BINARY EVOLUTION IN STELLAR SYSTEMS*

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Abstract. Aarseth has shown by means of n-body calculations that, in star systems with a range of particle masses, the most massive stars quickly form a binary which soon takes up a large fraction of the total binding energy of the cluster. Similar effects appear in other kinds of physical systems as well; mesic atoms behave in much the same way. The phase volumes of two otherwise equivalent stellar systems, each dominated by a tightly bound binary, favor exchange to incorporate the more massive star in the binary by a factor equal to the cube of the ratio of masses.

Self-gravitating n-body systems with a few hundred particles of different masses have been shown to form a tight binary rather quickly, with most of the total binding energy in the binary (Aarseth, 1971). The binary is usually formed of the most massive particles. This is similar to the state that yields the greatest phase volume in the microcanonical ensemble (Miller, 1973, 1974). It is remarkable, however, that the two most massive stars find their way into the binary so rapidly, and that the state dominated by a single binary is reached so quickly.

The situation is reminiscent of atomic processes when negative μ-mesons are stopped in condensed matter. The muons rapidly find their way to the ground state of an atom formed with the heaviest nucleus or with the nucleus having the greatest charge. A particularly interesting case occurs when muons are stopped in liquid hydrogen. The muon finds its way to a deuteron (even in hydrogen with 10^{-4} atomic contamination by deuterium), where it can cause a fusion $p + d \rightarrow He^3$. The muon is forcefully ejected after the fusion and is ready to start the process again. Several spectacular bubble chamber photographs show two or three fusion reactions during the 2.2 μs mean lifetime of a muon. The only difference between $(p\mu d)^+$ and $(ped)^+$ ions is the closer spacing caused by the much greater mass of the muon (207 electron masses); fusion can occur because the HD molecule is bound by the muon with a very small internuclear distance. A discussion of this process, together with some photographs of triple fusion reactions is given by Doede (1964).

There are essential differences between the muon-hydrogen system and stellar systems. While the μ-mesic atom is a quantum mechanical system, and, once in the ground state, can only become more tightly bound by transferring to a more massive or a higher-Z nucleus, a binary star can become more tightly bound without exchanging members. Still the analogy is suggestive, and illustrates how rapidly and effectively systems can make transitions toward states that afford a larger phase volume. In particular, the analogy suggests that a significantly larger phase volume must be available to states in which more massive particles are bound into a binary.

In a stellar system, the phase volume available within the energy interval from E to

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$E+\mathrm{d}E$, with different particle masses, is the sum of contributions due to each particle-pair (Miller, 1973). It is

$$\sigma_{\beta\gamma}(E)=D\left(G\sum_{\alpha=1}^{N} m_\alpha\right)^{3N-6}[Gm_\beta m_\gamma]^3\left\{\frac{\prod_{\alpha=1}^{N} m_\alpha}{\sum_{\alpha=1}^{N} m_\alpha}\right\}^{3/2}(-E)^{(1-3N)/2}, \tag{1}$$

for the pair of particles with indices β, γ, where D is a (large) numerical constant. The curly bracket comes from the integral over the $3N$-dimensional momentum space. The remaining coefficients come from the integral over the $3N$-dimensional configuration space, which is kept finite by placing a cutoff radius far beyond the present limits of the star cluster. The cutoff radius appears as a constant multiplying the virial theorem cluster dimension; the constant is absorbed into D. The configuration volume is the cartesian product of the configuration volume accessible to the star-pair (β, γ) as a binary (square bracket) and of the configuration volume available to the rest of the particles independently (round bracket). The total phase volume, $\sigma(E)$, is

$$\sigma(E)=\sum_{\text{pairs}} \mathbf{s}_{\beta\gamma}(E), \tag{2}$$

and is dominated by states in which two particles are bound into a binary with all the binding energy of the cluster while the remaining particles have low velocities at large distance. The total angular momentum can be set to the required value with a small velocity assigned to particles at great distances, leaving the entire energy in the binary.

The phase volume available with a certain pair of particles is proportional to $(m_\beta m_\gamma)^3$, if all other features of the system are the same. In particular, the ratio of phase volumes accessible to otherwise equivalent systems in which the binary is made out of particles α, β to that in which it is made out of α, γ is $(m_\beta/m_\gamma)^3$. This is the increase of phase volume accessible to the system by substitution of particle β for particle γ in a binary with particle α. It is independent of the number of particles in the system. The cutoff distance cancels in the ratio.

In Aarseth's (1971) 250-body Case I, the phase volume with the two most massive particles (mass 25) together is 125 times as great as that with one massive particle and one particle from the second most massive set (mass 5). The two most massive particles found each other to form a binary after only two crossing times and quickly absorbed about 80% of the energy in the cluster.

Aarseth's (1971) 500-body Case V has a mass spectrum such that the ratio of phase volumes does not favor the two most massive particles nearly as strongly. Relative to the phase volume accessible to a system dominated by a binary formed of particles 1 and 2 (particles are numbered in the order of decreasing mass), a system dominated by the pair 1-3 has 0.84, 1-4 has 0.72, and 1-5 has 0.62. The pair 2-3 has 0.70, 2-4 has 0.60, and so on. Experimentally, after a sequence of particle exchanges, particles 1-3

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formed a tight binary at about 12 crossing times, and soon about half the cluster energy was in that binary.

The general picture afforded by the comparisons of systems accords with Aarseth's results. Typically, systems with all masses equal do not form tight binaries nearly as rapidly – indeed, it is difficult to argue that binary formation plays much of a role in such systems. We stress that phase-volume considerations, while taking the $1/r^2$-force law explicitly into account, may indicate a preponderance of states that can only be reached after very long times when actual phase-space trajectories are taken into account.

Experimental results from n-body calculations dramatically confirm the picture provided by accessible phase-volumes in the microcanonical ensemble – that the final state of an n-body system is a tight binary with all other particles removed to infinity. It has long been suspected that this is the only stable final state, but the microcanonical ensemble arguments strengthen the conjecture substantially. The remarkable feature of the experimental results is the rapidity with which the system tends toward that state.

References

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DISCUSSION

Wielen: I have studied recently the Ursa Major star cluster. It is a gravitationally bound, sparse open cluster, which has probably lost most of its members. The binary ζ UMa AB = Mizar may well be identified with the central binary or triple system (together with 80 UMa = Alcor) which is formed dynamically in n-body simulations.