6

D-brane tension and boundary states

We have already stated that since the D-brane is a dynamical object, and couples to gravity, it should have a mass per unit volume. This tension will govern the strength of its response to outside influences which try to make it change its shape, absorb energy, etc. We have already computed a recursion relation (5.11) for the tension, whcih follows from the underlying T-duality which we used to discovere D-branes in the first place.

In this chapter we shall see in detail just how to compute the value of the tension for the D-brane, and also for the orientifold plane. While the numbers that we will get will not (at face value) be as useful as the analogous quantities for the supersymmetric case, the structure of the computation is extremely important. The computation puts together many of the things that we have learned so far in a very elegant manner which lies at the heart of much of what will follow in more advanced chapters.

Along the way, we will see that D-branes can be constructed and studied in an alternative formalism known as the 'boundary state' formalism, which is essentially conformal field theory with certain sorts of boundaries included[33]. For much of what we will do, it will be a clearly equivalent way of formulating things which we also say (or have already said) based on the spacetime picture of D-branes. However, it should be noted that it is much more than just a rephrasing since it can be used to consistently formulate D-branes in many more complicated situations, even when a clear spacetime picture is not available. The method becomes even more useful in the supersymmetric situation, since it provides a natural way of constructing stable D-brane vacua of the superstring theories which do not preserve any supersymmetries, a useful starting point for exploring dualities and other non-perturbative physics in dynamical regimes which ultimately may have relevance to observable physics.

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6.1 The D-brane tension

6.1.1 An open string partition function

Let us now compute the D-brane tension T_p. As noted previously, it is proportional to g_s^{-1}. We can in principle calculate it from the gravitational coupling to the D-brane, given by the disk with a graviton vertex operator in the interior. However, it is much easier to obtain the absolute normalisation in the following manner.

Consider two parallel Dp-branes at positions $X'^{\mu} = 0$ and $X'^{\mu} = Y^{\mu}$. These two objects can feel each other's presence by exchanging closed strings as shown in figure 6.1. This string graph is an annulus, with no vertex operators. It is therefore as easily calculated as our closed string one loop amplitudes done earlier in chapter 3.

In fact, this is rather like an open string partition function, since the amplitude can be thought of as an open string going in a loop. We should sum over everything that goes around in the loop. Once we have computed this, we will then change our picture of it as an open string one-loop amplitude, and look at it as a closed string amplitude for propagation between one D-brane and another. We can take a low energy limit of the result to focus on the massless closed string states which are being exchanged. Extracting the poles from graviton and dilaton exchange (we shall see that the antisymmetric tensor does not couple in this limit) then give the coupling T_p of closed string states to the D-brane.

Let us parametrise the string world-sheet as $(\sigma^2 = \tau, \sigma^1 = \sigma)$ where now τ is periodic and runs from 0 to $2\pi t$, and σ runs (as usual) from 0 to π. This vacuum graph (a cylinder) has the single modulus t, running

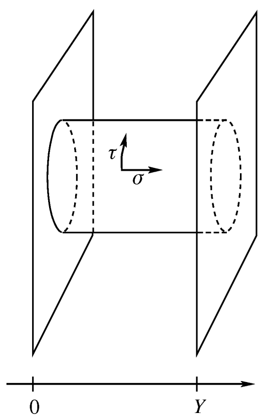

Fig. 6.1. Exchange of a closed string between two D-branes. This is equivalent to a vacuum loop of an open string with one end on each D-brane.

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from 0 to ∞. If we slice horizontally, so that $\sigma^2 = \tau$ is world-sheet time, we get an open string going in a loop. If we instead slice vertically, so that σ is time, we see a single closed string propagating in the tree channel.

Notice that the world-line of the open string boundary can be regarded as a vertex connecting the vacuum to the single closed string, i.e. a one-point closed string vertex, which is a useful picture in a 'boundary state' formalism, which we will develop a bit further shortly. This diagram will occur explicitly again in many places in our treatment of this subject. String theory produces many examples where one-loop gauge/field theory results (open strings) are related to tree level geometrical/gravity results. This is all organised by diagrams of this form, and is the basis of much of the gauge theory/geometry correspondences to be discussed.

Let us consider the limit $t \to 0$ of the loop amplitude. This is the ultra-violet limit for the open string channel, since the circle of the loop is small. However, this limit is correctly interpreted as an *infrared* limit of the closed string. (This is one of the earliest 'dualities' of string theory, discussed even before it was known to be a theory of strings.) Time-slicing vertically shows that the $t \to 0$ limit is dominated by the lowest lying modes in the closed string spectrum. This all fits with the idea that there are no 'ultra-violet limits' of the moduli space which could give rise to high energy divergences. They can always be related to amplitudes which have a handle pinching off. This physics is controlled by the lightest states, or the long distance physics. (This relationship is responsible for the various 'UV/IR' relations which are a popular feature of current research[315].)

One-loop vacuum amplitudes are given by the Coleman–Weinberg [35, 36] formula, which can be thought of as the sum of the zero point energies of all the modes (see insert 6.1):

$$\mathcal{A} = V_{p+1} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \int_0^\infty \frac{dt}{2t} \sum_I e^{-2\pi\alpha' t(k^2+M_I^2)}. \tag{6.1}$$

Here the sum I is over the physical spectrum of the string, i.e. the transverse spectrum, and the momentum k is in the $p+1$ extended directions of the D-brane world-sheet.

The mass spectrum is given by a familiar formula

$$M^2 = \frac{1}{\alpha'}\left(\sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i - 1\right) + \frac{Y \cdot Y}{4\pi^2\alpha'^2}, \tag{6.2}$$

where Y^m is the separation of the D-branes. The sums over the oscillator modes work just like the computations we did before (see insert 3.4 (p. 92)), giving

$$\mathcal{A} = 2V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi^2\alpha' t)^{-\frac{(p+1)}{2}} e^{-Y\cdot Y t/2\pi\alpha'} f_1(q)^{-24}. \tag{6.3}$$

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Insert 6.1. Vacuum energy

The Coleman–Weinberg[35, 36] formula evaluates the one-loop vacuum amplitude, which is simply the logarithm of the partition function $\mathcal{A} = Z_{\text{vac}}$ for the complete theory:

$$\ln\left(Z_{\text{vac}}\right) = -\frac{1}{2}\text{Tr}\ln\left(\Box^2 + M^2\right) = -\frac{V_D}{2}\int\frac{d^D k}{(2\pi)^D}\ln\left(k^2 + M^2\right).$$

But since we can write

$$-\frac{1}{2}\ln(k^2 + M^2) = \int_0^\infty \frac{dt}{2t}e^{-(k^2+M^2)t/2},$$

we have

$$\mathcal{A} = V_D\int\frac{d^D k}{(2\pi)^D}\int_0^\infty\frac{dt}{2t}e^{-(k^2+M^2)t/2}.$$

Recall finally that $(k^2 + M^2)/2$ is just the Hamiltonian, H, which in our case is just L_0/α' (see equation (2.64)).

Here $q = e^{-2\pi t}$, and the overall factor of two is from exchanging the two ends of the string. (See insert 6.2 for news of $f_1(q)$.)

In the present case (using the asymptotics derived in insert 6.2),

$$\mathcal{A} = 2V_{p+1}\int_0^\infty\frac{dt}{2t}(8\pi^2\alpha' t)^{-\frac{(p+1)}{2}}e^{-Y\cdot Y t/2\pi\alpha'}t^{12}\left(e^{2\pi/t} + 24 + \cdots\right). \quad (6.4)$$

The leading divergence is from the tachyon and is the usual bosonic string artifact not relevant to this discussion. The massless pole, from the second term, is

$$\begin{aligned}\mathcal{A}_{\text{massless}} &\sim V_{p+1}\frac{24}{2^{12}}(4\pi^2\alpha')^{11-p}\pi^{(p-23)/2}\Gamma((23-p)/2)|Y|^{p-23}\\ &= V_{p+1}\frac{24\pi}{2^{10}}(4\pi^2\alpha')^{11-p}G_{25-p}(Y),\end{aligned} \quad (6.5)$$

where $G_d(Y)$ is the massless scalar Green's function in d dimensions:

$$G_d(Y) = \frac{\pi^{d/2}}{4}\Gamma\left(\frac{d}{2} - 1\right)\frac{1}{Y^{d-2}}. \quad (6.6)$$

Here, $d = 25 - p$, the dimension of the space transverse to the brane.

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Insert 6.2. Translating closed to open

Compare our open string appearance of $f_1(q)$, for $q = e^{-2\pi t}$ with the expressions for $f_1(q)$, $(q = e^{2\pi\tau})$ defined in our closed string discussion in (4.44). Here the argument is real. The translation between definitions is done by setting $t = -\mathrm{Im}\,\tau$. From the modular transformations (4.46), we can deduce some useful asymptotics. While the asymptotics as $t \to \infty$ are obvious, we can get the $t \to 0$ asymptotics using (4.46):

$$
\begin{aligned}
f_1(e^{-2\pi/s}) &= \sqrt{s}\, f_1(e^{-2\pi s}), \;\; f_3(e^{-2\pi/s}) = f_3(e^{-2\pi s}), \\
f_2(e^{-2\pi/s}) &= f_4(e^{-2\pi s}).
\end{aligned}
$$

6.1.2 A background field computation

We must do a a field theory calculation to work out the amplitude for the exchange of the graviton and dilaton between a pair of D-branes. Our result can the be compared to the low energy string result above to extract the value of the tension. We need propagators and couplings as per the usual field theory computation. The propagator is from the bulk action (2.106) and the couplings are from the D-brane action (5.21), but we must massage them a bit in order to find them.

In fact, we should work in the Einstein frame, since that is the appropriate frame in which to discuss mass and energy, because the dilaton and graviton don't mix there. We do this (recall equation (2.109)) by sending the metric $G_{\mu\nu}$ to $\tilde{G}_{\mu\nu} = \exp(4(\Phi_0 - \Phi)/(D-2))G_{\mu\nu}$, which gives the metric in equation (2.111). Let us also do this in the Dirac–Born–Infeld action (5.21), with the result:

$$
S_p^E = -\tau_p \int d^{p+1}\xi\, e^{-\tilde{\Phi}} \det{}^{1/2}(e^{\frac{4\tilde{\Phi}}{D-2}}\tilde{G}_{ab} + B_{ab} + 2\pi\alpha' F_{ab}), \tag{6.7}
$$

where $\tilde{\Phi} = \Phi - \Phi_0$ and $\tau_p = T_p e^{-\Phi_0}$ is the physical tension of the brane; it is set by the background value, Φ_0, of the dilaton.

The next step is to linearise about a flat background, in order to extract the propagator and the vertices for our field theory. In fact, we have already discussed some of the logic of this in the introductory chapter, in section (1.2), where we came to grips with the idea of a graviton, so the reader is presumably aware that this is not really a daunting procedure.

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We simply write the metric as $G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(X)$, and this time expand up to second order in $h_{\mu\nu}$. Also, if we do this with the action (6.7) as well, we see that the antisymmetric fields $B_{ab} + 2\pi\alpha' F_{ab}$ do not contribute at this order, and so we will drop them in what follows*.

Another thing which we did in section (1.2) was to fix the gauge degree of freedom (1.21) so that we would write the linearised (first order) Einstein equations in a nice gauge (1.22). We shall pick the same gauge here:

$$F_\mu \equiv \eta^{\rho\sigma}(\partial_\rho h_{\sigma\mu} - \frac{1}{2}\partial_\mu h_{\rho\sigma}) = 0, \tag{6.8}$$

and introduce the gauge choice into the Lagrangian via the addition of a gauge fixing term:

$$\mathcal{L}_{\rm fix} = -\frac{\eta^{\mu\nu}}{4\kappa^2}F_\mu F_\nu. \tag{6.9}$$

The result for the bulk action is:

$$\begin{aligned} S_{\rm bulk} = -\frac{1}{2\kappa^2}\int d^DX\Bigg\{&\frac{1}{2}\left[\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{D-2}\eta^{\mu\nu}\eta^{\rho\sigma}\right]h_{\mu\nu}\partial^2 h_{\rho\sigma} \\ &+\frac{4}{D-2}\tilde\Phi\partial^2\tilde\Phi\Bigg\}, \end{aligned} \tag{6.10}$$

and the interaction terms from the Dirac–Born–Infeld action are:

$$S_{\rm brane} = -\tau_p\int d^{p+1}\xi\left(\left(\frac{2p-D+4}{D-2}\right)\tilde\Phi - \frac{1}{2}h_{aa}\right), \tag{6.11}$$

where the trace on the metric was in the $(p+1)$-dimensional world-volume of the Dp-brane.

Now it is easy to work out the momentum space propagators for the graviton and the dilaton:

$$\begin{aligned} \langle h_{\mu\nu}h_{\rho\sigma}\rangle &= -\frac{2i\kappa^2}{k^2}\left[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}\right]; \\ \langle\tilde\Phi\tilde\Phi\rangle &= -\frac{i\kappa^2(D-2)}{4k^2}, \end{aligned} \tag{6.12}$$

for momentum k. The reader might recognise the graviton propagator as the generalisation of the four dimensional case. If the reader has not encountered it before, the resulting form should be thought of as entirely consistent with gauge invariance for a massless spin two particle.

* This fits with the intuition that the D-brane should not be a source for the antisymmetric tensor field. The source for it is the fundamental closed string itself. We shall come back to this point many times much later.

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All we need to do is compute two tree level Feynman diagrams, one for exchange of the dilaton and one for the exchange of the graviton, and add the result. The vertices are given in action (6.11). The result is (returning to position space)

$$\begin{aligned}\mathcal{A}_{\text{massless}} &= V_{p+1}T_p^2\kappa_0^2 G_{25-p}(Y)\left\{\frac{D-2}{4}\left(\frac{2p-D+4}{D-2}\right)^2\right.\\ &\qquad\qquad\left.+\frac{1}{2}\left[2(p+1)-\frac{2}{D-2}(p+1)^2\right]\right\}\\ &= \frac{D-2}{4}V_{p+1}T_p^2\kappa_0^2 G_{25-p}(Y),\end{aligned} \tag{6.13}$$

and so after comparing to our result from the string theory computation (6.5) we have

$$T_p = \frac{\sqrt{\pi}}{16\kappa_0}(4\pi^2\alpha')^{(11-p)/2}. \tag{6.14}$$

This agrees rather nicely with the recursion relation (5.11). We can also write it in terms of the physical value of the D-brane tension, which includes a factor of the string coupling $g_{\text{s}} = e^{\Phi_0}$,

$$\tau_p = \frac{\sqrt{\pi}}{16\kappa}(4\pi^2\alpha')^{(11-p)/2} \tag{6.15}$$

where $\kappa = \kappa_0 g_{\text{s}}$, and we shall use τ_p this to denote the tension when we include the string coupling henceforth, and reserve T for situations where the string coupling is included in the background field $e^{-\Phi}$. (This will be less confusing than it sounds, since it will always be clear from the context which we mean.)

As promised, the tension τ_p of a Dp-brane is of order g_{s}^{-1}, following from the fact that the diagram connecting the brane to the closed string sector is a disc diagram, and insert 2.4 (p. 57) shows reminds us that this is of order g_{s}^{-1}. An immediate consequence of this is that they will produce non-perturbative effects of order $\exp(-1/g_{\text{s}})$ in string theory, since their action is of the same order as their mass. This is consistent with anticipated behaviour from earlier studies of toy non-perturbative string theories[100].

Formula (6.14) will not concern us much beyond these sections, since we will derive a new one for the superstring case later.

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6.2 The orientifold tension

The O-plane, like the D-brane, couples to the dilaton and metric. The most direct amplitude to use to compute the tension is the same as in the previous section, but with $\mathbb{RP}^2$ in place of the disc; i.e. a crosscap replaces the boundary loop. The orientifold identifies X^m with $-X^m$ at the opposite point on the crosscap, so the crosscap is localised near one of the orientifold fixed planes. However, once again, it is easier to organise the computation in terms of a one-loop diagram, and then extract the parts we need.

6.2.1 Another open string partition function

To calculate this via vacuum graphs, the cylinder has one or both of its boundary loops replaced by crosscaps. This gives the Möbius strip and Klein bottle, respectively. To understand this, consider figure 6.2, which shows two copies of the fundamental region for the Möbius strip. The lower half is identified with the reflection of the upper, and the edges $\sigma^1 = 0, \pi$ are boundaries. Taking the lower half as the fundamental region gives the familiar representation of the Möbius strip as a strip of length $2\pi t$, with ends twisted and glued. Taking instead the left half of the figure, the line $\sigma^1 = 0$ is a boundary loop while the line $\sigma^1 = \pi/2$ is identified with itself under a shift $\sigma^2 \to \sigma^2 + 2\pi t$ plus reflection of σ^1: it is a crosscap. The same construction applies to the Klein bottle, with the right and left edges now identified. Another way to think of the Möbius strip amplitude we are going to compute here is as representing the exchange of a closed string between a D-brane and its mirror image, as shown in figure 6.3. The identification with a twist is performed on the two D-branes, turning the cylinder into a Möbius strip. The Möbius strip is given by the vacuum

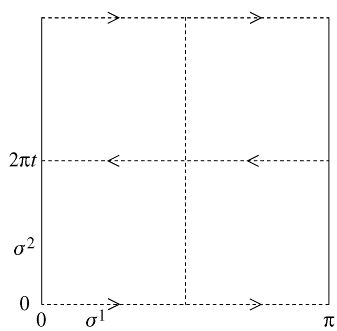

Fig. 6.2. Two copies of the fundamental region for the Möbius strip.

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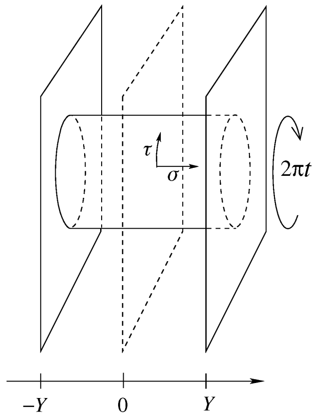

Fig. 6.3. The Möbius strip as the exchange of closed strings between a brane and its mirror image. The dotted plane is the orientifold plane.

amplitude

$$\mathcal{A}_{\mathrm{M}} = V_{p+1} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \int_0^\infty \frac{dt}{2t} \sum_i \frac{\Omega_i}{2} e^{-2\pi\alpha' t(p^2+M_I^2)}, \tag{6.16}$$

where Ω_I is the Ω eigenvalue of state i. The oscillator contribution to Ω_I is $(-1)^n$ from equation (2.94). Actually, in the directions orthogonal to the brane and orientifold there are two additional signs in Ω_I which cancel. One is from the fact that world-sheet parity contributes an extra minus sign in the directions with Dirichlet boundary conditions (this is evident from the mode expansions we shall list later, in equations (11.1)). The other is from the fact that spacetime reflection produces an additional sign.

For the $SO(N)$ open string the Chan–Paton factors have $\frac{1}{2}N(N+1)$ even states and $\frac{1}{2}N(N-1)$ odd for a total of $+N$. For $USp(N)$ these numbers are reversed for a total of $-N$. Focus on a D-brane and its image, which correspondingly contribute ± 2. The diagonal elements, which contribute to the trace, are those where one end is on the D-brane and one on its image. The total separation is then $Y^m = 2X^m$. Then,

$$\begin{aligned}\mathcal{A}_{\mathrm{M}} &= \pm V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi^2\alpha' t)^{-\frac{(p+1)}{2}} e^{-2\vec{Y}\cdot\vec{Y}t/\pi\alpha'} \\ &\quad \times \left[q^{-2} \prod_{k=1}^{\infty} (1+q^{4k-2})^{-24}(1-q^{4k})^{-24} \right].\end{aligned}$$

The factor in braces is

$$\begin{aligned} f_3(q^2)^{-24} f_1(q^2)^{-24} &= (2t)^{12} f_3(e^{-\pi/2t})^{-24} f_1(e^{-\pi/2t})^{-24} \\ &= (2t)^{12} \left(e^{\pi/2t} - 24 + \cdots \right). \end{aligned} \tag{6.17}$$

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One therefore finds a pole

$$\mp 2^{p-12} V_{p+1} \frac{3\pi}{2^6} (4\pi^2\alpha')^{11-p} G_{25-p}(Y). \tag{6.18}$$

This is to be compared with the field theory result

$$\frac{D-2}{2} V_{p+1} T_p T'_p \kappa_0^2 G_{25-p}(Y), \tag{6.19}$$

where T'_p is the O-plane tension. A factor of two as compared to the earlier field theory calculation (6.13) comes because the spacetime boundary forces all the flux in one direction. Therefore the O-plane and D-brane tensions are related by

$$\tau'_p = \mp 2^{p-13} \tau_p. \tag{6.20}$$

A similar calculation with the Klein bottle gives a result proportional to τ'^2_p.

Noting that there are 2^{25-p} O-planes (recall that one doubles the number every time another new direction is T-dualised, starting with e single D25-brane), the total charge of an O-plane source must be $\mp 2^{12}\tau_p$. Now, by Gauss's law, the total source must vanish because the volume of the torus T^p on which we are working is finite and of course the flux must end on sinks and sources.

So we conclude that there are $2^{(D-2)/2} = 2^{12}$ D-branes (times two for the images) and that the gauge group[37] is $SO(2^{13}) = SO(2^{D/2})$. For this group the 'tadpoles' associated with the dilaton and graviton, representing violations of the field equations, cancel at order g_s^{-1}. This has no special significance in the bosonic string due to the tachyon instability, but similar considerations will give a restriction on allowed Chan–Paton gauge groups in the superstring.

6.3 The boundary state formalism

The asymptotics (6.4) can be interpreted in terms of a sum over closed string states exchanged between the two D-branes. One can write the cylinder path integral in a Hilbert space formalism treating σ_1 rather than σ_2 as time. It then has the form

$$\langle B| e^{-(L_0+\tilde{L}_0)\pi/t} |B\rangle \tag{6.21}$$

where the 'boundary state' $|B\rangle$ is the closed string state created by the boundary loop.

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Let us unpack this formalism a little, seeing where it all comes from. Recall that a Dp-brane is specified by the following open string boundary conditions:

$$\begin{aligned}\partial_\sigma X^\mu|_{\sigma=0,\pi} &= 0, \qquad \mu = 0,\ldots,p;\\ X^m|_{\sigma=0,\pi} &= Y^m, \qquad m = p+1,\ldots,D-1.\end{aligned} \tag{6.22}$$

Now we have to reinterpret this as a closed string statement. This involves exchanging τ and σ. So we write, focusing on the initial time:

$$\begin{aligned}\partial_\tau X^\mu|_{\tau=0} &= 0, \qquad \mu = 0,\ldots,p;\\ X^m|_{\tau=0} &= Y^m, \qquad m = p+1,\ldots,D-1.\end{aligned} \tag{6.23}$$

Recall that in the quantum theory we pass to an operator formalism, and so the conditions above should be written as an operator statement, where we are operating on some state in the Hilbert space. This defines for us then the boundary state $|B\rangle$:

$$\begin{aligned}\partial_\tau X^\mu|_{\tau=0}|B\rangle &= 0, \qquad \mu = 0,\ldots,p;\\ (X^m|_{\tau=0} - Y^m)|B\rangle &= 0, \qquad m = p+1,\ldots,D-1.\end{aligned} \tag{6.24}$$

As with everything we did in chapter 2, we can convert our equations above into a statement about the modes:

$$\begin{aligned}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu)|B\rangle &= 0, \qquad \mu = 0,\ldots,p;\\ (\alpha_n^m - \tilde{\alpha}_{-n}^m)|B\rangle &= 0, \qquad m = p+1,\ldots,D-1;\\ p^\mu|B\rangle &= 0, \qquad \mu = 0,\ldots,p;\\ (x^m - Y^m)|B\rangle &= 0, \qquad m = p+1,\ldots,D-1.\end{aligned} \tag{6.25}$$

As before, we either use only $D-2$ of the oscillator modes here (ignoring $\mu = 0,1$) or we do everything covariantly and make sure that we include the ghost sector and impose BRST invariance. We shall do the former here.

The solution to the condition above can we found by analogy with the (perhaps) familiar technology of coherent states in harmonic oscillator physics (see insert 6.3).

$$|B\rangle = \mathcal{N}_p \delta(x^m - Y^m)\left(\prod_{n=1}^{\infty} e^{-\frac{1}{n}\alpha_{-n}\cdot\mathcal{S}\cdot\tilde{\alpha}_{-n}}\right)|0\rangle. \tag{6.26}$$

The object $\mathcal{S} = (\eta^{\mu\nu}, -\delta^{mn})$ is just shorthand for the fact that the dot product must be the usual Lorentz one in the directions parallel to the brane, but there is a minus sign for the transverse directions.

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Insert 6.3. The boundary state as a coherent state

Let us recall that all we are playing with are creation and annihilation operators with a slightly unusual normalisation, as noticed at the beginning of section 2.3. Working with one set of the standard operators, a and $a^\dagger$, for the left and an *independent* set $\tilde{a}$ and $\tilde{a}^\dagger$ for the right, in essence we are trying to solve the equation

$$a|b>= \mp\tilde{a}|b\rangle.$$

Now recall how coherent states are made. We have

$$[a, a^\dagger] = 1, \qquad a|0\rangle = 0,$$

and so we can define a conjugation operation which shifts a by z, by defining

$$a(z) = e^{-za^\dagger} a e^{za^\dagger}.$$

It is easy to see that $a(z) = a + z$, since by elementary differentiation and the use of the commutator, we have

$$\frac{\partial a(z)}{\partial z} = 1.$$

Therefore the state

$$|z\rangle = e^{za^\dagger}|0\rangle$$

is an eigenvalue of the annihilation operator a, since

$$a|z>= e^{za^\dagger} e^{-za^\dagger} a e^{za^\dagger}|0\rangle = e^{za^\dagger}(a+z)|0\rangle = z|z\rangle.$$

We can therefore use as a solution to our first equation above, the coherent state with the choice $z = \mp\tilde{a}^\dagger$,

$$|b\rangle = N e^{\mp\tilde{a}^\dagger a^\dagger}|0\rangle,$$

where N is a normalisation constant.

The normalisation constant is determined by simply computing the closed string amplitude directly in this formalism. The closed string is prepared in a boundary state that corresponds to a D-brane, and it propagates for a while, ending in a similar boundary state at position $\vec{Y}$:

$$\mathcal{A} = \langle B|\Delta|B\rangle, \tag{6.27}$$

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where Δ is the closed string propagator. How is this object constructed? Well, we might expect that it is essentially the inverse of $H_{\rm cl} = 2(L_0+\bar{L}_0-2)/\alpha'$, the closed string Hamiltonian, which we can easily represent as:

$$\Delta = \frac{\alpha'}{2}\int_0^1 d\rho \rho^{L_0+\bar{L}_0-3},$$

and we must integrate over the modulus $\ell = -\log\rho$ of the cylinder from 0 to ∞. We must remember, however, that a physical state $|\phi\rangle$ is annihilated by $L_0 - \bar{L}_0$, and so we can modify our propagator so that it only propagates such states:

$$\Delta = \frac{\alpha'}{2}\int_0^1 d\rho\frac{d\phi}{2\pi}\int_0^{2\pi}\frac{d\phi}{2\pi}\rho^{L_0+\bar{L}_0-3}e^{i\phi(L_0-\bar{L}_0)},$$

which, after the change of variable to $z = \rho e^{i\phi}$, gives

$$\Delta = \frac{\alpha'}{4\pi}\int_{|z|\leq 1}\frac{dzd\bar{z}}{|z|^2}z^{L_0-1}\bar{z}^{\bar{L}_0-1}.$$

Computing the amplitude (6.27) by using this definition of the propagator is a straightforward exercise, similar in spirit to what we did in the open string sector. We get geometric sums over the oscillator modes resulting from traces, and integrals over the continuous quantities. If we make the choices $|z| = e^{-\pi s}$ and $dzd\bar{z} = -\pi e^{-2\pi s}dsd\phi$ for our closed string cylinder, the result is:

$$\mathcal{A} = \mathcal{N}_p^2 V_{p+1}\frac{\alpha'\pi}{2}(2\pi\alpha')^{-\frac{25-p}{2}}\int_0^\infty\frac{ds}{s}s^{-\frac{25-p}{2}}e^{-Y\cdot Y/s2\pi\alpha'}f_1(q)^{-24}. \quad (6.28)$$

Here $q = e^{-2\pi/s}$.

Now we can compare to the open string computation, which is the result in equation (6.3). We must do a modular transformation $s = -1/t$, and using the modular transformation properties given in insert 6.2, we find exactly the open string result if we have

$$\mathcal{N}_p = \frac{T_p}{2},$$

where T_p is the brane tension (6.14) computed earlier.

This is a very useful way of formulating the whole D-brane construction. In fact, the boundary state constructed above is just a special case of a sensible conformal field theory object. It is a state that can arise in the conformal field theory with boundary. Not all boundary states have such a simple spacetime interpretation as the one we made here. We see therefore that D-branes, if interpreted simply as resulting from the introduction of

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open string sectors into closed string theory, have a world-sheet formulation which does not necessarily always have a spacetime interpretation as its counterpart. Similar things happen in closed string conformal field theory. There are very many conformal field theories which are perfectly good string vacua, which have no spacetime interpretation in terms of an unambiguous target space geometry. It is natural that this also be true for the open string sector.