

## TWISTED TOPOLOGICAL GRAPH ALGEBRAS

HUI LI

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The field of  $C^*$ -algebras has been influential because of its close relationship to mathematical physics and geometry and its deep classification programme. Graph algebras (see [8, 9]) which are based on graph theory provide a large class of examples of  $C^*$ -algebras which are classifiable. The advantages of graph algebras are that the structure of a graph  $C^*$ -algebra, including the ideal structure and the  $K$ -theory, can be read off directly from the underlying graph.

Many authors have studied various generalisations of graph algebras. On the one hand, Kumjian and Pask in [7] constructed  $k$ -graph algebras which are the higher-dimensional version of graph algebras. On the other hand, Deaconu in [1] investigated topological graph algebras which are the continuous version of graph algebras associated with single local homeomorphisms of second countable locally compact Hausdorff spaces. Katsura gave a complete concept of topological graph algebras in [3–6].

Twisted  $C^*$ -algebras, which incorporate suitable cohomological data into the existing construction of  $C^*$ -algebras, provide new examples of  $C^*$ -algebras frequently exhibiting strong connections with the twisting cohomology data. The survey paper [13] provides many interesting examples and gives a detailed motivation for studying twisted  $C^*$ -algebras. For graph algebras, it seems that there are two interesting types of twisted  $C^*$ -algebras. On the one hand, Kumjian *et al.* in [10, 11] studied twisted  $k$ -graph algebras. On the other hand, Deaconu *et al.* in [2] investigated twisted groupoid  $C^*$ -algebras obtained from single local homeomorphisms.

In my PhD thesis, I incorporate a 1-cocycle from the sheaf cohomology group into Katsura’s topological graph algebra and obtain a new  $C^*$ -algebra which is called the twisted topological graph algebra. I provide examples to demonstrate that the

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twisted topological graph algebras can differ from the untwisted ones. I investigate the twisted topological graph algebra by introducing a new type of representation called a covariant twisted Toeplitz representation and showing that the Cuntz–Pimsner algebra of the twisted graph correspondence is generated by a universal covariant twisted Toeplitz representation of the graph. I expand on Katsura’s ideas to prove fundamental results about the twisted topological graph algebra. In particular, I establish a version of the Cuntz–Krieger uniqueness theorem and study the ideal structure for the twisted topological graph algebra.

The construction of twisted topological graph algebras includes the twisted groupoid  $C^*$ -algebras in [2] and gives a complete twisted theory for Katsura’s topological graph algebras. Another remark is that when applying the constructions of twisted  $k$ -graph algebras or twisted topological graph algebras to one-dimensional discrete graphs, one will only gain ordinary graph algebras.

A succinct account of the construction of twisted topological graph algebras can be found in the paper [12].

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HUI LI, Research Center for Operator Algebras,  
 Department of Mathematics, East China Normal University (Minhang Campus),  
 500 Dongchuan Road, Minhang District, Shanghai 200241, China  
 e-mail: [hli@math.ecnu.edu.cn](mailto:hli@math.ecnu.edu.cn)