Magnetic Fields in the Solar Nebula and the Angular Momentum Transfer

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1. Introduction

We are now carrying out the project to investigate how the primodial solar disk is formed and evolves. One of the central problems of this project is to study the angular momentum transfer in the nebula caused by the magnetic braking or the turbulent mixing in the boundary layer between the surface of the disk and its high temperature corona envelope.

Here, we summarize the result of analysis for the magnetic braking in 1-dimensional approximation, which will make the basis to interpret the full numerical calculations which we are now carrying out.

2. Angular Momentum Transport in the Radial Direction

As pointed out by Hayashi (1981), when seed magnetic fields which are generated by turbulences have radial component, azimuthal component grows by rotation and decays by Joule loss. For this process, the basic equation is given by

$$\{\frac{\partial}{\partial t} + \Omega_0[r_0 - \frac{3}{2}(r - r_0)]\frac{\partial}{r_0 \partial \phi} - \frac{c^2}{4\pi\sigma_e}\nabla\} \begin{pmatrix} H_r \\ H_\phi \\ H_z \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{3}{2}\Omega_0 H_r \\ 0 \end{pmatrix}. \quad (1)$$

The azimuthal component H_ϕ takes the maximum value at $t \sim t_{\mathrm{decay}}$, $H_\phi^{\max} \simeq (t_{\mathrm{decay}}/t_{\mathrm{Kepler}})H_r(0)$, where t_{decay} is the decay time $t_{\mathrm{decay}} = 4\pi\sigma_e z_0^2/c^2$, z_0 is the scale height of the disk, and t_{Kepler} is the Kepler time. The amplification factor $t_{\mathrm{decay}}/t_{\mathrm{Kepler}}$ takes the value about $10^{-1} - 10^1$ at Jupiter. The angular momentum is transferred by the r-ϕ component of the Maxwell stress tensor along the r-direction. The dynamical viscosity coefficient in this case is given by $\nu \sim (t_{\mathrm{decay}}/t_{\mathrm{Kepler}})c_s z_0$.

3. Angular Momentum Transport by Alfvén Wave

Recently, Terasawa et al. (unpublished) has pointed out the following mechanism: When the magnetic fields are uniform in the z-direction, the ϕ-component of magnetic fields which are generated by rotation is transmitted as the Alfvén wave in the z-direction and the the angular momentum escapes along the z-direction.

E. Falgarone et al. (eds.), Fragmentation of Molecular Clouds and Star Formation, 441–442.

In 1-dimensional approximation, it is assumed that $\partial/\partial r = 0$, $v_z = 0$, $H_z =$ const. , where v_z is the z-component of the fluid velocity. The basic equation in this approximation is given by

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial t}(v_\phi + D\frac{\partial h}{\partial z}) \qquad \frac{\partial v_\phi}{\partial t} = v_{\rm A}^2\frac{\partial h}{\partial z}, \tag{2}$$

where $h = H_\phi/H_z$ is the ϕ-component of magnetic fields normalized by the uniform fields, $D = c^2/4\pi\sigma_e$ is the magnetic diffusion coefficient, σ_e is the electric conductivity, and $v_{\rm A} = \sqrt{H_z^2/4\pi\rho}$ is the Alfvén velocity. The boundary conditions are given as, at $z = 0$, $h = 0$, $\partial v_\phi/\partial z = 0$, and at $z \to \infty$, where the Alfvén velocity is constant and the diffusion coefficient vanishes, $v_\phi/v_{\rm A} = -h$ (out-going wave condition).

The Alfvén velocity $v_{\rm A}$ and the diffusion coefficient D are different in the disk and in the envelope. In the case of thin transition layer, i.e., $D = D_0$, $v_{\rm A} = v_{\rm A,disk}$ for $z < z_0$, and $D = 0$, $v_{\rm A} = v_{\rm A,ext}$ for $z > z_0$, we can easily find a solution implicitly which has the functional form (compare with the solution by Mouschovias and Paleologou) of $h = -h_0 e^{-\lambda t}\sinh(kz)$, $v_\phi = -v_0 e^{-\lambda t}\cosh(kz)$ in the disk. From the differential equation, we get $\lambda/k^2 = v_{\rm A,disk}^2/\lambda - D_0$ and $h_0/v_0 = k/\lambda$. From the boundary condition, we get $\tanh(kz_0) = v_0/h_0 v_{\rm A,ext} = \lambda/kv_{\rm A,disk}$. If the density contrast between disk and envelope is very large, so that $v_{\rm A,disk}/v_{\rm A,ext} << 1$, the solution becomes

$$\lambda = \frac{v_{\rm A,disk}^2}{z_0 v_{\rm A,ext} + D}, \quad k^2 = \frac{v_{\rm A,disk}^2}{z_0 v_{\rm A,ext}(z_0 v_{\rm A,ext} + D)}, \quad \frac{v_\phi(z = z_0 + \epsilon)}{v_\phi(z = z - \epsilon)} = 1 - \frac{\lambda D}{v_{\rm A,disk}^2} = \frac{z_0 V_{\rm A,ext}}{z_0 V_{\rm A,ext} + D}. \tag{3}$$

The solution for $z > z_0$ is given by $h = h_0 k z_0 e^{-\lambda(t - z/v_{\rm A,ext} - z_0/v_{\rm A,ext})}\theta(-z + v_{\rm A}t + z_f)$, $v_\phi = v_0 e^{-\lambda(t - z/v_{\rm A,ext} - z_0/v_{\rm A,ext})}\theta(-z + v_{\rm A}t + z_f)$, where θ is the step function and z_f is the position of a wave front at $t = 0$. From these results, we can express the in-fall velocity of the gas in Kepler disks as

$$t_r = -\frac{r}{v_r} = \frac{D + v_{\rm A,ext} z_0}{2 v_{\rm A,disk}}. \tag{4}$$

Narita et al. (in preparation) has studied the characteristic features of this mechanism in the proto-solar disk. In actual disks, the surface part is ionized by cosmic rays but the inner part is less ionized, so that the electric conductivity σ_e depends on z. The density ρ in a gravitationally equilibrium disk varies exponentially, so that the Alfvén velocity depends on z. In this case, it is not clear how we can modify eq.(4), nor it dose not depend on time. Narita et al. studied these problems numerically.

Numerical results show that

$$\alpha(z) = \frac{1}{1 + \frac{D}{z v_{\rm A,ext}}}, \tag{5}$$

which describes a velocity gap $v_{\phi,\rm ext}/v_{\phi,0}$, varies timely. If the magnetic fields are scarcely frozen ($\alpha << 1$), the damping time scale for $v_\phi(z)$ is given locally by

$$\tau(z) = \frac{c^2\rho(z)}{H_z^2\sigma_e(z)}. \tag{6}$$

This means that when $\alpha << 1$, not only the magnetic braking dose not depend on the external density, but also it does not depend global structure of the disk. From this equation, it is expected that the magnetic braking is effective in the transition region between disk and envelope. This means that the in-fall flow of the gas is fast at the surface region of the disk.

References

Hayashi, C., 1981, Prog. Theor. Phys., suppl. **70**, 35.

Mouschovias, T.C., Paleologou, E.V., 1979, Ap. J. **230**, 204.