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ON A RANDOM SEARCH TREE: ASYMPTOTIC ENUMERATION OF VERTICES BY DISTANCE FROM LEAVES – CORRIGENDUM

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Abstract

We correct typographical errors in our original paper.

Keywords: Root; rank; distance; distribution

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In our paper 'On a random search tree: asymptotic enumeration of vertices by distance from leaves' (*Adv. Appl. Prob.* **49** (2017), 850–876), there are a number of typographical errors in mathematical expressions. These errors typically involve calligraphic letters. Below we have listed the correct expressions, with references to the places where the errors occurred in the original paper.

1. Page 854, displayed equation (2.4):

$$\mathcal{H}_n(\rho) = h_n(\rho) + \frac{1}{n}\sum_{j=0}^{n-1}\big(\mathcal{H}_j(\rho) + \mathcal{H}_{n-1-j}(\rho)\big), \qquad n > 1.$$

Same page, second displayed equation from the bottom:

$$\mathcal{M}_j(x) := \sum_{n\ge 1} \mathbb{E}[\mathcal{X}_{n,j}]\, x^n = \frac{2^j}{j!}\left(\log \frac{1}{1-x}\right)^j, \qquad j > 0.$$

2. Page 857, proof of Lemma 2.3, first displayed equation:

$$\lim_{n\to\infty} n^{-1}\mathcal{H}_n(\rho) = \lim_{n\to\infty} n^{-1}\mathbb{E}\left[\sum_{v\in[n]} \rho^{R(v)}\right] = \lim_{n\to\infty} n^{-1} \sum_{k\le n-1} \rho^k\, \mathbb{E}_{n,k}.$$

3. Page 860, first displayed equation in proof of Lemma 2.4:

$$F(x, y) = \sum_{n\ge 0} y^n \mathbb{E}\left[x^{\mathcal{V}_{n,k}}\right], \qquad \mathcal{V}_{0,k} := 0.$$

Displayed statement (2.15), last inequality: > 0.

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Next displayed equation, last expectation: $\mathbb{E}[x^{\mathcal{V}_{n-1-j,k}}]$.
Next line: $\mathcal{V}_{n,k}=0$ for $n\le k$, $\mathcal{V}_{k+1,k}=1$ (resp. 0) with probability $2^k/(k+1)!$ (resp. $1-2^k/(k+1)!$), and for $n>k+1$,

$$\mathbb{E}[x^{\mathcal{V}_{n,k}}]=\frac{1}{n}\sum_{j=0}^{n-1}\mathbb{E}[x^{\mathcal{V}_{j,k}}]\cdot\mathbb{E}[x^{\mathcal{V}_{n-1-j,k}}].$$

4. Page 868, bottom line: $\mathcal{B}_k(x)=\mathcal{B}_{k-1}(x)-\mathcal{B}_{>k}(x)$.
5. Page 869, Lemma 4.1 should read as follows:

Lemma 4.1 *For all nonnegative integers k, the equalities*

$$\frac{d}{dx}\mathcal{A}_k(x)=\frac{2}{1-x}\mathcal{A}_k(x)+\frac{d}{dx}\mathcal{B}_k(x),$$
$$\frac{d}{dx}\mathcal{B}_{>k}(x)=2\left(\frac{1}{1-x}-B_{\le k-1}(x)\right)\mathcal{B}_{>k-1}(x)$$

hold. Here $\{B_{\le t}(x)\}$ *is the sequence determined by the recurrence (3.3),* $B_{\le -1}(x):=0$, *and* $\mathcal{B}_{>-1}(x)=\mathcal{B}_{\ge 0}(x)$ *is the generating function of the expected numbers of leaves, that is,*

$$\mathcal{B}_{>-1}(x)=\frac{x-1}{3}+\frac{1}{3(1-x)^2}.$$

Consequently

$$f_k=2\int_0^1(1-x)\mathcal{B}_k(x)\,dx.$$

6. Page 870, second displayed equation from the top:

$$f_k=\lim_{x\uparrow 1}(1-x)^2\mathcal{A}_k(x)=\int_0^1(1-x)^2\frac{d}{dx}\mathcal{B}_k(x)\,dx=2\int_0^1(1-x)\mathcal{B}_k(x)\,dx.$$

Proof of Lemma 4.2, first displayed equation:

$$\begin{aligned}\mathbf{1}_{\{R(\text{root})=k\}}\mathcal{L}_n&=\mathbf{1}_{\{R(\text{root'})=k-1\}}\mathbf{1}_{\{R(\text{root}'')=k-1\}}\mathcal{L}_n\\&\quad+\mathbf{1}_{\{R(\text{root'})=k-1\}}\mathbf{1}_{\{R(\text{root}'')>k-1\}}\mathcal{L}_n+\mathbf{1}_{\{R(\text{root'})=k-1\}}\mathbf{1}_{\{R(\text{root}'')=k-1\}}\mathcal{L}_n\\&=\mathbf{1}_{\{R(\text{root'})=k-1\}}\mathbf{1}_{\{R(\text{root}'')=k-1\}}(\mathcal{L}'+\mathcal{L}'')\\&\quad+\mathbf{1}_{\{R(\text{root'})=k-1\}}\mathbf{1}_{\{R(\text{root}'')>k-1\}}\mathcal{L}'+\mathbf{1}_{\{R(\text{root'})>k-1\}}\mathbf{1}_{\{R(\text{root}'')=k-1\}}\mathcal{L}''.\end{aligned}$$

The next line should read as follows: '... to the expectation of $\mathbf{1}_{\{R(root)=k\}}\mathcal{L}_n$ conditional on the event "the vertex set of T' is a given set J of j elements from $[n]\setminus$ root" is....' The bottom displayed equation should read as follows:

$$\mathbb{E}[\mathbf{1}_{\{R(\text{root})=\text{k}\}}\mathcal{L}_n|J]=g_{j,k-1}p_{n-1-j,>k-1}+g_{n-1-j,>k-1}p_{j,\ge k-1}.$$

7. Page 871, first displayed equation:

$$g_{n,k}=\mathbb{E}[\mathbf{1}_{\{R(\text{root})=k\}}\mathcal{L}_n]=\frac{1}{n}\sum_{j=0}^{n-1}g_{j,k-1}p_{n-1-j,\ge k-1},$$

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The first line of the proof of Theorem 4.1 should read as follows: ‘Consider $\mathcal{L}_{n.k}/V_{n,k}$, for instance.’

The third display in the proof of Theorem 4.1 should read as follows:

$$\mathbb{E}_3 = \frac{1}{n(c_k + O(\varepsilon))}\mathbb{E}[\mathcal{L}_{n,k}\mathbf{1}_{\{V_{n,k}\geq 0.03an\}}\mathbf{1}_{\{V_{n,k}/n-c_k|\leq\varepsilon\}}]\ldots$$

The last line of page 871 should read as follows: ‘. . . Therefore, $\mathcal{L}_{n,k}/V_{n,k} \to f_k/c_k$. . ..’

Competing interests

There were no competing interests to declare which arose during the preparation or publication process for this article.

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