Reduction of Astrometric Binaries for Low Mass Companions

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ABSTRACT. Described is the plan of a study just underway to assess the progress so far in the astrometric search for low mass companions. The range of periods which would be discernible, based on the observations' distribution in time, and the range in amplitude of the detectable orbits, based on careful analysis of the accuracy of the observations, can be used to determine the confidence level of any perturbations as well as to specify what possible companions and orbits for a given star have been ruled out. This study, when completed for several nearby stars, should yield the first comprehensive progress report for the search for astrometric companions.

This report summarizes the plan of a study just underway to assess the progress to date in the search for unseen companions of nearby stars. With the amount of and accuracy of data currently available for study in most cases, it will be mostly a determination of the search for stellar companions. More data with higher accuracy and consistent coverage are needed to detect and/or rule out planets.

It should be noted at the outset that the determination of progress in the search is much more than simply counting the number of possible companions already reported. A summary of the nearby stars by van de Kamp (1975b) and updated by Lippincott (1978) already includes that information. But these summaries do not include the reported errors of any of the quantities listed, i.e. position, proper motion, and parallax, as well as those of any orbital parameters. Without knowing more about the accuracy of the study, it is impossible to tell whether a given star with no companions yet reported really has none or that the observations are too few, too inaccurate, or too badly distributed in time to reveal companions. However from the accuracy of the observations and their distribution in time, it should be possible to estimate the range of periods and semi-major axes which were detectable for a given star. Then masses in the orbits determined from the latter parameters have been detected or can be ruled out. As other studies are made, a sort of "checklist" can be made, marking the progress in the search for companions. Also, as this information is compiled for several stars, results for stars can be compared and statistically analyzed for quantities such as the occurrence rate of binaries of various mass ratios.

One way of marking the progress in the search for companions is to do it graphically. Quantitatively, orbital motion is represented by

$$\frac{(A+a)^3}{\pi^3} = P^2(M+m) \qquad (1)$$

and

$$MA = ma \qquad (2)$$

where M and m are the masses of the primary and secondary, respectively, expressed in solar masses. A and a are the semi-major axes of the primary and secondary orbits in arcsec; P is the period in years; and π is the star's parallax in arcsec. In the study of an individual star, π can be measured astrometrically and M can be estimated from the mass-luminosity relationship. If a perturbation is evident, A and P are the observable quantities. Figure 1a illustrates the case for Barnard's

Star, assuming circular orbits and no blending from the companion. The straight lines represent values of m for possible combinations of A and P. Figure 1b is similar to 1a but for Lalande 21185 (+36°2147). The differences between 1a and 1b are from the values of π and M used; for consideration of any unseen secondary mass, stellar or planetary, a similar figure would be necessary to illustrate the case for each star. Some generalization is possible if $m \ll M$. Then the same plot applies to all stars with the same value of the detection index

$$I = \pi M^{-2/3} \quad . \qquad (3)$$

The detection index was defined by Gatewood et al. (1980) and is used to calculate the ease with which planets can be found around a specific star. Figure 1a illustrates a sort of "best case" in planetary searches. Figure 1b is a less sensitive case because of the larger primary mass and its greater distance from the sun.

If the ranges of detectable A's and P's for a study of a given star are known, one can tell with a glance at Figure 1 which masses in which orbits would have been found if those companions indeed existed. For example a study of Barnard's Star with a minimum A value of 0.02 arcsec and detectable periods between 2 and 30 years should see the perturbation caused by any mass greater than $0.005 \mathfrak{M}_\odot$ within those periods and any mass greater than $0.002 \mathfrak{M}_\odot$ with periods between 10 and 30 years.

The very short period orbits are more favorable for discovery spectroscopically and the cases for the same two stars discussed previously are shown in Figure 2. Assuming circular orbits inclined along the line of sight, the diagonal lines represent values of m for various combinations of P and K, the radial velocity variation because of orbital motion. Of course the plots here are dependent only on M, since spectroscopic accuracy is not limited by a star's distance. Also, for shorter periods smaller masses are detectable for the same K, the inverse of the astrometric technique, so that the methods complement each other.

The plan of the present study is to determine what progress has been made in searching each of several nearby stars for companions, based on analysis of published data or data from the original sources whenever possible. Fourier analysis would be applied first to search for any periodicities. No new discoveries are expected, i.e. the authors of the original studies would likely have found any variations, but Fourier transforms are powerful tools and may suggest periodicities for further searching. The power spectral window would also be calculated to reveal any periods to which the data are aliased by their distribution in time. For example most sets of observations have a one year periodicity in the data spacing because observations are made seasonally. The one year period would be evident in the Fourier transform. The spectral window would also show a one year periodicity, warning of interference from the data's distribution in time at that period (see Deeming, 1975).

The spectral window will show the periods to which a data set is aliased. Otherwise, estimating the range of detectable periods from the distribution of data depends on the individual star since the spacing of observations within data sets varies greatly from star to star. The most crude estimate is that the longest noticeable period would be somewhat less than the length of the observations and the shortest would be somewhat longer that the interval between most observations. These somewhat vague guidelines will be further defined following computer simulations to see what effects are caused by large gaps in or varying degrees of unevenness in the data spacing.

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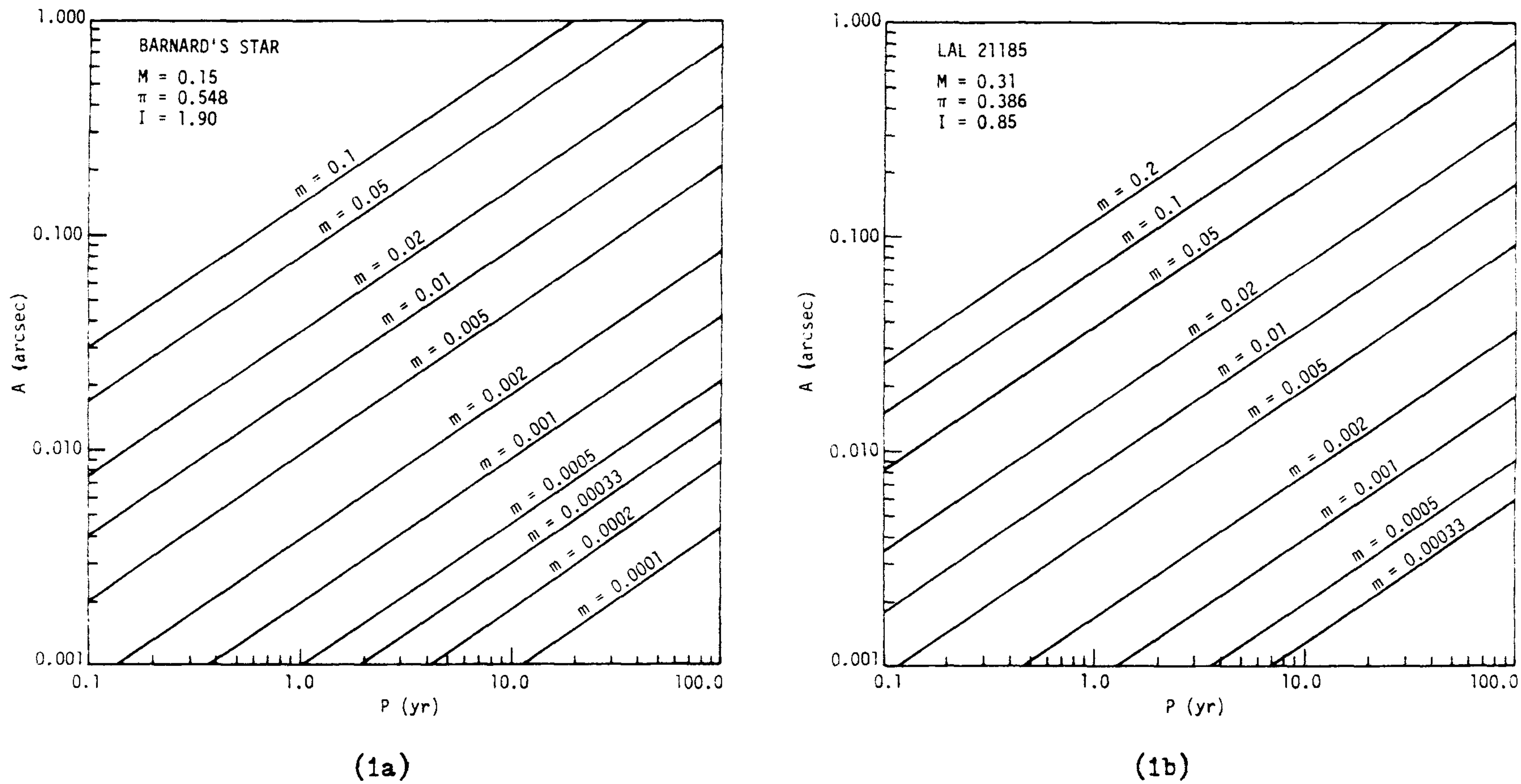

Figure 1. The relationship between the period P, the semi-major axis A, and the secondary mass m for astrometric orbits around Barnard's Star and Lalande 21185. Dark companions, i.e. no significant blending, and circular orbits are assumed.

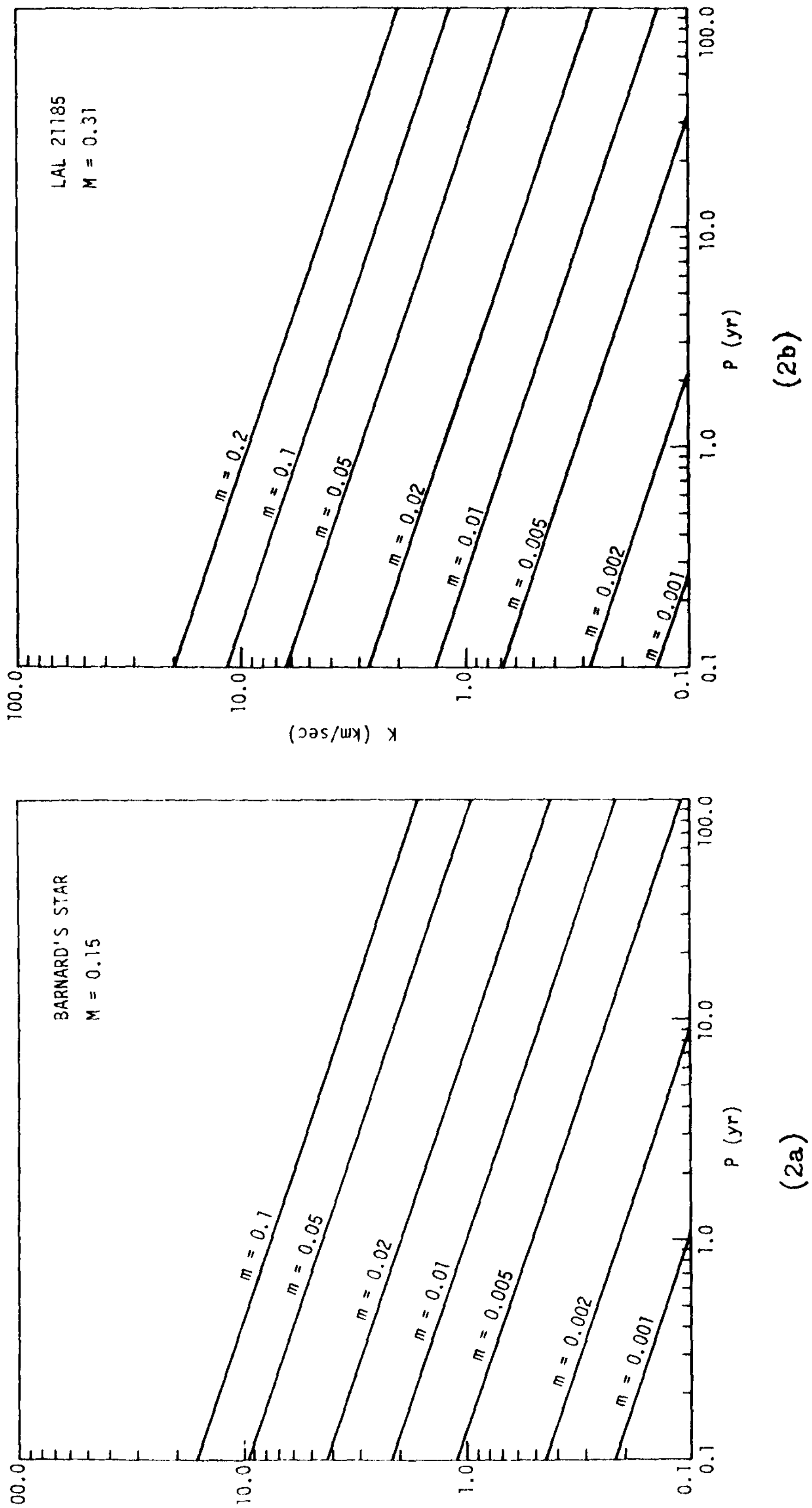

Figure 2. The relationship between the period P, the K velocity, and the secondary mass m for spectroscopic orbits around Barnard's Star and Lalande 21185. Circular orbits inclined along the line of sight are assumed.

In estimating the minimum A detectable in astrometric studies, consider a circular orbit in the plane of the sky. The orbital motion would introduce a scatter with the standard deviation

$$\sigma_{orb} = 0.707 \text{ A} \tag{4}$$

into the observations along either axis. Combined with the observational error σ_{obs} , the total error observed σ_t would be

$$\sigma_t^2 = \sigma_{obs}^2 + \sigma_{orb}^2 \quad . \tag{5}$$

If σ_{obs} were used to estimate the minimum value of A

$$\sigma_t^2 = \sigma_{obs}^2 + (0.7\ \sigma_{obs})^2 \tag{6}$$

$$\sigma_t = 1.2\ \sigma_{obs} \quad . \tag{7}$$

Thus to assume that the minimum detectable A is equal to the observed error of the observations without a perturbation assumes that the error is well enough known to recognize an increase of 20% over the expected value. More conservatively, an increase of 50% in the scatter of the observations would be measured if the minimum A were assumed to be 1.6 σ_{obs}.

If the orbit were non-circular, the observable variations would of course be affected. However, even an eccentricity of 0.5 would reduce the semi-minor axis only 13%. Heintz (1978) has stated that double star orbits show some preference for smaller eccentricities, but have values of e which average about 0.5. Su-Shu Huang (1973) has suggested that planetary orbits in general will have small eccentricities and those in our system confirm this estimate. The simplest approach to allow for eccentric orbits is to make calculations for cicular orbits and then state what limitations apply to the results if the orbits were eccentric.

The final consideration in determining the observable range of A is the orientation of the orbit. If the orbit is highly inclined to the plane of the sky, we would observe oscillatory motion along a straight line which in the worst case would be oriented 45° from either of the axes of observation and have a maximum displacement of only 0.7 A. In the case of a non-circular orbit which presents only its semi-minor axis, this would be further reduced; for e=0.5 the displacement would be 0.6 A. In that case the minimum detectable A would be 2.7 σ_{obs} to observe a 50% increase in the scatter of the observations. Again, computer simulations will be run to confirm that these limits are practical.

Spectroscopic analysis has similar limitations. The observed scatter because of orbital motion is 0.7 K for circular orbits and will be affected similarly to the astrometric orbits by the orientation, inclination, and eccentricity of the orbits.

One other factor not mentioned yet but which must be taken into account is the possibility of systematic error in the observations. Simply ignoring them leads at best to a false confidence in the data, ruling out companion masses which could be masked by the systematic error. In some cases systematic errors may actually mimic a perturbation, as was the case with the first planet reported around Barnard's Star (van de Kamp, 1975a). Each set of observations must be analyzed for any systematic errors before it is used to determine the presence or absence of any unseen companions.

This is the plan of the study which will of course concentrate on the nearby stars. It should produce a summary of the cases and whatever statistics can be wrung from the results. It hopefully will be updated with each new study and will act as a progress report in our efforts to learn more about our nearest stellar neighbors.

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