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ABSTRACT

Powerful, extragalactic radio sources might be fuelled by energy release near a massive black hole. In this article we describe some relativistic effects which may be relevant to this process. We use Newtonian language so far as possible and illustrate the effects with "simple" analogies. Specifically, we describe the gravitational field near a black hole, Lens-Thirring and geodetic precession, electromagnetic energy extraction of the spin energy of a black hole and the structure of accretion tori around a black hole.

1. INTRODUCTION

Ever since 1964 massive black holes have been recognized as possible prime movers for quasars and extragalactic radio sources. With the subsequent discovery of rapid X-ray and optical variability, small scale radio jets and apparent superluminal expansion, this notion has become more appealing. However, we still lack direct observational evidence for the existence of massive black holes; and the chain of theoretical argument linking them to the observations reported at this symposium is consequently rather long. Some of these observed properties, in particular the alignment and apparent re-alignment of radio jets, may be explained by crucially general relativistic effects. It is our purpose here to describe some of these effects in a manner that is accessible to an astronomer unfamiliar with the full relativistic formalism.

2. THE GRAVITATIONAL FIELD OF A BLACK HOLE

We shall adopt the "3+1" point of view on relativistic gravity (e.g. Thorne and Macdonald 1981). In this view, which is mathematically equivalent to the usual 4-dimensional one, spacetime is split up into a curved 3-dimensional space with metric g_{jk} ($j=1,2,3$) and a universal time t which differs from "proper time" by a gravitational redshift factor.

Because space is curved, as we move from one equatorial circle

surrounding a black hole to another, the circumference increases less rapidly than Euclid would have demanded: $d(\text{circumference})/d(\text{radius}) < 2\pi$. If we extract the equatorial plane from around the hole and embed it in Euclidean 3-space, then we obtain a trumpet-horn surface. For a nonrotating hole (and far outside a rotating hole) the equation of this surface in cylindrical coordinates $(\tilde{\omega}, z)$ is $(z/m) = 4(\tilde{\omega}/2m - 1)^{1/2}$ where $m = GM/c^2$.

The gravitational field around the hole can be described using an electromagnetic analogy. We can decompose it into a gravitoelectric field \vec{g} and a gravitomagnetic field \vec{H} . In the limit of weak (Newtonian) gravity, \vec{g} reduces to the usual gravitational acceleration and \vec{H} disappears. For systems with weak gravity and low velocity ($v \ll c$), the Einstein equations for \vec{g} and \vec{H} become almost identical to Maxwell's equations, and the geodesic equation for the motion of a freely falling particle becomes equivalent to the Lorentz force law:

$$\begin{aligned} \nabla \cdot \vec{g} &= -4\pi G\rho, \quad \nabla \cdot \vec{H} = 0, \quad \nabla \times \vec{g} = 0, \quad \nabla \times \vec{H} = 4[-4\pi G\rho\vec{v}/c + 1/c(\partial\vec{g}/\partial t)] \\ (d\vec{v}/dt) &= \vec{g} + (\vec{v}/c) \times \vec{H} \end{aligned} \tag{1}$$

See, e.g., Braginsky et al. (1977), where the equations are written to second order in the source velocity rather than just first order as here. \vec{g} and \vec{H} can be derived from scalar and vector potentials: $\vec{g} = -\nabla\phi$, $\vec{H} = \nabla \times \vec{\gamma}$. (ϕ and $\vec{\gamma}$ can be related to the time-time and time-space components of the metric tensor by $\phi = \frac{1}{2}(g_{00} + c^2)\gamma_j = cg_{0j}$.) Note the minus signs in the "Poisson" and "Ampere" equations expressing the attractive character of gravity and the extra factor of 4 in the equation for $\nabla \times \vec{H}$ which is related to the spin-2 nature of the gravitational field in contrast to the spin-1 character of the electromagnetic field.

From our electromagnetic experience, we can infer immediately that, at a distance $r \gg m$ where the space is nearly flat, a spinning hole will be surrounded by a radial gravitoelectric field $\vec{g} = -(GM/r^2)\hat{r}$, and a dipolar gravitomagnetic field $\vec{H} = (2G/c)[\vec{S} - 3(\vec{S} \cdot \hat{r})\hat{r}]/r^3$. The gravitoelectric monopole moment is the hole mass M . Using the gyromagnetic ratio familiar from classical mechanics, the gravitomagnetic dipole moment is $\frac{1}{2}G\vec{S}/c$ where \vec{S} is the hole's spin angular momentum. The gravitomagnetic field is then -4 times the usual dipolar field. Note that \vec{g} and \vec{H} communicate information to a distant observer about the two parameters M and \vec{S} which characterize completely an uncharged spinning black hole.

3. RELATIVISTIC PRECESSION AND JET ALIGNMENT

A. Gravitomagnetic Precession and the Bardeen-Petterson Effect

Consider a ring of gas in orbit about a hole with spin angular momentum \vec{S} . Let the specific orbital angular momentum $\vec{\ell}$ be slightly misaligned with \vec{S} . A torque $\vec{r} \times (\vec{v} \times \vec{H})/c$ per unit mass will act upon the ring. Using the formula for the dipolar gravitomagnetic field and

averaging over azimuth gives a mean torque per unit mass of $2(\vec{S} \times \vec{\ell})/r^3$. The ring will therefore undergo gravitomagnetic or Lens-Thirring precession with angular frequency $2\vec{S}/r^3$. Adjacent rings precess at different rates, and friction between them drives the inner rings into the equatorial plane (Bardeen and Petterson 1975; Rees 1978). The disk should become equatorial when the inflow time becomes comparable to the precession time; i.e. at a radius $r_{BP} \sim (S/S_{max})^{2/3} (v_r/v_K)^{-2/3} m$ where $S_{max} = m^2 c^3/G$ is the maximum spin angular momentum that the hole can have, v_r is the inward drift speed and v_K is the Keplerian velocity. According to this mechanism, any model that produces a jet near a black hole must align it parallel to \vec{S} regardless of the initial orbital angular momentum of the accreted gas. A corollary is that the hole can change its spin direction substantially only when it has increased its mass by a fractional amount $\sim (S/S_{max})(m/r_{BP})^{1/2}$ (Rees 1978).

B. Spin-Orbit Coupling, Space-Curvature Precession and Jet Precession

Consider a gyroscope moving in a circular orbit with velocity \vec{v} through the gravitoelectric field \vec{g} of a massive body. This motion induces a gravitomagnetic field $\vec{H}' = -(\vec{v}/c) \times \vec{g}$ in the frame of the gyroscope, which in turn drives it to precess with $\vec{\Omega}_{SO} = -\vec{H}'/2c = (m/2r) \vec{\omega}$ where $\vec{\omega}$ is the orbital angular velocity. This precession is the analog of electromagnetic "spin-orbit coupling" in a hydrogen atom. By contrast with the hydrogen atom there is no "Thomas precession" to be added to $\vec{\Omega}_{SO}$ because the gyroscope is "freely falling" rather than accelerated relative to local inertial frames. There is however an additional contribution to the acceleration, with no electromagnetic analog, attributable to the space curvature. In Fig. 2a, we see an embedding diagram for the curved space around a massive body. The gyroscope is only sensitive to the local geometry so we may replace the trumpet horn by a cone tangent at the radius of the orbit. We can imagine constructing this cone from a flat piece of paper on which the gyroscope maintains a fixed direction (Fig. 2b). In one orbit the gyroscope moves from A to A' and therefore precesses through an angle $\psi = 2\pi[1 - \sqrt{1 - 2m/r}] \approx 2\pi m/r$. The

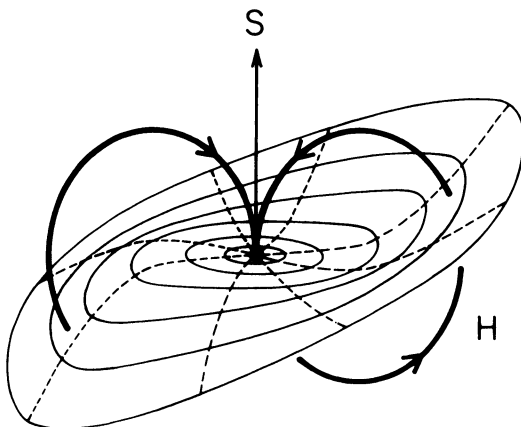


Fig. 1. The Bardeen-Petterson Effect

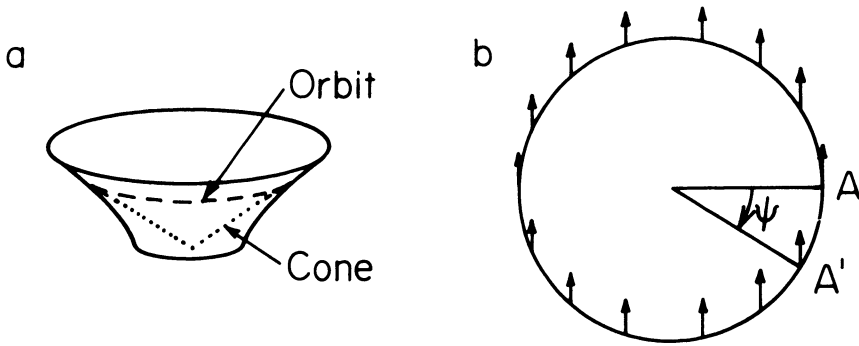


Fig. 2. Precession of a gyroscope induced by space-curvature.

space curvature precession rate is then seen to be $\vec{\Omega}_{SC} = 2\vec{\Omega}_{S0}$. The two components of precession together bear the name geodetic precession and their total precession rate is $\vec{\Omega}_{geo} = 3/2(m/r)\vec{\omega}$.

For a gyroscope the gravitomagnetic precession is smaller than the geodetic precession. However for a disk or ring, only the gravitomagnetic precession leads to an observable effect.

If our gyroscope is a spinning black hole in an orbit with radius a about another black hole of mass M , then the spin and any jet aligned with it will precess geodetically with period $2\pi/\Omega_{geo} \sim 10^4 (a/0.01 \text{ pc})^{5/2} (M/10^8 M_{\odot})^{-3/2} \text{ yr}$. Geodetic precession may be responsible for the inversion symmetric radio sources (Begelman, Blandford and Rees 1980).

4. ELECTRODYNAMICAL EXTRACTION OF ROTATIONAL ENERGY

A. Energy Extraction from a Black Hole

The mass of a spinning black hole (as measured by its gravitational force on a distant test particle) need not increase with time. Up to 29 per cent of the mass may be extracted as usable energy by classical processes. We may think of this removable mass as "gravitomagnetic" or "rotational" energy stored outside but near the hole's horizon. A particle on a suitable orbit outside a spinning hole can exert slow-down torques on the hole (Penrose 1969). However, if this energy is to be usable, this particle must be coupled dynamically to the external world. One way of achieving this is to replace the particle by an electromagnetic field.

B. Electromagnetic Energy Extraction

Consider a rotating black hole surrounded by a magnetized accretion disk (Fig. 3). Although the magnetic field threading the disk may be rather chaotic, the field lines threading the hole and unconstrained by gas pressure should be rather well ordered: if made chaotic, they will

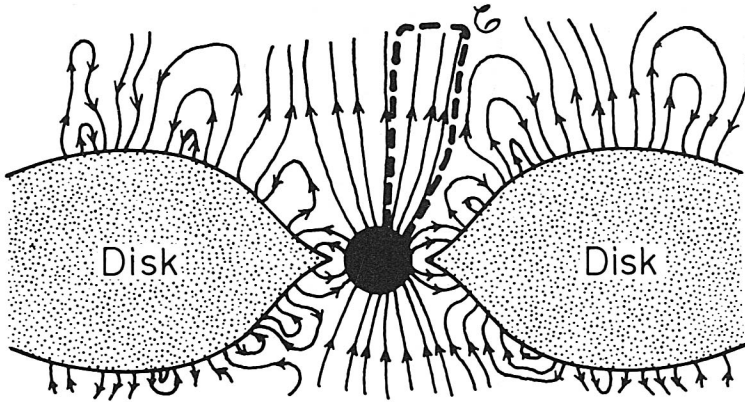


Fig. 3. Electromagnetic Extraction of Rotational Energy of a Black Hole

rearrange themselves on a timescale $\sim m/c$ so as to minimize the electromagnetic energy near the horizon while holding fixed the flux through the horizon (Macdonald and Thorne 1981). These magnetic field lines are held on the hole by Maxwell pressure from surrounding field lines, which in turn are anchored in the disk by currents. If the disk were suddenly removed, the field would convert itself into radiation and fly away in a time $\sim m/c$.

Consider a closed curve \mathcal{C} at rest in the curved space. The general relativistic version of Faraday's law of induction gives for the EMF around the curve

$$\oint_{\mathcal{C}} \vec{E} \cdot d\vec{\ell} = - \frac{1}{c} \frac{d}{dt} \Phi - \oint_{\mathcal{C}} (\vec{\gamma} \times \vec{B}) \cdot d\vec{\ell} \tag{3}$$

where $\vec{\gamma}$ is the gravitomagnetic potential and Φ is the magnetic flux bounded by \mathcal{C} (Macdonald and Thorne 1981). The fields \vec{E} and \vec{B} should be regarded as defined by the Lorentz force law for a charge q , $d\vec{p}/dt = q(\vec{E} + \vec{v} \times \vec{B}/c)$, where t is the universal time of §2.

Now choose for the curve \mathcal{C} the dashed line in Fig. 4, which connects the horizon of the hole with the "acceleration region" where charged particles can cross the field lines. This curve is a path along which current can flow. (Current which enters the horizon can be regarded as flowing along the horizon until it exits again; see Znajek 1978.) If the magnetic field is stationary and axisymmetric, the EMF is produced

solely by the gravitomagnetic potential and is of magnitude $EMF \sim (S/S_{\max}) B_m \sim 10^{20} (S/S_{\max}) (B/10^4 \text{ G}) (M/10^8 M_{\odot}) \text{ V}$. In effect the B field becomes a DC transmission line for transporting power in the form of a Poynting flux from the hole to the acceleration region. (It is, however, necessary that charges of both sign be supplied to the magnetosphere. This can be effected by cross field diffusion or more probably by a discharge process involving the production of electrons and positrons by γ -rays.) The field lines will be approximately equipotential, but both the hole and the acceleration region will possess resistances $R \sim 30\Omega$ (Znajek 1978; Damour 1978). Since the load and "battery" are impedance matched, the power transmitted to the load is

$$L \sim (EMF)^2/4R \sim 10^{45} (S/S_{\max})^2 (B/10^4 \text{ G})^2 (M/10^8 M_{\odot})^2 \text{ ergs}^{-1} \quad (4)$$

This power will probably be deposited ultimately in a flux of high energy particles. This constitutes a possible origin for the relativistic plasma responsible for radio jets and for other non-thermal activity associated with active galactic nuclei. Final collimation of the jets would probably occur at large distances from the hole either by external pressure or toroidal magnetic fields.

This mechanism of energy extraction by magnetic torques can also operate in an accretion disk (e.g. Blandford and Payne 1981). However, here the electromagnetic energy extraction occurs by a slightly different mechanism. Far enough from the hole, $\vec{\gamma}$ is negligible and the EMF around a stationary circuit is zero. However the gas in the disk should have a very low resistivity and thus the electric field in the instantaneous rest frame of the gas is zero. In an inertial frame there will be a potential difference $\sim \int (\vec{v}_K \times \vec{B}) \cdot \vec{d}\ell$ between field lines. This same potential difference is available in the magnetosphere for the heating and bulk acceleration of plasma. Note that a magnetic torque extracts angular momentum as well as energy from the disk, and under extreme conditions can replace the viscous torque invoked in standard accretion disk theory.

5. ACCRETION TORI

As described by Dr. Rees in these proceedings, the accretion disk surrounding a black hole may thicken close to the hole either as a result of radiation pressure when the accretion rate is supercritical, or as a result of ion pressure when it is subcritical. The gas flow is then usefully idealized as a torus with a black hole at its center. The shape of this torus is governed by the balance between gravitational and centrifugal forces, just like the shape of a whirlpool.

At the inner, equatorial edge of the disk, if Newtonian gravity were valid, force balance would say $\ell^2/r_I^3 = GM/r_I^2 + (\text{inward pressure force})$; or $r_I \leq \ell^2/GM$ where ℓ is the specific angular momentum and r_I is the inner radius. Because centrifugal forces always overwhelm gravity at $r < r_I$, matter can never fall out of the Newtonian disk and into the Newtonian "black hole" at $r = 0$.

General relativity changes this in a crucial way: the gravitational force becomes infinite at a finite radius, the black hole's horizon. As a result there is a region of non-stationarity close to the hole, where centrifugal forces can no longer counterbalance gravity; and if the disk encroaches upon this region, matter will fall out of it and into the hole.

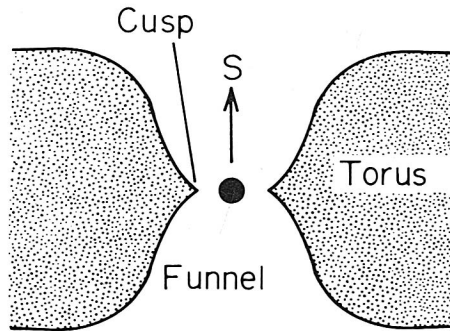


Fig. 4. Accretion torus around black hole and funnel.

This effect of relativity can be mocked up in Newtonian theory by replacing the Newtonian gravitational potential by $\phi = -mc^2/(r - 2m)$ (Paczynski and Wiita 1980). Consider the centrifugal plus gravitational energy, $\epsilon = \ell^2/2r^2 + \phi$, for motion of a gas particle in the equatorial plane. This "effective potential for radial motion" has a minimum only if $\ell > 3\sqrt{3/2} mc$. For larger values of ℓ , gas will trickle out of the disk and into the hole if the minimum radius of the torus is at the maximum of the potential. This maximum corresponds to a cusp in the surface of the torus. However, if $\ell > 4mc$, the energy at the cusp becomes positive and gas can escape to infinity. There is a region of non-stationarity ($2m < r < 4m$ in the equatorial plane) within which gas must either fall into the hole or escape to infinity. For a large torus of negligible binding energy and constant specific angular momentum, force balance guarantees that $\epsilon = \ell^2/2\tilde{\omega}^2 + \phi = \text{constant} \approx 0$ over the torus' surface, which implies that the surface has the shape of a paraboloid of revolution out of radii $\tilde{\omega} \gg m$: $z = (\tilde{\omega} - 4m)(\tilde{\omega} + 4m)/8m$ (Fig. 4). The resulting funnel may collimate any wind that blows off the inner part of the disk (e.g. Lynden-Bell 1978). However, for a radiation-driven wind, radiation drag makes it hard to achieve the high terminal Lorentz factors demanded by observations of superluminal radio sources.

6. CONCLUSION

Black holes appear to be capable of accounting for the alignment, precession, energizing and collimation of radio jets by the mechanisms outlined above. It is quite possible that virtually all of the relativistic particle energy of most of the powerful radio sources that we see be derived electromagnetically from the hole's spin--either a spin

existing from birth or one built up in an early phase of high mass accretion. Such electromagnetic energy extraction would likely be accompanied by a subcritical mass accretion rate, as the modest optical outputs of powerful radio sources suggest. Supercritical accretion produces a high-luminosity, radiation-supported torus, which may explain optically brilliant but radio-quiet quasars in the case of high-mass holes associated with elliptical galaxies, and may explain Seyfert nuclei in the case of modest-mass holes in spiral galaxies.

Of course, many quite different mechanisms have been proposed for the explanation of quasars and radio sources. It is not unreasonable to expect that this diversity of theory finds a counterpart in a heterogeneity of the real objects. We are certainly a long way from a direct proof of the presence of a black hole in any extragalactic radio source, and we must once again turn to our observational colleagues for further enlightenment.

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