# POLE POSITION STUDIED WITH ARTIFICIAL EARTH SATELLITES*

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**Abstract.** Long-arc orbit computation of highest accuracy can provide pole positions. Optical Baker-Nunn and laser range observations of several satellites are combined. The accuracy of the pole position is comparable to that of the mean satellite-tracking station coordinates ($\pm 5$ m) when sufficient tracking data are available. Exploitation of the technique requires more accurate tracking data.

## 1. Introduction

Tracking data generally available in the past have been of 20-m accuracy. Classical observation of the pole position has been included in the data reduction. The consistency of the polar motion as determined by the International Polar Motion Service (IPMS) and the Bureau International de l'Heure (BIH) is 1 to 2 m. Clearly, using 20-m data to determine 10-m polar-motion effects will involve averaging, and the validity of such averaging will depend on the absence of systematic errors in the data and analysis.

Future tracking data promise to be of the accuracy of 2 to 10 cm. Today there are laser satellite-tracking systems routinely making 50-cm observations. There is reasonable expectation that 30-cm accuracy will be realized routinely in 1 or 2 yr, and the ultimate accuracy, within the decade.

The processing of tracking data to determine pole position requires establishing the coordinates of observing stations to comparable accuracy, much as the latitude service must do. Similarly, orbital trajectories must be computed with the same accuracy. These geodetic and analytical studies are the major accomplishments necessary, and the inherent accuracy of the polar motion will rest on these subjects.

It is not the purpose of this paper to discuss geodesy and celestial mechanics, and we confine ourselves to a few remarks. The current values for the coordinates of observing stations and the Earth's gravity field are to be found in Gaposchkin and Lambeck (1970) and were determined primarily with 20-m optical data. The accuracy of the coordinates is 5 to 10 m RMS for the fundamental stations, obtained with averaging many thousands of observations. The geopotential is represented in spherical harmonics with 316 terms, complete to degree and order 16. Orbital accuracy is between 10 and 20 m RMS. Only with sufficient laser data can we establish a 10-m accuracy. Other tests indicate we are in agreement with this.

To improve on the determination of pole position, we are limited by the data and other physical constants currently available. This analysis is therefore preliminary and serves to point out the critical areas.

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*P. Melchior and S. Yumi (eds.), Rotation of the Earth,* 128–130. *All Rights Reserved.*

## 2. Methods

The motion of the pole has two effects detectable by satellites. First, there is a change of orientation of the axis of figure $\bar{P}$, or of some arbitrary reference, with respect to an inertial reference (see Figure 1). The inertial reference is specified by the angular momentum $\bar{L}$, which we assume to be the same as the spin axis $\bar{\varpi}$. The difference is not yet observable, being of the size of the flattening (1/298) times the motion ($0.5 \times \times 10^{-6}$), i.e., $10^{-8}$ or 1.7 cm.

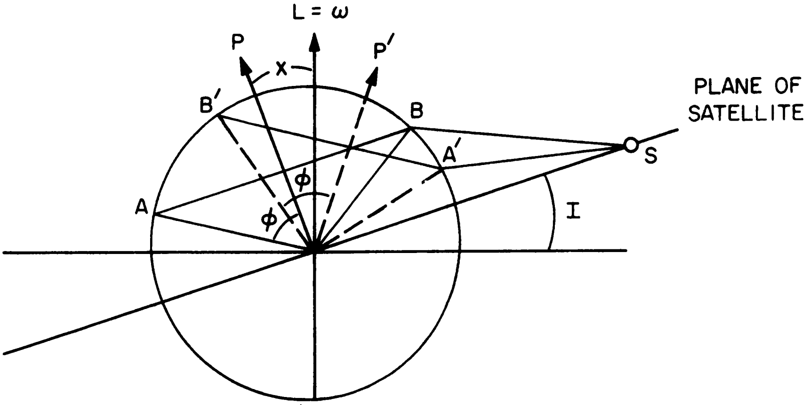


Fig. 1. Kinematic determination of pole position.

The adopted positions of the observing stations define a reference pole. The rotation in 12 hr changes the direction to the satellite by $2\times$. So observations taken at $t$ and $t'$ (from $B$ and $A'$), i.e., when the pole is at $P$ and $P'$, are sufficient to determine both the angle $x$ (the pole position) and the inclination $I$ of the satellite. In general, the position of the pole $(x, y)$ is determined at the same time as the orbital elements.

Second, the axis of figure changes in the gravity field as a result of two effects. It gives rise to the harmonics $\bar{C}_{2,1}$ and $\bar{S}_{2,1}$ by the rotation of the principal $J_2$ term to the inertial reference system. Owing to the elasticity of the Earth, the change in rotation axis with respect to the body axis causes a shift of mass also resulting in harmonics $\bar{C}_{2,1}$ and $\bar{S}_{2,1}$.

In Figure 2 we take the axis of figure $\bar{I}$ as the origin of a coordinate system, $\bar{\varpi} = \bar{L}$ as the spin axis, and $\bar{P}$ as the defined pole with coordinates $(l, m)$ and $(\xi, \eta)$. The pole $\bar{P}$ is arbitrary, and it is that system we adopt for a reference.

A satellite measuring the gravity field in the coordinate system $\bar{P}$ would sense the harmonics

$$\bar{C}_{2,1} = \frac{\bar{C}_{2,0}\eta}{\sqrt{3}} - \frac{k\Omega^2 a_e^3 l}{\sqrt{15GM}}$$

$$\bar{S}_{2,1} = \frac{\bar{C}_{2,0}\xi}{\sqrt{3}} - \frac{k\Omega^2 a_e^3 m}{\sqrt{15GM}},$$

where $\bar{C}_{2,0}$ is the principal oblateness term in the Earth's gravity field ($C_{2,0} = -J_2/\sqrt{5}$).

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It is interesting to note that if we choose as our reference system $\bar{L}$ (i.e., $l=\xi, m=\eta$), which is the most convenient, then $\bar{C}_{2,1}=\bar{S}_{2,1}=0$ for $k=0.313$. In this case, the Earth is spinning about an axis of maximum moment of inertia; i.e., the axis of figure is not an axis of maximum moment of inertia. The value $k=0.313$ is in agreement with other measurements of $k$ but not with the value determined by the period of the Chandler period ($k=0.28$). The values of $\bar{C}_{2,1}$ or $\bar{S}_{2,1}$ will be of the order of $\bar{C}_{2,0}\eta \times \times (0.31-k)=10^{-3}\times 10^{-6}\times 10^{-2}=10^{-11}$ and very difficult to detect.

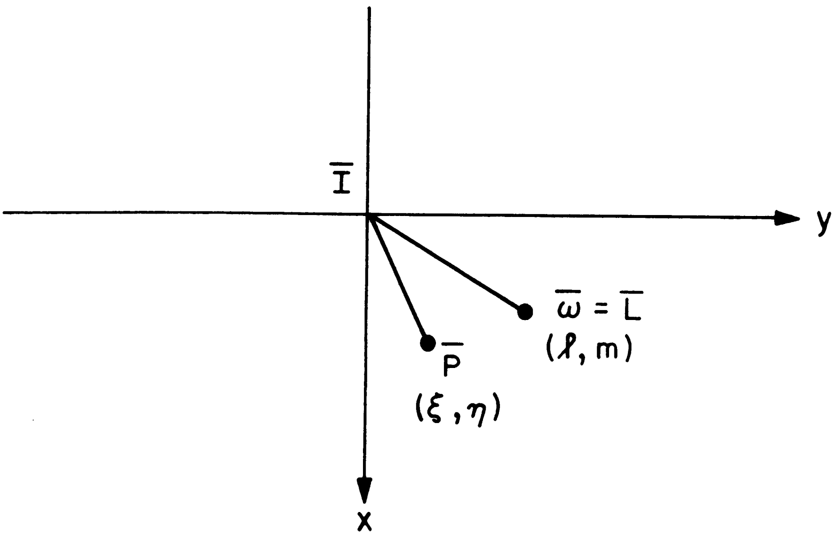


Fig. 2. Gravitational determination of pole position.

By using the gravitational effects, we can determine the principal axis, given the measurements of pole position from kinematic measurements ($\xi-l$, $\eta-m$) and satellite gravity-field measurements of $\bar{C}_{2,1}$ and $\bar{S}_{2,1}$. Other aspects of using $\bar{C}_{2,1}$ and $\bar{S}_{2,1}$ are discussed by Gaposchkin (1968).

## References

Gaposchkin, E. M.: 1968, 'The Motion of the Pole and the Earth's Elasticity as Studied from the Gravity Field of the Earth by Means of Artificial Earth Satellites', in *Proceedings of the Symposium on Modern Questions of Celestial Mechanics*, Centro Internazionale Mathematico Estiva.

Gaposchkin, E. M. and Lambeck, K.: 1970, 1969, *Smithsonian Astrophys. Obs. Spec. Rep.* No. 315, Cambridge, Mass.

## DISCUSSION

*S. Débarbat:* In your abstract you wrote "when sufficient tracking data are available". What do you mean exactly?

*E. M. Gaposchkin:* Tracking of 4 laser satellites by 10 stations, operating 24 hours a day.