to consider the properties of galaxies that are likely to be relics of their formation. Most astronomers would agree that the list of important parameters should be headed by the total mass M, energy E and angular momentum J. Next on the list should probably be the relative contributions to these quantities from the disc and bulge components of galaxies and denoted D/B for the mass ratio. They can be estimated from the median (i.e. half-mass) radius R, velocity dispersion σ and rotation velocity v of each component, either through the virial theorem or through the luminosity L and an assumed value of M/L. As a first approximation, it is reasonable to suppose that galaxies of a given disc-to-bulge ratio or morphological type form a sequence with mass as the fundamental parameter. The comparison of theory with data is further simplified by considering the extreme cases of ellipticals, with D/B << 1, and late-type spirals, with D/B >> 1. The approach outlined below is to explore the consequences of relaxing in succession the constraints that E, J and M be conserved during the collapse of proto-galaxies. In this article I concentrate on theories that are based on some form of hierarchical clustering because the pancake and related theories are not yet refined enough for a detailed confrontation with observations.

II. A HIERARCHY WITH E, J AND M CONSERVED

The starting point for each of the theories discussed here is a primordial spectrum of density perturbations with a power-law form, $\delta\rho/\rho \propto M^{-\frac{1}{2}-n/6}$, and an index in the range $-1.5 \lesssim n \lesssim 0$. Such a distribution of matter would evolve by gravitational forces alone into a nested hierarchy that might resemble the distribution of galaxies on scales from about 0.1 to 10 Mpc (Fall 1979, Peebles 1980). In the process, individual structures would be given a small amount of rotation by the tidal torques of their neighbours, which can be quantified in

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terms of the dimensionless spin parameter, $\lambda \equiv J|E|^{\frac{1}{2}} G^{-1} M^{-5/2}$ (Peebles 1969, Efstathiou & Jones 1979). The dependence of the spin and median radius on mass have been difficult to predict analytically as a result of the broad spectrum of structures in the hierarchy at a fixed density contrast. Recently Barnes & Efstathiou (private communication) have analysed a 20,000 body simulation with white noise initial conditions ($n \approx 0$) and a critical density parameter ($\Omega = 1.0$). By fitting the results to power-laws over the range $0.1M^* \lesssim M \lesssim 10 M^*$, they find

$$\lambda_H \propto M_H^{-0.03\pm0.02}, \qquad R_H \propto M_H^{0.52\pm0.02}, \qquad \text{(H for hierarchy)} \qquad (1)$$

and a median spin $\lambda_H = 0.07$ near the characteristic mass M^*. These relations may depend weakly on n and Ω but the exact dependence will not be known until more simulations are available. The important feature of eqns. (1) is that the exponent in the radius-mass relation differs significantly from the often-quoted value $(n+5)/6$.

Elliptical galaxies are usually taken to be the prototypes for non-dissipative formation because their density profiles and velocity anisotropies can be reproduced in N-body collapse models (Aarseth & Binney 1978, van Albada 1982). It is therefore of interest to compare the predictions above with the data on 44 ellipticals compiled by Davies et al. (1982). By fitting the results to power-laws over the range $0.1L^* \lesssim L \lesssim 10L^*$, they find

$$\lambda_E \propto M_E^{-0.33\pm0.09}, \qquad R_E \propto M_E^{0.50\pm0.03}. \qquad \text{(E for elliptical)} \qquad (2)$$

These relations have been derived from the formulae appropriate for self-gravitating bodies with de Vaucouleurs profiles: $\lambda_E \approx 0.4(v_m/\overline{\sigma})$, $M_E \approx 5.0(\overline{\sigma})^2 r_e/G$ and $R_E \approx 1.35 r_e$ where v_m is the maximum rotation velocity along the major axis, $\overline{\sigma}$ is the average velocity dispersion within $\frac{1}{2}r_e$ and r_e is the effective radius. A comparison of eqns. (1) and (2) shows that the visible bodies of elliptical galaxies are not typical of the structures that form in a clustering hierarchy with E, J and M conserved. An even stronger objection to this idea is that the luminosity densities of L* galaxies, both ellipticals and spirals, are 3 or 4 orders of magnitude higher than those implied by an extrapolation of the pair-correlation function for galaxies down to their median radii (Fall 1981). Thus, if ellipticals reached their present states by violent relaxation, this must have been preceeded by some dissipation. One possibility along these lines is that elliptical galaxies formed by the merging of spiral galaxies or subgalactic structures.

III. A HIERARCHY WITH E DISSIPATED AND J AND M CONSERVED

A currently popular notion is that the visible bodies of galaxies formed by the dissipative collapse of residual gas in a hierarchy of dark haloes (White & Rees 1978). Several recent investigations have shown that this provides a natural explanation for the masses, radii and spins of disc galaxies (Fall & Efstathiou 1980, Silk & Norman 1981, Burstein & Sarazin 1982, Faber 1982, Gunn 1982). As an illustration, consider a spherical halo with a constant circular velocity v_c out to some truncation radius r_t. Without having to specify the rotation curve of the dark material, its angular momentum and mass can be expressed as

$$J_H = \sqrt{2}\, \lambda_H\, v_c^3\, r_t^2/G, \qquad M_H = v_c^2\, r_t/G, \qquad \text{(H for halo)} \tag{3}$$

when terms of order λ_H^2 are neglected in the energy $E_H = -v_c^4\, r_t/2G$. Now suppose the gas that collapses in such a halo arranges itself into an exponential disc with a scale-radius α^{-1} determined by the circular velocity v_c and the specific angular momentum $J_D/M_D = 2v_c/\alpha$. Since the disc material would have experienced the same tidal torques as the halo material before dissipating any energy it seems reasonable to set $J_D/M_D = J_H/M_H$; this implies

$$R_H/R_D = 0.30(\alpha r_t) = 0.42/\lambda_H \approx 6. \qquad \text{(D for disc)} \tag{4}$$

The first equality follows from the expressions $R_H = \frac{1}{2}r_t$ and $R_D = 1.67\,\alpha^{-1}$ for the median radii of the halo and disc and the last equality is appropriate for $\lambda_H \approx 0.07$. These results agree nicely with the more exact calculations of Fall & Efstathiou (1980), which include the effects of a bulge and the gravity of the disc.

The picture outlined above has several interesting consequences. For a bright spiral such as the Milky Way, with $\alpha^{-1} \approx 4$ kpc, the extent of the surrounding halo is predicted to be $r_t \approx 80$ kpc. This is comparable with the radius of the proto-galaxy deduced by Eggen, Lynden-Bell & Sandage (1962) from the motions of old stars in the solar neighbourhood. When eqns. (3) and (4) are combined with eqns. (1) from the Barnes-Efstathiou simulations, the results are

$$\mu_o \propto (M_D/M_H)^{0.98} M_D^{\,0.02}, \qquad v_c \propto (M_D/M_H)^{-0.24} M_D^{\,0.24}, \tag{5}$$

where $\mu_o = \alpha^2 M_D/2\pi$ is the central surface density of an exponential disc. For relatively isolated disc-halo systems, it seems natural to expect $M_D/M_H \approx$ const because this should be roughly the ratio of mass in primordial gas to that in dark matter. In this case, eqns. (5) take forms that are reminiscent of Freeman's (1970) law and the Tully-Fisher (1977) relation. The coefficients of proportionality in these expressions are determined by eqns. (1) and therefore by the scaling of the N-body simulations to astronomical units. A simple way to do this is by means of the empirical relation $v_c^2 \approx \alpha M_D G$, which implies $M_H/M_D \approx 1.4/\lambda_H$ when combined with eqns. (3) and (4). For $\lambda_H \approx 0.07$, the result is $M_H/M_D \approx 20$ and this agrees with current estimates of the average ratio of dark to luminous material, $\Omega(\text{dark})/\Omega(\text{lum}) \sim 10$ (Faber 1982). Thus, a hierarchy with E dissipated but J and M conserved appears to account for the most important properties of disc galaxies.

If elliptical and spiral galaxies formed in the same kind of haloes,

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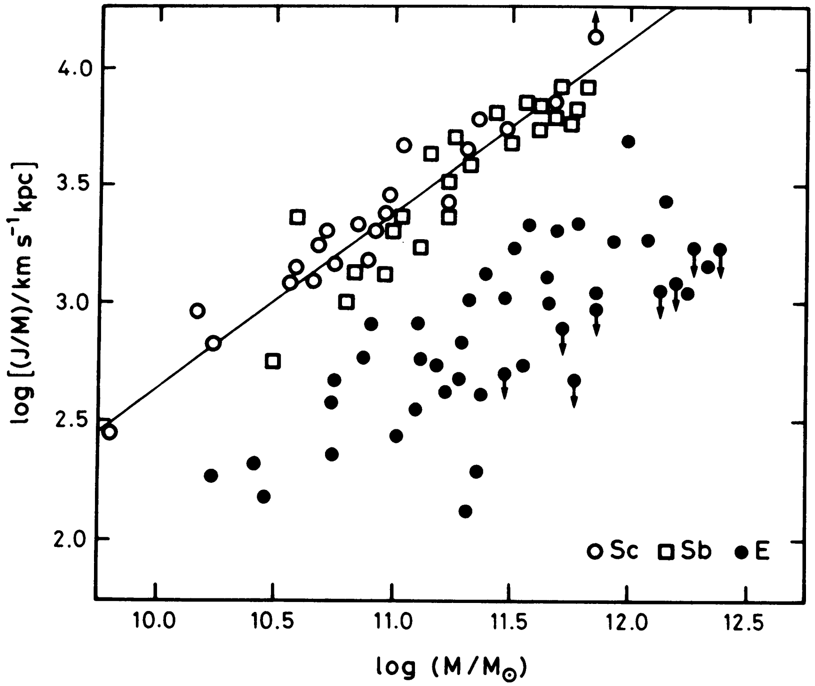

Figure 1. Specific angular momentum against luminous mass for galaxies of different morphological types. Spirals: $J/M = 2v_c\alpha^{-1}$ with $v_c = V(R_{25}^{ib})$ and $\alpha^{-1} = 0.32\ R_{25}^{ib}$; $M = (M/L)L(B_T^{ib})$ with $M/L = 3.0$ for Sb and $M/L = 1.5$ for Sc; data from Rubin et al. (1980, 1982). Ellipticals: $J/M = 2.5\ v_m\ r_e$ as appropriate for a de Vaucouleurs profile, a flat rotation curve and random orientations; $M = (M/L)L(B_T^0)$ with $M/L = 6.0$; data from Davies et al. (1982). A Hubble constant of 50 km s^{-1} Mpc^{-1} is adopted but the relative positions of points do not depend on this distance-scale.

they would occupy similar positions in the J/M – M plane, irrespective of how much energy they dissipated. As Fig. 1 shows, however, the visible bodies of spirals have about 6 times as much angular momentum as ellipticals of the same mass. The values of J/M in this diagram were computed from rotation velocities and photometric radii and the values of M were computed from total luminosities and M/L ratios for the mean B–V colour of each type in the population synthesis models of Tinsley (1981). This procedure avoids the awkward question of whether the luminous parts of galaxies are self-gravitating and the spurious correlations that may arise when v_c is used to compute both J/M and M for disc galaxies (Freeman 1970). It is not yet possible to plot galaxies with D/B $\approx$ 1 on the diagram because of the lack of coordinated photometric and kinematic data needed to estimate the contributions of each component to J/M and M. Nevertheless, the few data that are available indicate that intermediate types fill the gap between Sb/c and E galaxies and that J/M increases with D/B at each value of M. This is a refinement of the suggestion by Sandage, Freeman and Stokes (1970) that the relative prominence of the disc and bulge components of galaxies should reflect the proportions of material with high and low angular momentum. The immportant point about Fig. 1 in the present context is that it may contain useful information on the haloes of galaxies.

IV. A HIERARCHY WITH E DISSIPATED AND J AND M STRIPPED

The simplest explanation for the different angular momenta of elliptical and spiral galaxies would be in terms of a spread in the tidal spins of their haloes (Kashlinski 1982). In this case, low λ haloes must be produced preferentially in regions of high density and high λ haloes in regions of low density to account for the strong correlation between the clustering environments of galaxies and their morphological types (Dressler 1980). Such a coupling of long and short wavelength modes seems unlikely in the linear phase of growth when tidal torques are induced. Instead, some sort of non-linear process is probably needed if the initial spectrum of density perturbations had random phases. One suggestion along these lines is that the outer envelopes of haloes in clusters were stripped off by tidal interactions with their neighbours whereas isolated haloes were not disturbed in this way. The morphologies of galaxies might then be a reflection of the degree to which the gas of high specific angular momentum was dispersed before it could collapse (Binney & Silk 1978, Larson, Tinsley & Caldwell 1980). This scheme requires that the stripping and collapse time-scales be comparable in the outer parts of proto-galaxies and that the mass of the diffuse gas and the visible bodies of galaxies be comparable in rich clusters. To see how the second condition has a bearing on the properties of galactic haloes, it is instructive to consider the following simple model.

The spatial and projected density profiles of the 'isothermal' halo discussed in Section III are

$$\rho_H(s) = (M_H/4\pi r_t^3)(r_t/s)^2, \quad \mu_H(r) = (M_H/2\pi r_t^2)(r_t/r)\cos^{-1}(r/r_t), \quad (6)$$

where s denotes distance from the centre and r denotes distance from the rotation axis. Prior to stripping, the fraction of mass with specific angular momentum in the interval (h, h+dh) is assumed to be $f_H(h)dh$ with

$$f_H(h) = (\beta/h_t)(h/h_t)^{\beta-1}\cos^{-1}(h/h_t)^\beta \qquad \text{for } h \leq h_t. \quad (7)$$

Here β is an adjustable parameter and h_t is related to r_t and λ_H by eqns. (3) and the normalization

$$J_H/M_H = \int_0^{h_t} dh\, f_H(h)h = \tfrac{1}{2}\sqrt{\pi}\beta h_t(1+\beta)^{-2}\Gamma(\tfrac{1}{2\beta})/\Gamma(\tfrac{1}{2} + \tfrac{1}{2\beta}), \quad (8)$$

where Γ denotes the usual gamma function. If the halo rotates on

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cylinders, its velocity is determined by the condition

$$M_H f_H(h)dh = 2\pi r \mu_H(r)dr \qquad \text{with} \quad h = r\, v_H(r), \tag{9}$$

hence
$$v_H(r) = (h_t/r_t)(r/r_t)^{1/\beta - 1}. \tag{10}$$

The surface density of a disc with the same distribution of specific angular momentum is determined by the condition

$$M_D f_H(h)dh = 2\pi r \mu_D(r)dr \qquad \text{with} \quad h = r\, v_c \,, \tag{11}$$

hence
$$\mu_D(r) = (\beta M_D/2\pi r_o^2)(r/r_o)^{\beta-2}\cos^{-1}(r/r_o)^{\beta} \quad , \tag{12}$$

where $r_o = h_t/v_c$ is the corresponding truncation radius. It is straightforward to show that the angular momentum of the halo within the radius s is

$$J(<s) = 4\pi\int_o^s dr v_H(r) r^2 \int_r^s ds' \rho_H(s') s'(s'^2 - r^2)^{-\frac{1}{2}} = J_H(s/r_t)^{1/\beta+1} \tag{13}$$

If the halo is stripped on spherical shells, the angular momentum and mass of the remaining material are related by the simple power-law

$$J(<s)/J_H = [M(<s)/M_H]^{1/\beta+1} \quad \text{or} \quad M \propto (J/M)^{\beta} \tag{14}$$

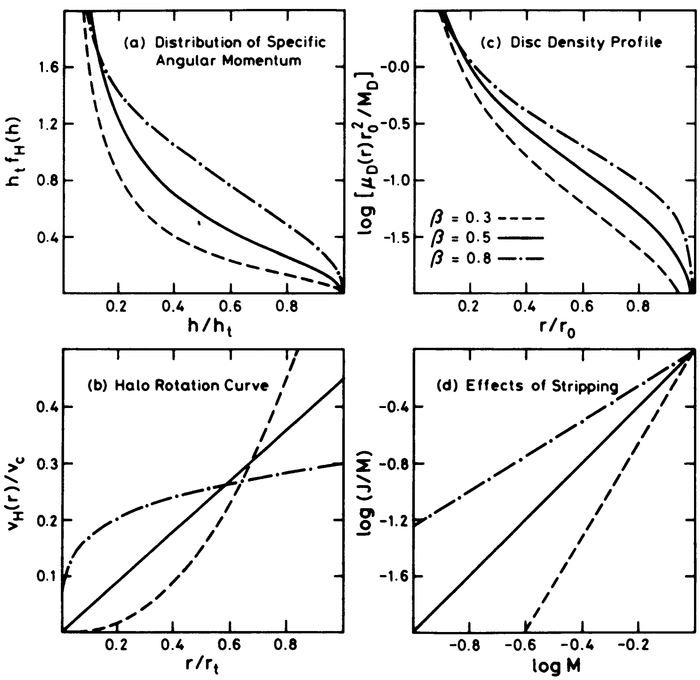

Figure 2. Illustrative model of a disc-halo system with three values of the parameter β. (a) from eqn. (7); (b) from eqns. (3), (8) and (10) with $\lambda_H = 0.07$; (c) from eqn. (12); (d) from eqn. (14).

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Several properties of this model are shown in Fig. 2 for $\beta = 0.3$, 0.5 and 0.8. In each case, a disc that is roughly exponential over 3.5 scale-radii could form in the halo if no stripping occurred and each element of gas conserved its angular momentum during the collapse. Smaller values of β imply steeper rotation curves in the halo and therefore more angular momentum lost when a given fraction of mass is stripped. The significance of this effect can be seen by comparing Fig. 2(d) with Fig. 1. To convert the haloes of spirals into the haloes of ellipticals requires that their masses be reduced by factors of about $(6\pm2)^{\beta}$. In the core of the Coma cluster, the ratio of the mass in X-ray emitting gas to the mass in optically visible material probably lies in the range 1 to 4 depending on the value of the Hubble constant and other uncertainties (Faber 1982). This implies $0.4 \lesssim \beta \lesssim 0.9$, which should be compared with the rotation curves induced by tidal torques. In the absence of any detailed calculations, a reasonable model for a proto-halo during the linear phase of growth might be a uniform density sphere with solid body rotation (Gunn 1982). This corresponds to an 'isothermal' halo with a nearly flat rotation curve and therefore to $\beta \simeq 1$ if $f_H(h)$ is conserved in the approach to dynamical equilibrium. The slow rotation of ellipticals then requires the stripping of so much material that it becomes hard to understand why they are more massive than giant spirals.

V. CONCLUSIONS

The preceeding arguments are summarized in the table below.

Hierarchy	Spirals	Ellipticals
E conserved J conserved M conserved	Radii too small. Spins too high. Etc.	Radii too small. Spin-mass relation too steep.
E dissipated J conserved M conserved	Radii, masses and spins about right	Location in clusters hard to explain.
E dissipated J stripped M stripped	As above in small groups and the field.	Small spins imply large mass-loss.

The strength of the picture outlined here is its success with spirals and the weakness is its difficulty with ellipticals. This is not, however, a firm conclusion because some combination of effects might explain the differences between early and late types and their correlation with environment. For example, the factor of 6 spread in the angular momenta of galaxies at a fixed mass might reflect both the distribution of tidal spins in the linear regime and some tidal stripping in the non-linear regime. Finally, it must be emphasized that all of these remarks pertain to the general framework of hierarchical clustering with scale-free initial conditions. A test of this hypothesis is not likely to come from studies of the internal properties of galaxies because the theoretical interpretation is usually ambiguous. For more direct tests, we will probably have to rely on studies of the large-scale distribution of galaxies and fluctuations in the cosmic background radiation.

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DISCUSSION

DEKEL : With regard to the idea of spin-loss : there are indications that the rotation curves of ellipticals are flat. If so, the specific angular momentum indeed rises with radius. If the gas was contracted from a r^{-2} distribution, the original halo rotation curve must have been rising exponentially and to obtain a factor of 3 decrease in total spin, 30 % mass loss may have to occur. Tidal encounters in clusters might easily account for this. The spin-loss can be naturally combined with a scenario in which all galaxy types are formed by gas contraction in dark halos. The structure of the halo can determine the type : halos that are puffed-up by slow tidal encounters in protoclusters allow star formation at larger radii, i.e. more bulge.

FALL : As you say, the main constraint on this scheme is that the rotation curves of the haloes must rise steeply to avoid the removal of too much mass as the angular momentum of proto-galaxies is reduced by stripping. The density profiles and rotation curves of the visible bodies of galaxies might be used to infer the corresponding properties of the haloes if one has reason to suppose that each element of mass conserved its angular momentum during the collapse. But why would you use ellipticals for this purpose when the hypothesis being tested is that their masses and angular momenta were not conserved ? When these arguments are applied to relatively isolated spirals on the grounds that their haloes are more likely to have remained intact, the inferred rotation curves are fairly flat. In any case, the real question is whether tidal torques would endow the structures that form in a clustering hierarchy with this distribution of specific angular momentum. To my knowledge, the necessary calculations have not yet been made but simple arguments suggest that exponentially rising rotation curves are too steep, especially after virial equilibrium is reached. This is one of the reasons I am sceptical about halo stripping as the sole explanation for all of the morphological and environmental differences between galaxies.