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Lattice models for solar flares and coronal heating

Invited Review

O. Podladchikova[1] **and B. Lefebvre**[2]

[1]Royal Observatory of Belgium, Av. Circulaire-3, Brussels, 1180
email: Elena.Podladchikova@oma.be

[2] The Blackett Laboratory, Imperial College London, London SW7 2BW, UK
email: b.lefebvre@imperial.ac.uk

Abstract. Solar coronal heating is a complex problem due to the variety of scales and physical phenomena involved, and intricacy of "boundary conditions". Lattice models and self-organized criticality provide means to model phenomenologically some of the physics involved over a wide range of scales, and reproduce certain statistical features of solar flares. Furthermore, these models offer a basis for the study of Parker's hypothesis of coronal heating by nanoflares. We provide a short review of this approach pioneered by Lu & Hamilton (1991) and related more recent works involving lattice models.

Keywords. Sun: corona, Sun: flares, Sun: magnetic field, methods: numerical

1. Coronal heating and solar flares

The heating of solar corona still poses a number of fundamental questions about its detailed mechanisms which are difficult to answer by theory and direct observations (see Klimchuk 2006, Walsh & Ireland 2003 for recent reviews).

Parker (1988) proposed that the solar corona could be heated by the dissipation at many small-scale tangential discontinuities arising spontaneously in the coronal magnetic fields braided and twisted by random photospheric footpoint motions. These events are related to a magnetic energy release, sudden changes of magnetic field topology, heating of plasma and acceleration of particles to high energies. Parker has called this elementary energy release events 'nanoflares'. This idea stimulated the intensive search of observational signatures of microflares and nanoflares as well as many theoretical developments on the contribution of small scales to energy dissipation in the solar corona.

Microflares, were first detected in soft X-rays in a balloon experiment by Lin *et al.* 1984. The development of new instrumentation allowed performing the multi-wave satellite and ground based high-resolution observations of smaller-scale (about a thousand of kms) lower energy phenomena. They were observed in active regions but also in the quiet regions of the Sun and in coronal holes (Shimizu *et al.* 1994). Soft X-ray observations (Benz *et al.* 1997) and EUV observations (Harrison 1997) have revealed enhanced emission and thus intense heating above the magnetic network borders. A similar phenomenon that forms small X-ray jets at the limb was reported by Koutchmy *et al.* (1997). Berghmans *et al.* (1998) and Benz *et al.* (1998) have found the heating impulsive events exhibiting a lower level of fluctuations above the magnetic cell interiors. The number of observed events increases with the sensitivity.

The distribution of solar flares intensities in soft X-rays or EUV, have been thoroughly investigated for quite a long time being in particular important for the coronal heating problem (Hudson, 1991). An intriguing experimental fact is that the frequency distributions of solar flare energy, peak rate and several other characteristics exhibit power-law dependencies (Lin *et al.* 1984, Datlowe *et al.* 1974, Dennis, 1985, Biesecker *et al.* 1994) in quite large range of intensities covering several decades down to the smallest detectable energies (Aschwanden *et al.* 2000). For regular flares that occur mainly in active regions, Crosby *et al.* (1993) have found that the total energy in the flare electrons observed in hard X-ray bremsstrahlung has distribution $f(E) \sim E^{\alpha}$ with index $\alpha = -1.53 \pm 0.02$. But the energy supplied by the flares in the active regions is not sufficient for the corona heating. Recent space instruments demonstrate that for smaller energies PDF dependence upon the energy has also a power law distribution. Krucker & Benz (1998) have found from the Yohkoh/SXT in X-Ray and from SOHO/EUV observations the PDF dependence upon the energy has a power law distribution in the energy range $10^{24} - 10^{26}$ ergs with the index about -2.59. Aschwanden *et al.* (2000) and Parnell & Jupp (2000) from TRACE/EUV observations have found also power law with significantly different value of this index -1.80. Found from distribution integral energy can be enough to explain the coronal temperature under condition of the same dependencies on the unresolved yet nanoflares energy up to 3×10^{23} erg (Krucker & Benz 1998). Difference of indexes however stress the question on which flaring energetic scales the heating mostly happens.

The found power-laws suggests a certain scale-invariance between flares, and is also compatible with Parker's hypothesis that large flares could consist of a multitude of nanoflares. Such a point of view therefore suggests an "organization" or cooperation between flares. Such phenomena are difficult to approach theoretically or numerically with traditional plasma physics models due to the very wide range of spatial and temporal scales involved, which makes some coarse-grained description still necessary.

2. Self-organized criticality and solar flares

2.1. *Self-organized criticality*

Bak, Tang & Wiesenfeld (1987) (hereafter BTW) have introduced Self-Organized Criticality (SOC) as a unifying mechanism providing scale-invariant fluctuations in many non-equilibrium (driven) complex systems. In analogy with critical phase transitions, SOC systems develop a state with long-range correlations (power-law decaying correlation functions), but without requiring the fine tuning of an external parameter such as the temperature. Certain general ingredients are common to many systems exhibiting SOC, and even though the precise necessary conditions for SOC are not fully understood (Jensen 1998; Sornette 2000):

- a large number of 'elements' or 'degrees of freedom',
- the evolution of each degree of freedom is prescribed by the interactions with its neighbors rather than by its own intrinsic dynamics,
- the interaction mechanism involves a small number of elements (such as nearest neighbors) and is non-linear (often threshold-dependent),
- the system is weakly perturbed (driven) on a time scale much greater than that of the relaxation mechanism,
- (possibly) the relaxation mechanism is conservative.

Under such conditions SOC establishes as a state where the slow perturbations imposed to the system can trigger responses of arbitrary size (avalanches in the sandpile analogy originally used by BTW) whose observable parameters (e.g. size or duration) obey power-law

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statistics indicative of scale-invariance. Thanks to the slow driving of the system, the system's responses are usually well separated in time and exhibit 'bursty' time series.

2.2. *The Lu and Hamilton model*

Lu & Hamilton (1991) have pointed out interesting parallels between solar flares and SOC (see also Lu *et al.* 1993 and Lu 1995). Solar flares are localized and bursty energy release events in response to slow external stressing (footpoint motion by photospheric convection). In the spirit of Parker's conjecture, they can be described as an ensemble of many elementary small-scale dissipative events (nanoflares), possibly locally triggered one by another in analogy to an avalanche. These relaxation events occur on time scales much smaller than the buildup of the unstable situation which they originate from, and are believed to depend on a threshold. And finally, solar-flare peak-fluxes (count rate), integrated fluxes and duration follow power-law statistics with scaling factors independent of features such as the solar cycle or active region considered.

Elaborating on this parallel, Lu & Hamilton (1991) and Lu *et al.* (1993) (hereafter LH) have proposed a 3D model of solar flares quite similar to the BTW sandpile model. A comprehensive review of this (and related) model is given by Charbonneau *et al.* (2001), and we shall give here only a much briefer account. The model is defined on a cubic lattice, representing a part of the solar corona of linear size comparable to that of an active region. To each cell or lattice node is attributed a quantity F_i, akin to an average magnetic field inside the cell (following Charbonneau *et al.* (2001), we simplify the original model using a scalar F_i). At every 'time step', a perturbation is added at a random location on the lattice, which is small compared to the instability threshold and whose average is positive so that the source tends to slowly increase the total field in the system. A site is considered unstable when the field 'curvature' exceeds a threshold

$$\delta_{\mathbf{i}} = F_{\mathbf{i}} - \frac{1}{6}\sum_{nn} F_{nn} > \delta_c \tag{2.1}$$

where nn denotes the 6 nearest neighbors and $\delta_c > 0$ is a threshold value. This instability criteria makes the difference between Lu and Hamilton's model and the BTW sandpile, being not a discrete version of a gradient but rather of a second order differential operator. The difference in coupling mechanisms result into power-laws with different indices, making the LH and BTW model fall into different 'universality classes' Edney *et al.* (1998).

The relaxation then consists of redistributing part of the field in cell $\mathbf{i}$ to its neighbors

$$F_{\mathbf{i}} \to F_{\mathbf{i}} - \frac{6}{7}\delta_c, \quad F_{nn} \to F_{nn} + \frac{1}{7}\delta_c. \tag{2.2}$$

As long as there are unstable sites on the lattice, the source is switched off (the relaxation being supposed to be much faster than the source's perturbation), and all unstable sites are relaxed simultaneously. The relaxation iteration is repeated until no more unstable sites are present on the grid, and the source is switched on again. The relaxation process of eq. 2.2 conserves $\sum_i F$ on the lattice. Therefore, the field can only be lost at boundaries, depending on boundary conditions whose importance in maintaining a global curvature over the lattice was pointed out by Galsgaard (1996). Nevertheless, the relaxation mechanism decreases $E = \sum_i F^2$, which corresponds to magnetic energy if B is interpreted as a magnetic field. On the other hand, Lu *et al.* (1993) have noted that interpreting F as a vector potential A such that $\mathbf{B} = \nabla \times \mathbf{A}$ has the advantage of ensuring that $\nabla \cdot \mathbf{B} = 0$, allows to interpret the driving as twisting magnetic field lines rather than

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increasing the magnetic field locally in the corona and the relaxation condition of eq. 2.1 as depending on a current.

Lu *et al.* (1993) have found that this simple model produces power-law frequency distributions ranging over a few decades of various parameters associated to the 'avalanches', and that these power-laws do not depend on the chosen threshold value, small modifications of the relaxation rule nor grid size for large enough grids.

2.3. *Comparison with solar flare observations*

Models such as the one described in the previous paragraph being much less computationally demanding than more traditional ones allow to make reliable statistics over many flare-like relaxation events. However, comparison of these statistics with X-ray and EUV observations of flare and microflares is far from being straightforward. As discussed by Lu *et al.* (1993) and Georgoulis *et al.* (2001), this comparison requires for instance to translate the model output into physical spatial/temporal scales and quantities such as dissipated energy, and then to relate these quantities to the measured X-ray or EUV fluxes. This involves hypotheses and non-trivial transformations which may affect the shape of the frequency distributions (McIntosh *et al.* 2002, Aschwanden & Parnell 2002, Battaglia *et al.* 2005).

Nevertheless, relating in a simple way the dissipated energy E from the model to an equivalent luminosity, Lu *et al.* (1993) have found good agreement between frequency distributions from their model and X-ray flare observations for peak luminosity, integrated luminosity and duration. They even argued that the exponential decay of the observed distributions for the most luminous flares can be modeled by a finite-size grid effect and thus can be attributed to the finite size of active regions.

These results, together with the empirical arguments that flare satisfy many of the conditions met by SOC models, give strongly support the view of solar flares are relaxation events of a self-organized system (the solar corona). This would suggest in particular that solar flare statistics do not depend sensibly on the detailed microphysics involved in flares nor on the driving mechanism which leads to unstable configurations. It also suggests solar flares are very difficult to predict, since flares of arbitrary sizes might randomly be triggered by small (possibly unobservable) perturbations.

However, this leaves a number of questions opened. First, the model is based on empirical rules which are difficult to justify physically (e.g., how to interpret eq. 2.1 if F is a magnetic field, or how to interpret F^2 as an energy if F is a vector potential). This question is particularly important if we want to predict the scales down to which SOC operates or is modified and build SOC-like models of coronal heating by nanoflares. One may also ask how compatible is the weak disorder of a SOC system regularly swept by avalanches with the long-lived structures observed in the corona. Second, it was noticed by Wheatland *et al.* (1998) and Boffetta *et al.* (1999) that conventional avalanche models have an exponential (Poissonian) distribution of waiting-times between events, which is at variance with the observed waiting time distribution for solar flares which have a long power-law like tail (Pearce *et al.* 1993, Crosby *et al.* 1998, Wheatland *et al.* (1998), Lepreti *et al.* (2001)). Moreover, it was noted by McIntosh *et al.* (2002) that the projected flaring area in the LH model has a frequency distribution flatter then observations. These two points have triggered a large body of work which we shall now briefly (and incompletely) summarize.

3. Lattice models and solar flare phenomenology

3.1. *Fields, lattices and model interpretations*

The magnetic or vector potential fields used by Lu *et al.* (1993) seem to be the natural variables for such models, given the importance of the magnetic field topology. In this context, it is natural to investigate the 'continuum' limit of these models and compare them with known partial differential equations from plasma physics.

The continuum limit of the Lu & Hamilton and comparable models was investigated by Vassiliadis *et al.* (1998), Isliker *et al.* (1998), Liu *et al.* (2002). This leads to a nonlinear hyperdiffusion equation which was argued by Liu *et al.* (2002) to be compatible with MHD turbulent diffusivity. This view of lattice models taken as discrete version of partial differential (MHD) equations was take further by Isliker *et al.* (2000), Isliker *et al.* (2001) who have constructed an avalanche-like lattice model compatible with MHD equations. Einaudi & Velli (1999) have constructed a 2D model based on externally forced reduced MHD (RMHD) equations. This approach was extend by Buchlin *et al.* (2003) who studied impulsive events in a magnetic loop modeled by a set of such lattices exchanging information through Alfvén-like fluctuations.

While taking the continuum limit, Vassiliadis *et al.* (1998) nevertheless associate their 2D cells to magnetic flux tubes and note that cellular automata (such as the original BTW sandpile) and similar lattice models are intrinsically discrete and deal with finite 'macroscopic' variables (see also the discussions in Lu *et al.* 1993, Einaudi & Velli 1999). Clearly illustrating this view, Zirker & Cleveland (1993) consider as a variable the internal torsion of the flux tubes. Vlahos *et al.* (1995), Georgoulis & Vlahos (1996) use a scalar magnetic field representing an average in each cell, and it was shown by Edney *et al.* (1998) that scalar and vector versions of the LH-type models yield similar results. Longcope & Noonan (2000) choose for variable currents which propagate in the grid. Krasnoselskikh *et al.* (2002) consider average magnetic fields in each cell of a 2D lattice and self-consistently computed currents on the cell borders satisfying Kirchoff's law (see also Vlahos *et al.* 1995, Vassiliadis et al. 1998). Although not based on a lattice, the model of Hugues *et al.* (2003) consider a set of randomly moving magnetic loops. Hamon *et al.* (2002) use the Olami-Feder-Christensen SOC model for earthquakes whose variable is simply the stored energy in each cell (see also de Arcangelis *et al.* 2006 for parallels between solar flare and earthquake statistics). MacKinnon *et al.* (1996) use a model inspired by the Forest-Fire SOC model, and deliberately do not specify the nature of the variable.

3.2. *Coupling between elements and energy dissipation*

Although energy dissipation in most models is associated to the dissipation of current sheets in the spirit of Parker's nanoflare model, this may not appear explicitly in the relaxation mechanisms, depending in particular on the model interpretation. Relaxation mechanisms take many different forms. For instance Vlahos *et al.* (1995) and Georgoulis & Vlahos (1996) consider anisotropic variants of the LH instability criteria, based on the magnetic field difference between 2 neighboring cells and redistribute the dissipated field on all neighboring cells. This results in steeper power-laws than in the isotropic case. MacKinnon & Macpherson (1997) have considered a non-local variant of the LH model, where the relaxation mechanism can affect another site at a random location on the lattice possibly via high-energy particles accelerated on the reconnection site. This also results in different flare statistics. Krasnoselskikh *et al.* (2002) and Podladchikova *et al.* (2002) totally dissipate currents which exceed a threshold value, and consider an additional requirement that magnetic fields in adjacent cells have opposite directions

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forming an X-point like topology. This extra requirement also affects relaxation events statistics.

When currents can be explicitly calculated in the model, the dissipated energy is generally calculated as the sum of the squared dissipated currents. When the magnetic field is the main variable, the dissipated energy is calculated as the change in total magnetic energy. It is usually from this dissipated energy that statistics for comparison with solar flare observations are built. Anastasiadis *et al.* (2004) increase the sophistication of their cellular automata model by discussing particle acceleration during the energy dissipation process and the radiation they produce.

3.3. *Driving methods*

Some variants of the source were proposed, and considerable freedom is left by our ignorance of its properties.

Georgoulis & Vlahos (1996) distribute the driver magnitude as a power-law, and show that this can result in 'double power-law' flare distributions which are steeper at lower energies and hence would better correspond to a nanoflare-heated corona. Einaudi & Velli (1999) drive their model with large scale vortices, similarly to turbulent cascade models which assume energy injection at large scales. A number of small-scale drivers were considered by Krasnoselskikh *et al.* (2002), Podladchikova *et al.* (2002), which were found to result in different large-scale structures but similar properties of the dissipated energy.

Most importantly, the driver was found to be a key element in the waiting-time statistics problem. Norman *et al.* (2001) use a non-stationary in time source, which allow them to find a distribution of the waiting-time between flares nearly in power-law. Similar results are obtained by Sánchez *et al.* (2002) with a 'running sandpile' model which uses a slow source correlated in time, allowing for overlapping avalanches. Hamon *et al.* (2002) achieve this result by driving their model at a finite (non-vanishing) rate, which as they argue breaks its SOC behavior and brings it on the 'edge of SOC'. Podladchikova *et al.* (2003a) find long power-law tails in the distribution of waiting times in their model when driven by a turbulent-like source with a power-law k-spectrum. Fragos *et al.* (2004) use a source modeled by a percolation process.

4. Conclusion

Lu & Hamilton (1991) and Lu *et al.* (1993) have opened a modeling approach to solar flares based on self-organized criticality (SOC) and an analogy with Parker's nanoflare hypothesis. The success of the model in reproducing certain solar flare statistics as well as its shortcomings have stimulated the appearance of number of similar lattice or cellular automata models. These models allow to change easily the physical elements they include, and allow detailed statistical study still beyond reach of theory or traditional numerical simulations.

These models where able to reproduce the most remarkable statistical properties of flares and microflares, including the power-law tails of the flare waiting time distribution which were used by Boffetta *et al.* (1999) as an argument against SOC-based models. However, to our knowledge there is no single model which is able to recover simultaneously all known flare statistics. It is also worth noting that comparison with experimental data may not be straightforward. For instance, direct comparison of model outputs and observations may require complex transformations which could modify flare statistics (e.g. McIntosh *et al.* 2002, Aschwanden & Parnell 2002, Battaglia *et al.* 2005). Furthermore, there is some disparity in observed frequency distributions (see Achwanden &

Parnell 2002 and references therein). This is in part due to the difficulty of separating events from the coronal 'background', difficulty which is increasing for smaller events such as micro- and nano-flares. The importance of event selection technique in particular for the waiting-time distributions was discussed by Wheatland *et al.* (1998), Lepreti *et al.* (2001), Buchlin *et al.* (2005), Paczuski *et al.* (2005). Further discrepancy might also be attributed to fitting techniques, making desirable the use of more systematic techniques (e.g. Podladchikova *et al.* 2003).

After Lu & Hamilton, some of the lattice models have to various degrees departed from the pure SOC concept. This was partly motivated by the need to connect purely empirical models to more physics-based models, connection which should manifest itself at smaller scales. Providing a finer description of small-scales is particularly important in extrapolating flare and micro-flare behavior down to nano-flare scales in order to study Parker's nanoflare heating idea. SOC may be less relevant at these scales than it could be at larger ones. It was for instance suggested that only relatively large flares (class M and above) might exhibit SOC by Bershadskii & Sreenivasan (2003). Another departure from pure SOC consist in considering specific properties of the source motivated to provide non-Poissonian waiting-time statistics. This suggest that some of the events statistics do not result from pure self-organization but reflect in part properties of the driving mechanism, as could be for example the case in Earth's magnetotail dynamics (Chapman & Watkins 2001 and references therein).

The phenomenological lattice models described in this review certainly do not provide complete model for coronal heating or solar flare statistics, and are clearly not the only possible approach to these problems (for a general discussion see Sornette 2002, and Klimchuk 2006, Walsh & Ireland 2003 for coronal heating). They nevertheless have met considerable successes, and provide a rich and extensible framework for statistical studies of impulsive in the solar corona which has not yet been fully explored.

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