De Interpretatione: Commented Biography of Euclid*

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For Vanessa, these interpretations

Prologue

It is said that all philosophy is nothing other than a commentary on Plato.

Maybe.

But was not Plato himself a commentary on Parmenides, Heraclitus, the Pythagoreans, and the Sophists, not to mention Socrates?

And conversely, too, the *Commentary* on Aristotle composed by St Thomas was not the personal philosophy of Thomas Aquinas? Or then again, do Proclus' *Commentarii in primum Euclidis Elementorum librum* not embody a new and original neoplatonic philosophy of mathematics?

Popular tradition attributes my first statement to Bernard Shaw. The sarcasm of the observation is in fact typical of his personal style. As for the term 'commentary' – already held in contempt by the Age of Enlightenment – it allows one to perceive clearly the spirit of romanticism which survived into the century of scientism: in that tradition, the 'commentary' was set in opposition to the originality of the 'work of genius'. The 'Work' has a standing of its own. The status of the 'commentary' is more modest; its role, that of teaching, is limited; it has only to make the Work, already in existence, more comprehensible and accessible to a wider public. From this perspective, in comparison with the original creation of the Work, the commentary seems like a subordinate work of mediation. The pejorative resonance accompanying the expression is unmistakable.

'Work' and 'commentary': right at the top – the Work – and at a much lower level on the value scale, at the very bottom – the commentary.

Yet the singular originality, I would even say the genius of philosophy, consists in what is commentary par excellence.

Commentary

Text and Commentary: the Self and the Other – servitude volontaire and freedom

The commentary brings the Other face to face with the author's Self. The text is the master, the commentary its servant. The duty of the commentary is to serve. Its

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ideal is *servitude volontaire,* its ethos is unconditional loyalty to the One, the given, the Work.

Commentary splits up the One.

Dialogue inverts these roles. The locus of the self is now in the commentary. Just as in the dance of words after the exhortation, 'Change the figure': μεταβαλεῖν τὸ σχῆμα – the order which Plato, that great choreographer of words, gives to Socrates and to Alcibiades at the end of his great *Alcibiades* makes play with the whole geometric-choreographic amphibology of the term 'figure' – and Alcibiades, too, changes his own geometric shape and takes on the features of Socrates. At once the essence of things undergoes an inversion: the handsome Alcibiades, handsome but somewhat naïve, the 'Other' who attempts to understand Socrates in the dialogue, becomes the wise and ugly Socrates, while 'the Self', Socrates, author of the Work and lover of wisdom, assumes the traits of the Other, Alcibiades, who had been until then a lover of Socrates and commentator on his discourse.

Through the exchange of words and forms in the dialogue, the Work becomes the Other. The commentary examines the work, asks it questions, discusses it, expresses its doubts, raises objections, and says 'no' to it.

Just like Étienne de la Boétie's famous discourse, commentary is a *Discours contr'Un* – a discourse against all domination, monarchic or not, of the One.

Even in voluntary servitude, freedom remains the hidden power of the self, and no slave can resist it. The slave becomes aware of the reality of his self. His condition of voluntary dependence makes him become conscious of his own autonomy. He defines himself – οἶον νόμος ὤν ἑαυτῷ – as Aristotle once put it, he recognizes himself in his freedom.

Conscious of his autonomy, he finds himself in dialogue with the equally autonomous being of the Other. He asserts himself alongside the other, but also against it, as a being enjoying the same rights in all respects. His fundamental right is freedom of speech. His discourse is free. He is free to interpret. He is also free to say 'No!'

Plato comments on Parmenides . . .

In the *Sophist,* Plato comments on the ontology of the great Parmenides in order to refute it. In the guise of the Stranger from Elea, he proposes submitting to a 'mild degree of torture' the fundamental thesis of Eleatic ontology: 'You will never assign Being to Non-Being' to compel the proposition 'to confess that, in one way or another, Non-Being is', and that it is indisputably endowed with its own nature and essence. With his typical irony and self-mockery, Plato speaks of the 'parricide' perpetrated against the father of philosophy, Parmenides. The intellectual torture inflicted on the text has, however, well and truly forced the Eleatic doctrine to deny itself and commit intellectual suicide. And with the elegant and stylish perfidy that was not completely alien to him either, with an exquisite, and fully Attic, urbanity, Plato invited Parmenides himself to come onto the stage and to submit his own ontology to the rack of the dialectic of his *Parmenides.*

The great metaphysical drama of Being and Non-Being represented in *Parmenides* was the result of the hermeneutic work done by Plato through the interpretation of the most significant – and also the most dramatic – event which had taken place in the geometrical research of his time. It was the problem of irrationals. More precisely, the real problem was that of the penetration of the irrational into the closed world of *ratio*.

For the *so-called Pythagoreans* had already demonstrated the fundamental proposition of *Metreti*, namely that it is impossible to measure the diagonal of a square with the unit of length of the side. This means that the diagonal is indisputably a segment of line which can be constructed with absolute precision – it is, moreover, the reason why it has a defined size, defined in a univocal manner, but that it is impossible to assign a measurement to it. The diagonal has a *magnitude* but it is impossible to assign a *measure*. For if it had a measure, it could only be expressed by the oxymoron 'non-measurable measure'. The measure of a specified, determined magnitude is a binary relationship: it is the relationship or the *ratio*, the λόγος of the given magnitude or the magnitude of the distance to which convention has attributed the unit of measurement, '1', for example, '1 foot'. But if the measure of the diagonal exists, it can only be expressed by means of a λόγος which is a ἄλογος. Consequently, the very name of this incommensurable measure can only be articulated by means of a word which is itself inarticulable: it is the 'inexpressible diagonal', ἡ διάμετρος ἄρρητος, of which Plato speaks in his *Republic*. The relationship between the diagonal and the segment of the unit of measure, its λόγος, its *ratio*, should therefore necessarily be an 'irrational *ratio*' – which is a *contradictio in adjecto*, the irrationality itself, the geometric incarnation of pure madness.

This was perhaps the most significant event ever in mathematical thought, as the great theoretician of numbers Paul Erdös always told me whenever we met. Granted, but the 'significance' – not in Frege's sense, but in the sense of great value, of 'axiological significance' – specific to this event of measurement consisted in what it represented – to use a term of Valéry's – a 'great event for the mind'. And what happened in the diachronic unfolding of history was nothing other than the entry of the speculative mind into the professional field of mathematical technicalities. Through this it became evident that the *esprit de géometrie* was an integral and organic part of the universal spirit, and that it was only thanks to the speculative mind that specifically mathematical rationality was able to transcend the immanent limits of a natural calculating machine and develop freely.

And the discovery of this demented irrationality, its integration into the world of the ratio, the universe of a λόγος extended by the irrational, was in fact the most important realization, revolutionary in the true sense of the word, of the geometricians working in and around Plato's Academy.

This was, without any doubt, an absolute and sensational novelty, as much in geometrical as in philosophical terms. 'Do you agree, my dear Glaucon, that the head of state is irrational like the diagonal of a square?', asks Plato in the *Republic*. The significant political tenet, 'reason should govern the state', is here articulated in reverse, in the form of a play on the amphibology of the word ἄλογος – a current colloquial term which had meanwhile invaded the science of geometry, to be raised there to the status of an eminent and cold 'technical term'. So, in other words, a theorem of geometrical science interpreted as a political metaphor: in view of the sophisticated professionalism of the event, it was indubitably a surreal play on words, in the manner of Boris Vian, but by reason of its absolute novelty and its enigmatic character, it was also an intellectual provocation whose intention of snubbing the reader by its cryptic allusion to a difficult theorem, as new as it was sensational, should not escape us.

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For the object designated by the inexpressible term of irrational *ratio* cannot exist, and it is possible to demonstrate that, in the universe of the *ratio*, its ontic status is that of absolute Non-Being. The name of the inexpressible diagonal denotes a non-being. Consequently, if one claims that the diagonal is also a valid segment of line, which has not only magnitude but also measure, one accepts simultaneously this amazing and intrinsically absurd thing – αὕτη φύσις ἄτοπός τις, 'the instantaneous' – τὸ ἐξαίφνης – this a-temporal movement, which occurs outside all time, a movement which, suddenly, transforms – μεταβάλλει – the non-being into being: the measure of the inexpressible diagonal. It is the suicidal confession that Father Parmenides was forced to make in Plato's *Parmenides*, at the end of an intellectual torture inflicted by the latter on Being and non-Being with as much stylistic elegance as meticulous sadism. Plato's cruel commentary is a 'theorem', θεώρημα, a geometrical proposition, but in Plato's *Laws* as in Pasolini's *Teorema*, it is a show presented on the stage: it is the tragedy of Eleatic ontology.[1]

. . . Russell and Quine write commentaries on Plato

Plato's parricide commentary on Parmenides has come down to us by the name of 'Plato's philosophy', and Voltaire, who writes a commentary on it in his *Dictionnaire Philosophique*, has only one word for it: *galimatias*. Bertrand Russell also composed commentaries on Plato's mathematical philosophy and his conception of Being and Non-Being on several occasions – in order to take issue with it. It was also this that led to the development of his own 'analytical philosophy of language' which – as he says in his *History of Western Philosophy* – 'clears up two millennia of muddle-headedness about "existence", beginning with Plato's *Theaetetus*'. Even the most important contemporary representative of this philosophy, Willard van Orman Quine, comments in his famous *On What There Is* – with the aseptic humour of sober analytical philosophy – 'the old Platonic riddle of non-being' and recommends subjecting this 'tangled doctrine' which 'might be nicknamed Plato's beard' to Ockham's razor. This undertaking, which analyses Plato with only cosmetic care, became what is known as Quine's own philosophy and continues to be commented on as such.

Commentary is a discourse: discourse *pro* and *contra*, discourse and counter-discourse. And everything is the product of disputation, as Heraclitus' saying, passed down to us via Aristotle, affirms: πάντα κατ' ἔριν γίνεσθαι. Polemic is father of all things.

Commentary is an *oratio continua* which unfolds along a strictly linear trajectory and which nevertheless – like Peano's curve – completely fills a whole space, the space of the mind becoming conscious of itself.

Commentary: dialogue with an author and soliloquy of the mind

Yet the dialogue of the commentary and the work is a soliloquy. It is the same mind which created the work which makes a commentary on itself. It is always the mind which speaks to itself in its commentary; it is the same mind which constantly denies itself.

Work and commentary: the two hypostases of one and the same mind.

Discourse, too, this conversation of the soul with itself, is *praxis*: it is the work of the mind concerned with bringing itself into the world, concerned with becoming aware of what it is.

And this dialogue that the mind continues with itself in order to rise to the state of being conscious of itself – the results of this work which is never definitively completed – is exactly what constitutes philosophy.

Non-Euclidean commentary on Euclid

But mathematics is not philosophy. Theorems are not commentaries; they are the inferential consequences of hypotheses and axioms that already exist. This is clearly why it is so peculiar that the historical event known as the 'non-Euclidean revolution' is the result of the work of commentary on geometry's Holy Scripture, *The Elements* of Euclid. The commentary started by asking Euclid questions, then it began to have doubts. Until the time came when the commentary said 'no' to the work.

That means that non-Euclidean geometry is itself the result of work done by the speculative mind of philosophy within mathematics.

However, this is only one of the peculiarities that give the non-Euclidean history a distinctive place within the vast network of the genesis of mathematics.

The first rigorously argued elaboration of a non-Euclidean system is to be found in Father Saccheri's famous work, *Euclides ab omni naevo vindicatus*. The work was meant to be purely and simply a commentary upon Euclid. Its aim was not to construct the non-Euclidean system – the original intention of the commentary was to destroy the non-Euclidean world.

Nothing less, in fact, than annihilating a world, the non-Euclidean world.

For one of the most significant propositions of his commentary contains this stupefying and purely ontological result that both – the ontic Euclidean realm as much as the non-Euclidean domain – were each in themselves a closed geometrical world: *si unus, omnis* – if *one* sole geometric object, a single line, a single triangle, is Euclidean, then *all* objects, in the strictest sense of the term, absolutely all are necessarily Euclidean; and if a single triangle is non-Euclidean, then absolutely *all* objects are non-Euclidean. This is the fundamental and truly great theorem of Saccheri's commentary. The two predicates, 'Euclidean' and 'non-Euclidean', are qualities which can only be attributed respectively to a very singular and distinct object: the infinite universe, the totality of being, or of non-being, as is the case. Whatever it may be, therefore, the geometry of the being or that of the non-being, all geometry is a cosmology.

The goal Father Saccheri set himself was modest and humble. He had quite simply wanted to write a commentary on a fundamental proposition of the *Elements*, the postulate called 'Euclid's parallels' and, after two millennia of abortive attempts, provide a correct mathematical interpretation for the Euclidean proposition.

The proposition on which he made a commentary, the falsehood of which he wished to demonstrate, Saccheri had himself found in a commentary, that of Proclus: *Comentarii in primum Euclidis Elementorum librum*.

In his neoplatonizing commentary, Proclus had devoted a lengthy exegesis to 'the most paradoxical statement', παραδοξότατον θεώρημα, of all geometry, and which he had himself found in the work conceived centuries before by Geminus of Rhodes under the strange title of *Philokalia*. The statement in question had been the necessary result of his own commentary on Euclid's postulate. Geminus expressed it in the form of an interrogation inspired by an evident doubt raised by the work: 'whether parallel lines that converge towards each other in the same way as the asymptotes of a hyperbola exist?' It is not difficult to detect behind the question posed by Geminus about the existence of asymptotic parallels the purely speculative enquiry which was born in the still-mute world of the transcendent: could another world exist, a non-Euclidean world? So here, on the secret screen of the One, the Euclidean world, there suddenly appeared the shadow of an odd stranger, the Non-Euclidean; in the consciousness of the Self, there emerged the worrying question of the existence of the Other.

But the commentary on Euclid by Proclus also contained something completely new and this time something specifically geometrical. In his pages, one can read for the first time a proposition which formally contradicts Euclid's postulate of the parallels. His text characterizes the asymptotic parallels with such precision that – through immanent necessity – the statement which it contains has ever since represented the basis and inferential premises of all attempts to refute the existence of a non-Euclidean world.

And it followed from this that it was undoubtedly Saccheri's intention to demonstrate the absurdity of this assertion: to wash every stain from Euclid. This was the object of the commentaries which Father Saccheri exposed with astounding obstinacy and attention to detail: the non-Euclidean *inimica hypothesis*, against which he directed the entire panoply of his extraordinary and brilliant logic. Moreover, he was able to finish his exhaustive commentary with a cry of joy: the desperate combat, *diuturnum proelium*, which the rigorously organized and disciplined army of his theorems had given to the adverse hypothesis, had ended in a decisive victory! The great work, the divine *corpus* of Euclid shone anew in its original purity.

The adverse hypothesis which had motivated Saccheri's desperate campaign was the equivalent of the most paradoxical of all of Geminus' theorems – and in his commentary Saccheri was to give it a formulation which was almost identical, word for word, to an axiomatic proposition on which Lobachevsky was later to construct his non-Euclidean geometry. This is, moreover, why it is very understandable that – after the event – Saccheri was unable to escape the increasingly insistent suspicion of having worked in collaboration with the enemy; secretly, his *Euclides vindicatus* was thought to have been conceived in agreement with the *inimica hypothesis*. These are unjust and completely indemonstrable accusations. What is, by contrast, demonstrable is that the death certificate made out by Saccheri turned into the birth certificate of non-Euclidean geometry.

Without Saccheri either wishing or knowing it, his commentary became the *Work*, a work whose originality by far surpassed the *original originality* of Euclid's great work. The commentator – servant of his master – became the Master; the mediator of the existing work became the creator of his own Work.

'Poetry is superior to history', we read in Aristotle's *Poetics*, 'for history only recounts events which have taken place. But poetry is more philosophical.' Euclid's *Historia* also only describes what is: the geometry of a world that exists in reality. But Saccheri's poetry is more philosophical, for it describes what is not: the geometry of a world which does not exist. And before him, nobody had thought of describing this world, of enquiring about its existence, not even in a dream.

Except, perhaps – if we can trust Dante's account – King Solomon.

Dante's commentary on the non-Euclidean dream of King Solomon on Mount Gibeon

To the question once posed by Geminus, Moses Maimonides added the following commentary in his *Dux perplexorum*: 'there are things which the imagination can undoubtedly not represent, but which the intellect can think'. This was an allusion to Geminus' asymptotes.

But – and this is remarkable – Maimonides' book also asks the question of knowing whether divine omnipotence, in its infinite freedom, could not have also 'in the beginning' created a non-Euclidean world *ex nihilo*? It is well known that his reply was 'no'.

But Maimonides characterizes the non-Euclidean world by means of a square whose diagonal is commensurable with its side. This strange four-sided shape, a shock to visual intuition, was cited in Aristotle's *De Caelo*, as an example of what Aristotle describes with the *hapax legomenon*, 'impossible *per hypothesin*'.

It is undoubtedly no exaggeration to say that Aristotle's example is surprising. Let us admit, he writes, that it is impossible for the sum of the angles of a triangle to be equal to two right angles. And if, therefore 'Euclidicity' is impossible then there must also be squares whose diagonal is commensurable with their side. Another reason why this example is strange is that here, in his book *On the heavens*, Aristotle refers to the commensurable diagonal with Olympian calm and not the least surprise, while by contrast in his *Metaphysics* he speaks of the surprise which the incommensurability of the – Euclidean – diagonal evokes among those who are unfamiliar with geometry, while nothing would surprise the geometrician – ἀνήρ γεωμετρικός – more than a commensurable diagonal. The example of a ἀδύνατον ἐξ ὑποθέσεος in Aristotle's treatise *On the Heavens* is a commentary on the original – even very original – work, the paradoxical shape of a non-Euclidean square, the result of the professional researches of the geometricians at the Academy: if the commensurability of their diagonal elicits no surprise in Aristotle, it is because it is the logical consequence of the impossibility that an ἀνήρ γεωμετρικός was assigned without any justification, *per hypothesin*, as a modal predicate, to an *Euclidean* statement.

This consequence, the commensurability of the non-Euclidean diagonal, is quite clearly no truism. It is a theorem of this 'new and different world' which was only to be 'created from nothing' two thousand years later by a young Austrian officer, 'Monsieur Jean Bolyai de Bolya, Lieutenant au Corps de Génie de Sa Majesté Impériale et Royal à Temesvár'.

As for Maimonides' question, it is clear that it was the result of a commentary and an exegesis: its initial intention was to understand and interpret Aristotle's text in conformity with the idea of creation *ex nihilo*.

In his great work of commentary, Thomas Aquinas mentions the name of Rabbi Moyses Aegyptus on more than one occasion and he often cites the *Dux perplexorum*, sometimes even without explicit reference to the author. In his commentaries on Aristotle in particular, the same question on non-Euclidean creation appears not once but often, and expressed very much more explicitly than in Maimonides. For Thomas, just as for Aristotle, the non-Euclidean world is always represented by the simple plane of a triangle, which is in theory entirely correct and sufficient. Thomas's reply also agrees with that of Maimonides, but his argumentation is richer, more detailed, and much more categorical. '*Deus facere non possit, quod triangulus rectilinaeus non habeat suos tres angulos aequales duobus rectis*', we read in his *Summa contra Gentiles*. And in the *Supplementum* to his *Summa theologiae* the categorical proposition is found: *'Non potest fieri per miraculum . . . quod triangulus non habeat tres angulos aequales duobus rectis.'*

St Thomas clearly declared himself ready to accompany Dante, his admirer and pupil, in his wanderings in the *Paradiso*. In Canto XIII, he reminds Dante of King Solomon's famous dream on Mount Gibeon. God appeared in a dream to Solomon and says to him: *'postula quod vis ut dem tibi'*. According to Dante's 'oneiromantic' commentary, the king, who was still young, was then pondering upon some problems which were visibly preoccupying him. They concerned the *primum movens*, the number of the planets, the relationship between the contingent and the necessary – all questions familiar to us through the scholastic *quodlibeta* and the *disputationes* on the *Sentences* of Petrus Lombardus.

But Solomon, recounts Dante, did not ask the Lord any of these questions, nor a fourth, namely *'Se del mezzo cerchio far si puote / triangol sì ch'un retto non avesse'*. The question is clearly about the existence of a non-Euclidean triangle, for only such a *triangol* is constructed so that the angle in the semi-circle *'un retto non avesse'*. This shape formally contradicts theorem III. 31 of Euclid's *Elements*, but its secret charm derives perhaps above all from the fact that it had never flourished in any garden of the School outside Dante's *Paradiso*.

But in his *regal prudenza*, Solomon chooses not to ask the Lord if it was possible to draw a non-Euclidean triangle in the semi-circle but prefers to ask him how to tell the just from the unjust.

To all appearances, Dante in his *Paradiso* was making a commentary on Thomas Aquinas's commentaries on Aristotle at the same time as the further commentaries of Thomas's later commentators.

Nearly two centuries after Dante, Leonardo, in his *Trattato della pittura*, sets the certainty of mathematical truth against the barren emptiness and sterility of the disputes of the scholastics, and he writes that, in a science 'of truth' such as geometry, *'il litigio resta in eterno distrutto, e posto silenzio alla lingua de' litiganti'*, for *'qui non si arguirà che un triangolo abbia i suoi angoli minori di due retti, ma con eterno silenzio resta distrutta ogni arguizione'*. Did the prolix tongues of the medieval commentators that were so greatly despised really debate the possibility and impossibility of the non-Euclidean triangle?

Whatever the case, the non-Euclidean triangle of which Leonardo speaks did not sink into the calm depths of *'l'eterno silenzio'*, nor was the *'lingua de' litiganti'* silenced. The *disputa e contende* did not cease. Quite on the contrary, the *arguizione* became increasingly lively and audible.

La cosa meravigliosa: the asymptotic parallels and Montaigne's commentary

Prompted, no doubt, by the Emperor Frederick II, the *stupor mundi*, the *marrano* Giovanni da Palermo attempted to explain Maimonides' proposition, somewhat obscure and difficult to understand, on the imagination and the intellect by means of a Latin translation of an Arabic text older than the asymptotes. Some centuries later, in 1549, his commentary was to be commented on in its turn by Rabbi Moses il Provenzale, in a text written in Hebrew. This text was translated into Italian in 1550 by Giusepho of Padua and published at Mantua – with the support of his patron, Don Diego Hurtado de Mendoza. The title of the work, *Cosa meravigliosa*, was about the same question: 'How to describe two lines on a plane which get closer and closer together but which never meet: *como possono uscire due linee sopra una superficie le quali si accostino sempre ne poseno incontrarsi mai?'* It is perhaps not without interest to mention that the *Cosa meravigliosa* remained the only non-Euclidean work written in a vernacular until the beginning of the nineteenth century.

Francesco Barozzi translated the commentary of Rabbi Moyses Narbonensis – this was how he referred to the Provençal scholar – into Latin; he added his own extremely detailed and critical commentary and published it at Venice in 1586 with the title *Admirandum illud geometricum problema*.

As the careful researches of Luigi Maierù have proved, the *cosa meravigliosa* was an immediate sensation, and not only in Italy.

The first echo of the event that was hardly noticed by the great mathematicians of the period is to be found in Michel de Montaigne. In his *Apologie de Raymond Sebond* he writes a commentary on the *cosa meravigliosa* that is remarkably sensitive, demonstrating once more the greatness of his mind.

> J'aymois mieulx suyure les effets, que la raison. Or ce sont choses, qui se choquent souuent: & m'a l'on dit qu'en la Geometrie, qui pense auoir gaigné le haut point de certitude parmy les sciences, il se trouue des demonstrations ineuitables, subuertissans la verité de l'experience: comme Iaques Peletier me disoit chez moy, qu'il auoit trouué deux lignes s'acheminant l'vne vers l'autre pour se ioindre, qu'il verifioit toutefois ne pouuoir iamais, iusques à l'infinité, arriuer à se toucher. Ptolomeus, qui à esté vn grand personnage, auoit estably les bornes de nostre monde: tous les philosophes anciens ont pensé en tenir la mesure. C'eust esté Pyrrhoniser, il y a mille ans, que de mettre en doubte les opinions qui en estoient receuës d'vn chacun; c'estoit heresie d'auouer des Antipodes, & voylà de nostre siecle une grandeur infinie de terre ferme qui vient d'estre descouuerte. Les Cosmographes de ce temps ne faillent pas d'asseurer que meshuy tout est trouué, & que tout est veu; il reste presentement à savoir, si Ptolomée s'y est trompé aultrefois sur les fondements de sa raison, si ce ne seroit pas sottise de me fier maintenant à ce que ceux-cy en disent: Et qui sait qu'une tierce opinion, d'ici à mille ans, ne renverse les deux précédens; et s'il n'est plus vraysemblable que ce grand corps, que nous appellons le monde, est chose bien aultre que nous ne jugeons.

A little later, François de La Mothe Le Vayer, his disciple and tutor of the Dauphin, wrote in his highly controversial work, *Discours pour montrer que les doutes de la philosophie sceptique sont de grand usage dans les sciences:*

> La Nature dépend absolument de la nûe volonté de son Créateur; & nous ne commettons pas une petite faute, quand nous la voulons assujettir aux regles des Mathematiques ou aux fines conclusions de la logique. De quel front pourrons-nous dénier à l'autheur de la Création, la faculté de faire agir contre les maximes & les regles soit d'Aristote soit d'Euclide?

Other commentaries were to follow.

Three decades after its first appearance, Saccheri's *Euclides vindicatus* was the subject of meticulous commentary in a review by Georg Simon Klügel. It did not escape the latter's severe analysis that the final conclusion of Saccheri's argumentation was false. The title he gave his work, in the heady foretaste of victory, did not correspond to the contents of his work. Euclid had no need of purification. The magnificent *corpus* of his *Elements* was not sullied by any flaw in logic. By contrast, *Euclides ab omni naevo vindicatus* is disfigured by a terrible logical *naevus*: contrary to what Saccheri asserted, the *inimica hypothesis* continued to stand unrefuted, like an eternal provocation. *Liber singularis* is Klügel's commentary on Saccheri. When Klügel opened Saccheri's book in his gothic chamber, he took fright. Just as Dr Faustus was later seized by fear as he opened *The Book of Nature*, Klügel had sensed, decades before him, in this *liber singularis* of geometry, the sign of a *macrocosm* as yet unknown, he saw the cosmology of an impossible world, without flaws, and all at *once he sensed on his face the smell* of sulphur from the geometrical other world. The *mirificium pentagramma* of the non-Euclidean plane 'pained him', but had not frightened him. And Klügel asked the dramatic question: 'what would happen if somebody asserted that there were asymptotic lines that converged without ever meeting at a single point?' And came immediately to the reluctant conclusion that the indestructible world of the *inimica hypothesis* could only refer to the tangible presence of a previously unsuspected *enigma ipsius genii humanii.*

The geometrical text and its commentary: creativity and sterility

In 1621 Sir Henry Saville devoted an opening lecture at the University of Oxford to commentary on the definitions and first propositions of the first book of Euclid. It was he who described the Euclidean definition and the postulate of the parallels as a blemish – '*naevus in pulcherrimo Geometriae corpore*' – and this because the truth of it was not rigorously demonstrated in the *Elements*.

But what is the blemish? And if it were one, whom would it have interested?

At the same time as Saville's *Praelectiones* were published a new era in mathematical research had dawned; a period such as had never been experienced before, rich in a whole spectrum of ideas, and abundantly fertile in heterodox methods. It was the era of the infinite, of negative, imaginary, and transcendental numbers, a period distinguished by – to cite just a few – the great names of the history of mathematics: Cavalieri and Kepler, Cardan, del Ferro and Descartes, Wallis and Newton, Leibniz and the Bernoulli, Desargues, Fermat and Euler. We can state without the least hesitation that, compared with Euclid's *Elements* – the arithmetic of the infinitely big and the geometry of the infinitely small, the new algebra of negative and imaginary numbers was entirely constructed from blemishes, as it were. The new theories were disfigured by a scandalous lack of proofs and, rather than proceeding *more geometrico* by rigorous demonstration – in the manner to which we are accustomed in the *Elements* – these brilliant authors tried to justify and found the pandemonium of their mathematical phantasms by ideas that were as inventive as they were terrifying, enough to make one's hair stand on end. Moreover, who had ever previously demonstrated the mystical existence of these phantasmagorical entities which manifestly did not exist and which had had to be sought in the other world, by magic? The new numbers openly appealed to an ontology which had from the very first denied their existence; through their names alone, they insolently proclaimed their illegitimate origin. For they were called false, sophisticated, fictitious, absurd, and impossible numbers. 'By you, sirs,' said Dr Faustus, addressing the Prince of Darkness, 'your essence can usually be read in your name.'

But who would then allow themselves to be diverted from enthusiastically welcoming all these 'wondrous strange' quantities that had come from the other world, which had no 'quantity' or at least only an imaginary quantity, or even less than nothing – or who, as

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Bishop Berkeley put it, were 'the ghosts of departed quantities'; as one welcomes a stranger – as Hamlet, at the same period, all but welcomed the ghost of his father. And like the Prince of Denmark, the mathematicians of his time knew that even in Plato's heaven of ideas, more things existed 'Horatio, than are dreamt of in your philosophy'.

For it is possible that all these fictional, imaginary, false, sophisticated, and absurd numbers could none the less comfort the mathematicians mourning their prenatal death, for even if they did not exist, they were no less abundant and utterly unexpectedly fertile for a non-entity. They were effective, and they were productive and virulent. Their *corpus* was admittedly not *pulcherrimum*, although they could not hold a candle to the *plus belle fille de Paris*, they could only give what they had: the fruits of their admittedly tainted but living body – fruits that were perhaps monstrous, but luxuriant. A varied abundance of results as unexpected as they were unsuspected.

In these conditions, who would have been embarrassed by the imperceptible blemish marring the marvellous, but petrified and lifeless, body of the *Elements* – a cold marble statue of antique divinity – as old as time?

The mathematical creative geniuses of the sixteenth, seventeenth, and eighteenth centuries showed no interest in 'Euclid's blemish'.

As a branch of scientific literature, the commentary did not enjoy a good reputation at that period.

Commentary? To what end? It was the *Work* which was the order of the day, nothing but the Work. What mattered was to create something new, original, and unexpected – create, always creating, but certainly not commenting and exhaustively interpreting absurd, outmoded, boring, classical texts, written by authors who were obsolete, indeed compromised. What sense would there have been in attempting to perceive the sense of non-sense, to comprehend the incomprehensible?

The term 'commentary' alone was sufficient to elicit irritation and distaste. It immediately evoked the sterile artifices of the *irrefragabili, angelici et subtilissimi doctores* of the Middle Ages, who could scarcely any longer pass for anything other than representatives of the hated School, which had disappeared in the meantime, personifications of the ridiculous. Who could have had any interest in writing a commentary on the text of Euclid, which had become a museum-piece?

Who, moreover, read Euclid?

Plenty of more interesting questions were the order of the day.

Research was then preoccupied with vital, complicated, and difficult problems, such as compound interest, ballistics, navigation and bridge-building, celestial mechanics, the conduction of heat, the construction of machines, or determining the volume of barrels of wine with a complex structure. And the methods developed to resolve these problems did not turn out to be important for practice only, but apparently also still more for theoretical thought, for they opened totally new and completely unsuspected ways for research, they endowed mathematics with the magic instruments of new methodologies whose productivity was very largely to surpass everything which had been known until that date.

This all demanded immense efforts and the full-blooded mathematicians set about the task with the requisite enthusiasm. They were all seized by the fever of discovery and invention. The ideal sought was that of originality, creativity, and productivity. They were all fired by the ambition to discover something unexpected, unforeseen, surprising, and exciting – to invent something new, always the new and nothing but the new! The efficacy and prospect of glory stimulated their ambitions. The competition to resolve sophisticated problems with a subtle formula, the revelation of great secrets, the obtaining of sensational results that were absolutely original, the introduction of concepts as astonishing as they were relevant – these were the aims that were established and which the mathematicians of these times attempted to achieve.

And they quarrelled. But it was no longer the contemptible semantic subtleties of the School which were the object of dispute. Their confrontations could no longer be reduced to absorbing and endlessly ruminating upon the puerile debates of yesterday. No! The mathematicians of the new era disputed bitterly, passionately, unbridled and without consideration on the question of establishing who invented something first, who was the true genius, to whom was due the glorious *jus primae noctis* of unveiling something new which no one else had thought of.

The exponentially increasing richness of concrete results contrasted with the pitiable poverty of spirit and the dessicated sterility of the commentaries on Euclid and Aristotle and with the repetitive monotony of the commentaries upon commentaries.

To active mathematicians it seemed that the texts of the exegetes of Euclid, including the most recent, gave off the same rancid smell of boredom as the obsolete robes of theological seminaries and the mouldy texts of the commentaries which, in the musty Middle Ages, had been the object of disputes as vigorous as they were futile.

From the eighteenth century onwards, it quite simply became a ritual to accompany commentaries on Euclid and the attempts which such-and-such an author made to demonstrate the postulate of the parallels with a stereotyped litany. The aim of these exercises in virtuosity had not changed, however: it was still a case of convincing the reader of the great value and the importance of the author's undertaking. In all these psalmodies, the same important argument inevitably recurred: since Antiquity all the great mathematicians, all the *celeberrimi et illustrissimi viri*, had attempted to prove the postulate of the parallels without achieving the hoped-for success, but where the greatest mathematicians had failed, the humble, and moreover unknown, author had finally won total victory and simultaneously legitimated his claim to well-earned universal fame.

Without even taking into account the fact that, like all his predecessors – humble or less humble – this humble author was to die of exhaustion in anonymity, the interminable funeral oration contained in the pious necrology of failed authors is disappointing. With the exception of the truly great Islamic mathematicians, who were concerned with the postulate of the parallels – at the period when the commentary still enjoyed scientific respectability and when what has been called 'Western culture' was concerned especially and specifically with the realm of Islam – the list of names consists of minor provincial masters, undistinguished and mathematically entirely unproductive, such as the court councillors Kaestner and Karsten, the professors Klügel, Gensichen, Hoffmann, Seyfert, and Schulz. Admittedly, all were to be numbered among the *illustrissimi et celeberrimi viri* of the universities of the period, but their true place was rather among οἱ νᾶνοι ἀπὸ τῶν κήπου – among the mathematical dwarfs in Snow White's garden, rather than among the priests and high priests of the god of geometry.

In these solemn evocations, the only man to escape this fate, the Jesuit Father Saccheri, a simple amateur mathematician, is never named. Even as a theologian, his name has remained totally unknown. For he published three big volumes of theological pamphlets on the sacrament of confession, directed against the views of Cardinal Pallavicin and another dignitary of the Church who hid behind the pen-name of *Eremito solitario*. The thesis which Saccheri defended was that lying was a mortal sin that no ecclesiastical or state interest could justify. And the fact that he adds to his argumentation some of the theorems of his *Euclides vindicatus* – which appeared later – certainly did not help his name to greater notoriety. No, Father Saccheri did not belong to the distinguished club of the university élite. As a marginalized amateur mathematician, he could only join the vast club of the insignificant unknown. Later, much later, Paul Valéry wrote, in the same context: '*Ce Saccheri soupçonnait, sans l'avouer, ce qu'il y a de convenu dans Euclide et entr'ouvrait une porte à bien des audaces futures de la géométrie. Ce n'était, il est vrai, qu'un jésuite.*'

The commentary on Euclid and the world of mathematics: geometry and metaphysics

In these circumstances, it is not surprising that the true mathematicians of the period received the commentators on Euclid without the least pity, and that they commented upon them with unconcealed scorn – if they happened so much as to glance in their direction.

In his *Recherche de la vérité*, a famous and much-read work, Nicolas Malebranche only voiced the general opinion of his mathematical friends when he commented, exasperated, upon Saville's *Praelectiones*:

> Voici un Sçavant Anglois qui parle de cette entreprise, comme si elle étoit fort grande & fort difficile & il remercie Dieu de ce que par une grace particulière, il a executé ce qu'il avoit promis. Quoi? La quadrature du cercle? Ce grand homme a expliqué que la définition et le postulat des lignes parallèles en Euclide sont défectueux. Voilà les desseins bizarres, dont la fausse érudition nous rend capables!

The commentaries of the mathematicians of the following generation were to be no different: '*Perdre le temps inutilement*', the Great Arnauld observed laconically and disdainfully. No more than '*un abus de la métaphysique en géometrie*', judged d'Alembert, visibly disgusted. And Laplace added: '*On doit abandonner ces discussions aux Métaphysiciens-géomètres*', the term 'metaphysician' here undoubtedly not intended as a compliment.

It would be difficult to put it better!

It was perfectly true: the commentaries did not lead to any really geometrical result. And one cannot overlook the fact that they were not guided by the *esprit de géométrie*, but by a strange, hidden, confused and purely metaphysical teleology.

For despite the close and rigorous net of definitions, lemmata, and theorems, the proposed proof of the postulate was no more than a means to attain an end which could not remain completely hidden. And the end which the individual subjects, the agents of the undertaking, pursued in reality was always the same: to refute the possibility of a non-Euclidean world – the intellectual annihilation before birth of a world which *ab initio* only represented for everybody – whether geometricians or not – the evident impossibility of itself.

For, even when they respected the rigorous morality of the *mos geometricus* and composed texts of theorems, bound together by the necessity of the logical inference, that could not hide the fact that in reality their hidden *telos* resided in the metaphysical realms, in transcendence, and not in the heavenly spheres of geometry.

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The question of motivation of the commentaries on Euclid

The question was also asked by the famous writer, essayist, and satirist, Georg Christof Lichtenberg, who taught experimental physics at the University of Göttingen and, as such, had been one of the teachers of Gauss: 'Convince the sceptics? But who could convince anyone who wanted to believe in absurdities at any price?'

Absolutely right. The refutation of the idea of a non-Euclidean world was not part of the current goals of experimental physics, nor even of geometry – neither in the past, nor in Lichtenberg's lifetime, and still less in our own time. But if the problem does not lie in geometry, where should we seek the motives, and above all the meanings, of the intellectual endeavours constantly put to work to refute this manifest absurdity: a non-Euclidean world?

To continue to develop geometry as a branch of mathematical science, to enrich Euclid's *Elements* with constantly new and interesting theorems, it is sufficient to accept its postulate. Demonstrating it is superfluous. And the truth of the Euclidean postulate has never been called into doubt by anyone, and, moreover, nobody could ever doubt it.

Just like God, Euclid has no biography.

The hieratical tetragram, the ineffable name of the One, means: 'I am what is'. And this already contains the entirety of the visible text of the world, written ἐν ἀρχῇ by the Eternal, its hidden author. Euclid, the God geometrician, is also present to the world in his name. And his name means that he enjoys a 'good reputation'. As for assertions to the contrary, they have always enjoyed a bad reputation.

This is undoubtedly why no one has ever really experienced the need to establish by means of a redundant proof the divine geometry and the good reputation established by its name alone, nor to refute an absurdity like that embodied in the asymptotic parallels.

There has been – and there is still – in mathematics an infinite number of open and unresolved questions. There was, for instance, the question of knowing whether, in a finite number of steps and by means of elementary constructions, one could construct a square whose area would be equal to that of a given circle. There is still, in our own time, the question of knowing whether all even numbers can be represented as the sum of two prime numbers.

There were good reasons for attempting to reply to these questions, by reason of the role which these propositions played in the subsequent development of the mathematical sciences. Moreover, we know the reply to the first question: it is negative. The second question remains open. Or, put differently, the reply is probably 'yes', but to this very day we have no absolute certainty on this subject.

As far as the question of the truth of the Euclidean postulate, on the other hand, is concerned, it is striking that there is no incentive to motivate any attempt to respond within the mathematical sciences themselves. For we knew from the very outset, with an absolute certainty, that Euclid's postulate is true. Similarly today, strangely enough, we know – and with that same absolute certainty – that the axiom of Euclid's parallels is still true, and this despite the fact that meanwhile the non-Euclidean axiom has also been admitted and integrated, with the same rights, into the world of the *épistémè*, where it enjoys the same dignity of truth today which has been the privilege of the Euclidean axiom from all eternity. For in the past as in our own time, the truth of Euclid's axiom of parallels has never been an open problem.

Étienne Montucla, the greatest historian of mathematics of the period, made this terse and somewhat irritated judgement on the totality of the work of the commentators on Euclid who sought to demonstrate the truth of his postulate by means of proofs: '*Des affectations qui ne facilitent la science qu'en l'énervant*'. And, again, at the beginning of the nineteenth century, in a work on elementary mathematics, the great mathematician, François Lacroix asked this rhetorical question destined simply to express his indignation: '*À quoi peut servir d'alambiquer les notions les plus clairs, obscurcir par des preuves inutiles ce qui est évident de soi-même?*'

The work of commentary on Euclid's postulate was decried, it was excluded from the sphere of true mathematical research, and it vegetated, marginalized, at the outermost bounds of true mathematical life.

The commentary as increment of knowledge and preservation of thought

This is the point to indicate that, because of successive setbacks, interest in the problems exposed decreased in general, the combatants became exhausted, weapons were laid down, and recollection of the problems sank into a saddening or comforting oblivion. The number of unresolved problems whose memory has dissolved into sedative amnesia is almost impossible to estimate. In these circumstances, it really seems very strange that work on the commentary on Euclid's postulate never stopped, despite repeated setbacks.

Quite the contrary.

It seems that, far from discouraging interest in the refutation of the abstruse non-Euclidean world, the poor reputation of the commentary contributed rather to stimulating it. There was no sign of weariness or exhaustion. The *diuturnum proelium* continued and the struggle became increasingly bitter. Not only did work on Euclid's commentary progress unchecked, but it was constantly growing, it broadened out and spread, and it increasingly grew in size. Thus the refutation of the asymptotes and of the non-Euclidean hypothesis became a specific branch – simultaneously unclassifiable – of the literature, and it reached an ever-growing number of authors with sufficient aesthetic sensibility to understand its exotic charm.

Among them was one of the most important mathematicians of the eighteenth century, Johann Heinrich Lambert. Among the consequences which stemmed from the hypothesis that the sum of the angles of a triangle is less than two right angles, there was one which particularly struck him: if it was true, then there was a natural and absolute measure of length, which would not therefore depend on any convention. 'This consequence', he wrote, 'has something exciting about it which easily awakes the wish that the hypothesis could be true!' But he rallied his forces and resisted temptation. For he had also reached the conclusion that the flat world which contained such a square would somehow be realizable on the surface of a two-dimensional sphere whose radius would have a measure that could only be expressed by an imaginary number. However, Lambert had never admitted the existence of these numbers: what is imaginary cannot possibly have a real existence; and he went so far as to use in his logic the sign '$\sqrt{-1}$' of imaginary unity as the symbol of logical absurdity. The same had to be true for the internal geometry of an imaginary surface of a sphere: it had to be the geometry of the impossible. Besides, it seemed that in this non-Euclidean geometry the calculations to determine length and area were much more complex than on the Euclidean plane. Lambert estimated that, running against the accepted practice in this heterodox geometry, 'there were only *argumenta ab amore & inuidia ducta,* which should be kept completely apart from geometry like the other sciences'. And as far as the impossibility of that geometry is concerned, he ultimately reluctantly accepted that he had not 'been able to give any proof of it'. And he did not publish his work.

Every commentary was the object of a veritable *anairesis,* an *Aufhebung* – in every sense of the word: every commentary submitted its predecessor to merciless critical analysis, took it apart, negated it, and apparently destroyed it. This commentary had then of course replaced its predecessor with a better commentary which, in the opinion of the author, should be the last word on the subject. Only, it in its turn became the object of the commentary of its successor, thus sharing the fate of its predecessor.

The authors of commentaries were most often completely unimportant mathematicians, sometimes even amateurs and dilettanti, excluded from the real scientific life of the established *universitas scientiarum,* a veritable mathematical subculture. But in the divine theodicy of thought they nevertheless played a very considerable role. For, in the diachronic flux of repeated refutations they all contributed to the preservation of the commentaries which they refuted. It was this which constantly kept interest in the problem alive, and the work of commentary, the diachronic flux of time, received it, transmitted it, and raised it.

It was thanks to them that the commentary did not disappear into the empty nothingness of oblivion. And memory is a specific ontic state: *anamnesis* is the *modus essendi,* the being of the past in the present.

Memory introduces what was deep within the present and includes it in the actual being of the here and now – in the ontic mode of its past essence. In the diachronic space of the *Historia* the essence of what has been is affirmed in the present. The essence of what has been is the knowledge we have of it in the present. Anamnesis is an ontic state of the conscious being.

According to our current knowledge of physics, the universe obeys a strict law of conservation: the quantity of mass and of energy in the universe remains constant. The quantity of material goods diminishes; it is even destroyed through consumption. On the other hand, the quantity of knowledge which fills the universe of the spirit constantly increases. And the mass of spiritual goods increases as they are consumed. When one spends money, one becomes poorer. Somebody who has spiritual possessions becomes the richer the more his goods are consumed. To dispossess the spirit of its goods amounts to increasing their value.

The universe of thought is in a state of permanent expansion. However, this world also obeys a law of conservation. Values accumulate like capital, the values grow, added value is added to the mass of existing values, and it is preserved. The consumption of spiritual values conserves and expands them.

The natural force which preserves the past in the present is memory, and it confines it in actual knowledge. Memory is the work of the mind which unfolds and actualizes itself in constant dialogue with the given – in the form of a commentary without end. It is this dialogic *praxis* which constantly adds new knowledge to the mass of existing knowledge.

The work of the mind which, by means of dialogue with the given, unceasingly produces supplementary knowledge – this dialogue with the past is the force which does not cease to maintain in activity the constant expansion of the spiritual universe.

The commentaries on Euclid were superimposed one upon the other, like geological strata of texts. The entirety of their contextual mass consists in a coherent verbal substance which has only a single author: the one and indivisible *esprit de géometrie*. The commentators, whether important or insignificant – that makes no difference – are all considered as willing servants of the *esprit de géometrie*. It is *its* text that they have copied and attempted to understand, and all have consciously allowed themselves to be guided by the same decision: to remain faithful – humbly and absolutely – to the Work.

Nevertheless, or precisely on account of this reason, they corrected the existing text and wrote there their own improved readings. Here and there, they even rewrote the text and amended the existing text by means of the new text with their own interpretations.

They betrayed *The Work*.

They freed themselves. They became autonomous authors.

The commentary and geometry's unhappy consciousness

The recurrent sequence of commentaries produced a constant increase in ever-new contents and theoretical insights. But in what transpired as a result, there were also texts which were diametrically opposed to the Euclidean text, in so far as the commentator's intention had been to demonstrate his loyalty to Euclid through the refutation of the *inimica hypothesis*, the non-Euclidean enemy.

The enemy, the non-Euclidean world, did not exist. *Ergo*: it had to be invented.

And it was invented.

The text of the commentary on Euclid described all the properties of this hostile, non-existent world of the non-Euclidean, with the sole aim of refuting and destroying it; and it did so with a precision as exact as that devoted by the text of the work commented upon to describe the world of its author, the world of Euclid.

And each commentary noisily heralded the decisive victory, while the *inimica hypothesis* not only remained intact, but the unrefuted *corpus* of its extravagant theorems continued to prosper, becoming increasingly rich and flourishing. 'That which does not exist has no properties', Aristotle wrote on several occasions. But this non-being, however, had quite specific, even though unusual, qualities. 'There is no place where the non-being could have lived. For where is the sphinx? And where, in what place, is the centaur to be found?' None the less, the non-Euclidean sphinx was certainly somewhere, this 'where' existed: down there, on the pages of the commentaries on Euclid. The geometrical centaur was similarly in a specific place. It lurked in the cursed enchanted garden of *Euclides ab omni naevo vindicatus*. Yes, it lurked there, in Father Saccheri's mythological zoological garden. There it provided proof of a very actual presence, as an unbearable provocation.

'The non-being cannot be known, nor even articulated in discourse' – οὔτε γὰρ ἄν γνοίης τό γε μὴ ἐον, οὔτε φράσαις – asserted Parmenides the Great. But, despite non-being, the hated *corpus* of the non-Euclidean world was just as fully and precisely cognizable as the magnificent bodies of Euclid's five Platonic solids. And these shapes were also cognizable, just like the properties of another that did not exist: the measure of the inexpressible diagonal. And it was to all appearances to the recognizable character of that irrational *ratio* that the proposition – as beautiful as it was mysterious – alluded, which the great Parmenides uttered, in Plato's dialogue bearing his name, against his own thesis: 'What is said to be non-being is no less cognizable than that from which it is different – γιγνώσκεται, τι τὸ λεγόμενον μὴ εἶναι.

In this way, in the commentaries, a constantly growing number of propositions and demonstrations were accumulating which were later to be known as true theorems of non-Euclidean geometry.

Later, in fact, very much later.

For in the past they were all unhesitatingly deemed false, despite the fact that they had not as yet been refuted. It was certainly clear that the texts contained absurdities, that they described geometrical monsters; however, nobody had succeeded in developing a method which could support, by acceptable argumentation, the conviction that the totality of these texts represented a logical impossibility.

The accumulated experience of the course of the centuries seemed much rather to point to another impossibility: once the thing was expressed, it seemed to the thinking mind to be impossible to rid itself of this centaur of geometrical mythology. For each commentary also contained within it the text on which it made a commentary, and consequently it added new non-Euclidean monstrosities to the monsters already present in the classical text and, in the absence of a valid refutation, it preserved them intact for the future and, counter to the explicit wish of the author, it transmitted them over time and always raised them to a higher level of knowledge.

The result of the attempts at refutation of the non-Euclidean world was that at the beginning of the nineteenth century there was a *palimpsest* whose superimposed strata contained the entirety of all the commentaries on Euclid. The verbal space of the geometrical universe of discourse presented two poles: Euclidean argument and non-Euclidean counter-argument recited there their uninterrupted monologues, one alongside the other – together, parallel, simultaneous.

The only logic which remained clear and intangible was this: if the non-Euclidean world was the being and the true, it necessarily followed, by logical necessity, from its negation that the non-Euclidean world had to be non-being and false. Only one, *at most*, of these two worlds could have actual being; to only one, *at most*, of these two texts that contradicted each other could the status of truth be accorded. This prohibition seemed perfectly reasonably, it even seemed a natural, indubitable, and incontestable necessity; since Parmenides and Aristotle it had even been universally recognized, without prompting the least opposition in the form of the logical axiom of the excluded contradiction.

The logical axiom of the excluded middle is, by contrast, an order: the value of truth has to be attributed to *at least* one of the two mutually contradictory assertions.

The conjunction of the two logical axioms leads to the exclusive alternative: 'either . . . or' – either one of these assertions is true and the other false, or the other way round. But one of the two always remains logically excluded. For if the alternative is valid, then truth is unique, and existence is also unique – just like the Universe, in so far as it is the embodied hypostasis of the truth.

But which way leads to truth? And what is this truth which heralds existence? Is it the Euclidean? Is it the non-Euclidean? No reply to this question. The commentary falls silent; its text is composed only of silence.

The constitution of two poles in the verbal space points to the internal division within the *esprit de géometrie*. Certainty about the logical alternative conceals in it the absolute uncertainty and despair of the geometrical mind. Geometrical reason fell into a state of inner conflict. 'Tossed hither and thither, like life's grasshopper', between the two poles of the contradiction, the *esprit de géometrie* sank deep into the state of mind of the unhappy consciousness: 'Two souls, alas, dwell in my breast', sighed Dr Faustus, the geometrician.

For he knew that the world of his knowledge was Euclidean, but his intellect was also in possession of the Other, the Alien. This was why he was also aware that his knowledge comprehended the non-being and the false as belonging to him as his own property and that he could not free himself from it, that he could not tear out from his Self the Alien who was present within him in the condition of veritable knowledge, his Self was consubstantial with the Stranger; the Stranger belonged to his essence: he had therefore become an alien to himself. The geometric, Euclidean world, became a 'world of the mind an alien to itself', to use Hegel's fine expression.

Saccheri, Lambert, the young and truly brilliant lawyer of Cologne, Franz Adolph Taurinus, had all three pursued their geometrical activity under the sign of this unhappy consciousness.

Friedrich Ludwig Wachter was the first, in the whole of non-Euclidean history, to assign in 1816 the value of truth to 'anti-Euclidean geometry', as he called it. He was one of Gauss's most gifted pupils. Constrained by the 'either, or' alternative, he was not, however, long in coming to the conclusion that in this case 'Euclidean geometry was false'. But subsequently he modified his point of view and oscillated between the two poles of the alternative. He realized that the alternative was indecidable, and he was himself incapable of saying to which of the two systems he assigned the status of truth. Undoubtedly, one could say with Hegel that as a result 'a combat was joined with the enemy against whom victory was rather more a defeat'. The unhappy consciousness of the spirit of geometry – 'self-consciousness like a double and contradictory being' – precipitated Wachter himself into tragedy: he committed suicide when still very young. The geometrical drama determined the personal tragedy of his existence.

Gauss himself had spent almost thirty years in the tribulations of the unhappy consciousness, and the same unhappy consciousness was also the origin of the profound split which divided Wolfgang Bolyai, the father, from Johann Bolyai, the son, opposing them and uniting them in mutual enmity. To cite Hegel's reflections on the unhappy consciousness once more: they had, both of them, experienced 'the experience which the divided consciousness makes in its unhappiness'.

In the paradise of geometry there grew also a tree. Once obtained, the forbidden knowledge could never be forgotten. 'Ignorance is like innocence', the great mathematician G.H. Hardy once wrote, commenting on the resistance to non-Euclidean geometry, '. . . once lost, it can never be regained'. The spirit of geometry also became aware that it was impossible for it to escape the fall into the original sin of knowledge. And – once committed – the sin of geometry also proved irreversible. In retrospect, however, it seemed that the sin of the spirit of geometry had been its greatest virtue.

Kant's commentary on Euclid and St Thomas

Kant valued Georg Simon Klügel's critical mind very highly. As Klügel had not published anything of note apart from an undistinguished *Mathematical Dictionary*, which appeared at Leipzig in 1808, we may assume that Kant's judgement relates to the critical commentary of the *Recensio* of 1763, referred to above.

But, unlike Klügel, Kant did not see in the irrefutability of a non-Euclidean triangle a mysterious *enigma ipsius genii humanii*, but quite conversely the manifestation of the self-criticism of pure reason, whose vocation was to render transparent its hidden structure to the speculative mind becoming conscious of its own self.

The intellect thus becomes directly aware that the fight with the non-Euclidean enemy hypothesis must inevitably end with a fatality, because the opposition between the Euclidean and the non-Euclidean represented an undecidable logical alternative, also inevitable, and that, for this reason, the discursive *ratio* had no means at all to decide between them: the alternative of the Euclidean or the non-Euclidean could only, in essence, be fundamentally undecidable by means of logical inference, because the two contradictory assertions are each a synthetic judgement. For we read in *The Critique of Pure Reason* that in such a judgement we can know 'neither truth nor falsehood': the alternative cannot therefore be decided upon and remains undecidable. Which means that the same term, 'triangle' – and one may undoubtedly understand by this the concept of the triangle of absolute geometry – can be consistently connected as well with the 'Euclidean' as well as with the 'non-Euclidean' predicate. The two 'concepts', the Euclidean triangle just as much as the non-Euclidean triangle, are manifestly endowed with the logical property of 'non-contradiction'.

Some time later, in 1790, Kant also commented on Eberhardt's assertion, according to which the negation of a geometrical axiom also implied the negation of its consequences: 'There thus now exists *licentia geometrica* just as there has long existed a *licentia poetica*.' It was obviously meant sarcastically, but, whether he was aware of it or not, Kant's metaphor was not only *vera*, it was also *ben trovata*; for, for the first time, this *bon mot* ascribed to poetic licence the non-Euclidean enterprise.

It indisputably follows that the non-Euclidean assertion could not be proved. But suddenly what had until then remained hidden appeared clearly. Namely, that at the same time as the irrefutable and indemonstrable character of the non-Euclidean assertion was established, the indemonstrable and irrefutable character of the Euclidean assertion was itself also automatically established.

And this was why the value of truth, tainted with the appearance of necessity, could not be settled by means of logical inference. But from that, it follows that to decide what is logically undecidable could not be the task of discursive mind; it could, in complete contrast, only be the task, the duty – the prerogative – of the subject alone. The grammatical subject of the verb 'to decide' could only be an agent other than the Subject itself – in this case, the Subject of geometry.

The mathematician Lazarus Fuchs, one of Kant's first followers, had already asked in 1786, in his text, *On Parallel Lines*, the bold question as to whether the negation of a geometrical axiom was not itself an axiom. And his reply to the question was noteworthy: 'I do not know.' Both reply and question were not so much the reflection of sceptical doubt (*Zweifel*) as the dramatic tension of despair (*Verzweiflung*).[2]

Kant was the first to have recognized in all clarity that what Klügel described as an *enigma* was not the sign of weakness in the finite capacities of the human intellect, but quite to the contrary represented a particular characteristic of the mind which revealed the unsuspected strength of pure reason. For it was only at the moment when it became

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aware of the logically undecidable character of the alternative that the cognitive subject of geometry recognized itself in its own autonomy, as the sole authentic source of geometrical truth. This is why knowledge of geometrical truth is not an ordinary *discovery* but an 'act of the autonomy of the subject' – an act of self-knowledge of the subject in its absolute autonomy.

It was a consideration which was to change the ultimate destiny of the development of geometry: the repeated setbacks to the attempts to demonstrate the Euclidean postulate, and to refute the non-Euclidean assertions, were not the mark of an immanent weakness but of a prodigious strength of mind – the capacity to know itself and, through this recognition, to stretch its own possibilities to the infinite.

Such was Kant's commentary on Euclid.

And his meta-commentary on Maimonides and St Thomas was to say: 'It is not inconsistent to maintain – *haud absonum esse* – that, in the metaphysical sense, many worlds – *plures mundos* – could exist, if God had so desired.' Assuredly, what characterizes the diversity of these universes is the number of dimensions of their spaces – more than three – but this is manifestly only another way of designating them as non-Euclidean. Now, what is noteworthy in Kant's conception is that for the first time in the history of thought the plurality of worlds is envisaged exclusively under the concrete aspect of their geometry, as a plurality of diverse geometries distinct from the three-dimensional Euclidean space: it is conceived as a plurality of structures of 'possible types of space'.

But this is far from being the whole story.

For Kant was probably also the very first to recognize that 'several worlds could exist together' – worlds characterized by non-Euclidean structures – and he considered it 'very probable that God would have realized this in reality somewhere'. The most surprising thing, however, was what he wrote as early as 1746 on the knowledge of these worlds: 'The science of all these possible types of space would indisputably be the highest geometry which human understanding could ever conceive.' But Kant saw, with the same clear-sightedness, that the ontological foundation of this 'supreme geometry' could never depend on the science of geometry itself, but only on metaphysical speculation: the possibility, he insisted, 'that numerous worlds really exist can only be envisaged in the metaphysical sense'. *Si vero admittantur plures,* erunt plures mundi, *in sensu strictissimo metaphysico* – he repeated in 1755.

This was Kant's commentary on St Thomas's repeated statements of the impossibility of a non-Euclidean creation.

And his commentary on Plato was contained within his famous thesis that the necessity of Euclidean geometry could find no basis in either *hyperouranos* or in logic (which Plato himself already knew), but solely within the interior of the subject. Admittedly, the source of the only truth (Euclidean, in this case) dwelt in the interior of the subject, but the subject succumbs to the immanent force of transcendental intuition which necessarily decides between the 'Euclidean/non-Euclidean alternative' in favour of the Euclidean.

In Kant's criticism, commentary on Euclid reached the highest point of its historical trajectory. The fog of confusions which surrounded the problem of the parallels was suddenly dispelled to make way for the crystalline form of a transparent theoretical concept. What had until then been hidden suddenly became visible and revealed its truth: suddenly it became possible to recognize the veritable locus of the question and understand the true problem; henceforth, the true problem could finally be articulated clearly, and simultaneously appeared what had always been at issue, behind the confused mass of technicalities of a two-thousand-year-long undertaking. It finally appeared in all clarity that the locus of the true problem did not lie in geometry but in the transcendent space of metaphysical speculation, in the space of the mind, which is also the true locus of the science of geometry. And the true problem was the subject.

Hic Rhodus, hic salta: what is the structure of the subject, what is its distinctive specificity which obliges it to consider, with the force of necessity, the Euclidean proposition as true, and why is it impossible for the subject to consider the non-Euclidean assertion as true?

Kant's reply is well known: the only authority which grants the value of truth to Euclid's postulate and to it alone is not logic but the transcendental intuition of space; transcendental intuition is the only dyke substantial enough to hold back the non-Euclidean current of thought, increasingly raging.

Kant was undoubtedly right when he asserted that 'the possibility of a thing could never be demonstrated by means only of the consistency of a concept of that thing'. He called 'problematic a concept which contained no contradiction, but the objective reality of which could not be known in any way', and he declared that 'the extension of such concepts was void'. However, understanding could 'extend further forwards' in the logical deployment of the hidden meanings of this concept – this concept was therefore thoroughly 'useful' and even 'inevitable, for limiting the arrogance of the senses'.

But as a consequence, the understanding does not continue to dwell any less within the substance of what is purely 'problematic', that is to say, within the realm of an open problem. It is clearly a case of an ontological problem questioning the existence of an object which was able to correspond to the concept. For without an object which the concept denotes – as its *significatum* – it seems impossible to 'make it understood' that such concepts 'signify something'. Only knowledge of the object which it denotes can make the concept itself 'objectively valid'.

As a result, 'understanding itself seems a problem', for purely discursive thought has no other means available 'to know its object'. But this cognitive power only characterizes the non-sensitive, purely transcendental intuition of space. Clearly, Kant's reflections concern the objectional hypostasis belonging to a concept, the strange act by which an object is assigned to a concept.

But the Kantian postulate of non-sensitive transcendental intuition whose task would be to assign an object to a non-self-contradictory concept is revealed as the Achilles' heel of his conception. For if empirical intuition cannot be brought into play in the operation, why would it be impossible for the subject – on the sole basis of its freedom – to assign a non-Euclidean triangle to its self-consistent concept, if it is possible for it to assign a triangular object to the Euclidean concept of the triangle in precisely the same way – that is to say, independent of all empirical observation and all discursive thought? And if the Euclidean concept is just as consistent as the non-Euclidean, then not only is the 'non-contradiction' of the non-Euclidean 'inevitable' in the refutation of the empirical origin of the Euclidean object, but conversely, moreover, the logical non-contradiction of the Euclidean amounts purely and simply to the purity which no sensitive contact with the non-Euclidean object sullies.

And if, however, the assignation of the object to the self-coherent Euclidean concept is an act of transcendental subjectivity, and if the source of truth is to be found within the subject, then the question is knowing what could have prevented the subject of geometry from constructing, in precisely the same way, the non-Euclidean object in its objectional hypostasis to assign it to the non-Euclidean concept, also free from logical contradiction? Should one not rather consider that the limitation of the constructive capacity of the subject reduced to Euclidean objects should be viewed as a self-limitation which contradicts the very essence of the subject, its freedom?

This question of the intimate construction of the subject has considerably broken down the barrier of transcendental intuition. In any case, to attain its immanent *telos* – the non-Euclidean future – the flow of the past, on its course towards the apparent surface of events, was destined to encounter the dyke erected by Kant and to go through *the eye* which would open the way for it.

Would Kant have developed the concept of transcendental intuition in order to prevent the birth of non-Euclidean geometry?

Yes, undoubtedly!

And it was against precisely this concept that his most faithful disciple, Gauss, was to rebel, as well as Bolyai and Lobachevsky; they, too, were well aware that with the foundation of non-Euclidean geometry they had supplied the peremptory proof of the non-existence of transcendental intuition of space.

This was a matter of fact: the idea of the transcendental intuition of space is irreconcilable with the simultaneous truth and presence of a Euclidean geometry and a non-Euclidean geometry with the same rights to existence, the same rights to citizenship in the realm of the *épistémè*.

And yet the assertion, 'without Kant, no non-Euclidean geometry', is no less true.

It was, moreover, 'the only possible case' of which Kant spoke in his *Prolegomena*, 'where *reason* manifested its *secret dialectic* against its own will'. And in the non-Euclidean developments, what manifested this secret dialectic of reason was that 'on a universally admitted principle, it founded an assertion, and yet from another principle, itself also universally assessed, it deduced an opposite assertion, with the most rigorous logic there could be'. For did the non-Euclidean axiom not have the same rights, in all respects, as the non-Euclidean assertion, 'was it not quite as well attested as the Euclidean axiom'?

Non-Euclidean geometry: the commentary denies the work

The founders of non-Euclidean geometry, Gauss, Bolyai, and Lobachevsky, and, immediately afterwards, Riemann – the four apostles of the New Testament of geometry – could have read all the fundamental theorems of the future non-Euclidean geometry on the surface of the palimpsest which lay before their eyes. But they developed these propositions and demonstrations autonomously, and independently from one another – just like many classical commentators. They belonged to distinct generations, very distant in time and geographical space, and yet the decisive new non-Euclidean idea appeared in their consciousnesses almost simultaneously: *Herr: es ist Zeit* – 'Lord, the hour has come', and the *Weltgeist* – which is in itself always and everywhere present in the strictest contemporaneousness, speaks all languages and dwells constantly in *Cosmopolis* – the Weltgeist cast its shadow on the sundials everywhere simultaneously: at Göttingen, at Kazan', at Marosvasarhely.

Thus the question is justified: what was the specific increment of knowledge which raised this simultaneous act to the status of a unique event? An event which took place in the mind, and for the mind: 'A great event for the mind', to quote Paul Valéry's formula, as fine as it is expressive.

The essential quality that made their personal contribution a singular and truly great event was quite simply the fact that they had read this text in an entirely new fashion, and they had commented upon it and interpreted it in a spirit entirely different from all their predecessors.

The great event was a *magna conjunctio* in the spheres of the meta-galaxy of the mind: the two great lines of thought – geometry and philosophy – met each other once more. Hence, as for Valéry ('These two great lines had to meet') the meeting at Erfurt between the two great predators, Napoleon, the great prince of the world, and Goethe, the great prince of poetry, so the meeting between these two great lines of geometry and philosophy was an event of the universal spirit, stamped with the seal of necessity.

The non-Euclidean text was composed in the ritual mother-tongue of geometry, in the Euclidean idiom of the *Elements*; all the architecture of the non-Euclidean construct obeyed the strict rules of the same Euclid, who had built his own temple to the god of geometry at Alexandria. No new technique of demonstration, no formulation, nor demonstration of some fundamental new theory: it was an entirely new and unexpected hermeneutics of the existing geometrical text which lead to the non-Euclidean *magna instauratio*. Thus, the new geometry had perfect grounds for triumphally bearing the word 'revolution' on its banner – a term which the excellent Otto Liebmann was the first to make out in 1871. And the interpretation did not enter into the field of competence of computational reasoning.

Hermeneutics was exclusively the work of philosophical speculation.

Karl Marx's view on Feuerbach is famous: 'Philosophers only interpret the world in different ways; but what matters is transforming it.' We shall leave open the question of knowing whose task it is to change the world of human relations. But for Gauss, Lobachevsky, and Bolyai it was precisely the opposite which mattered: interpreting the existing world of geometry.

The result of this philosophical interpretation was a radically new world, which no one would have dared suspect before: the non-Euclidean world.

Gauss resumed his commentary on Euclid in a single word, 'No!' – and it was he, too, who made the negation the programmatic prefix of the name which he was to choose for the new system, in 1824: 'non-Euclidean geometry'.

Gauss was renowned for his laconic character – his motto was *pauca sed matura* – and he effectively limited his commentary on Euclid to a single word, 'No!' This single word founded the whole genealogy of the new geometry.

The formal act of 'onomaturgy' quite simply sanctioned an objective event, an event which is always designated in everyday language by the word 'birth'. A new being was born; a new world, the non-Euclidean world, saw the day. The new exegesis wanted the death certificate drawn up by Saccheri to be read and interpreted in retrospect as the birth certificate of non-Euclidean geometry.

And the gospel of the new geometry brought the good news: in the beginning was the word: 'Let it not be! And it was!'

The hermeneutics of Gauss's geometrical 'onomaturgy' was based upon an entirely new philosophy. Just as the names 'imaginary', 'fictitious', 'false', and 'sophist' (which the sixteenth-century Italians had given to the entities they had created) heralded, in flagrant opposition to the habitual connotations of these terms, the real existence of worlds of other

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numbers, radically new, in the same way the negative prefix to geometry's own name did not in any way signify the elimination, and still less the destruction, of the Euclidean work. The philosophical credo of this strange negation was not to destroy the work but to create it anew, to build a new system which was itself a Work and represented a world – at the same time and alongside the other – which had its place in the same universe of the *épistémè*, was not content with coexisting with it but confirmed and justified its existence with new arguments. The commentary which affirmed the truth of the non-Euclidean axiom simultaneously confirmed the truth that the work of Euclid concealed at the outset: the axiomatic status, the logical independence of the proposition, which had already been postulated by Euclid as an indemonstrable axiom.

'Your first word was NO', is the last word of Valéry's *Mon Faust*. The new negation which founded non-Euclidean geometry was creative and resolutely constructive.

This single word 'No' resounded as if a double negative had been struck.

At first, it was the original content of the Euclidean assertion which was denied and replaced by its opposite. The first negation was a simple logical operation carried out in the verbal medium of the object-language.

But the value of the false was simultaneously denied, which, by virtue of the logical alternative, necessarily affected the negation of the non-Euclidean assertion. And, opposing the ever-present logical constraint, the non-Euclidean proposition saw itself conferred the status of truth. Manifestly, this second negation could only be realized in the highest spheres of the meta-language.

In this double furrow was manifest 'the enormous power of the negative', of which Hegel spoke in the preface to *The Phenomenology of Mind*: the enormous power of the negative which manifested itself in the non-Euclidean event as 'energy of thought, of the pure Self'. This power is the 'magic force' which 'converted this negative into being'; it is the 'absolute power' by which 'what is bound conquers its own existence and its separate freedom'.

The non-Euclidean commentary as confirmation and Aufhebung of the Euclidean work

Thus the great negation which heralded non-Euclidean geometry resolutely contradicted the resounding exclamation which sounded at almost the same moment: 'God is dead!' Scepticism never found allies in the revolutionaries of geometry, and relativism had never found admission into the metaphysical embedded space of geometry; nihilism could not count on the least sympathy.

No rebellion, no *coup d'état*, no usurpation – the non-Euclidean revolution was not a change in power. Admittedly, it had denied Euclid, but by preserving and prolonging him, and elevating him to an infinitely higher place.

The non-Euclidean revolution put an end, not to Euclidean truth, but to the exclusive reign of something still more sacred: the logical axiom of non-contradiction. This axiom certainly presented its validity, but this was henceforth confined to the interior of a universe: in the very interior of the Euclidean world on the one hand and the non-Euclidean world on the other, its power remained intact. It was only its absolute universality which had had to be renounced: it was no longer competent to govern the intercosmic spaces which enclosed the plurality of the opposed worlds. The axiom of non-contradiction could no longer exercise its power in the embedding space of the speculative mind; Euclidean truth and non-Euclidean truth were present there simultaneously, enjoying the same rights of citizenship in the world of truth, citizens of the world of the *universitas scientiarum*.

'Rights equal to citizenship': Gauss was the author of this metaphor, which Félix Klein and Henri Poincaré were to cite.

Its political dimension is evident.

But one would be mistaken to attribute this to mere chance; it is not the work of a verbal caprice. Gauss was very conservative and a convinced monarchist, but supporter of a strictly constitutional monarchy. At the period when he devised the metaphor the question which lay at the centre of the political confrontations in Germany was to determine whether civic rights should be accorded the Jews. Without the least hesitation and the greatest steadfastness, Gauss took the side of equality of civic rights: for all, without distinction.

The non-Euclidean reversal was a political revolution, a revolution whose covert spirit was guided by political sense.

But in this 'fair Polis of geometry' – γεωμετρίας Καλλίπολις – as Plato calls it in *The Republic*, in this cosmos of truth, the reign of the axiom of the excluded middle is supreme. The order of the excluded middle is the specific axiom of the subject. For the essence of the subject is freedom. And freedom can only assert and demonstrate its reality by proclaiming its actual presence through the act of choice and decision in the accomplishment of an action, that is, in manifesting itself as a true 'state of doing' (*Tat-sache*). And it is precisely the excluded middle that seeks the act. But the act which founds a geometry and thereby creates a world consists in assigning truth: the value of truth is assigned by the subject of the praxis to an axiomatic proposition whose truth is impossible to establish by means of logic – to say nothing of empiricism. What is excluded there is the third term: not acting, leaving the proposition indemonstrable and irrefutable in a state of undecidability, assigning it neither the true nor the false. But a subject which does not take decisions and which does not act is not one, it suppresses itself, it ceases to be a subject.

The excluded middle contains a commandment which only a free subject can carry out: to attribute truth to *at least* one of the two opposed and logically independent, logically undecidable statements. *At least* does not, however, mean *at the very most*: the order of the excluded middle does not contain any prohibition on simultaneously attributing the value of truth simultaneously to the two opposing propositions.

Whether the subject considers only one, or both, true, depends only on the decision, the freedom of the subject, and not remotely on the subject itself. But in order for the subject to realize that liberty which is its own, it has of necessity to attain a new state, a higher level of consciousness.

What the subject has become aware of is that the object of the decision does not concern geometry alone; the important thing is not to know whether truth should be assigned to the Euclidean statement or to the non-Euclidean statement, but to know that this question concerns the subject: is the subject going to remain in the unhappy state of fission, the eternal split between the Self and the Other, the Alien, or be reconciled with itself, return to itself?

To the Euclidean Self, the Other has long appeared in non-Euclidean form. This apparition placed the Other beneath the eyes of the Self, like a projection of the Self to the exterior. The Other therefore had its place outside the Self – its persistence was an exteriority. And as the Euclidean, whose subject is the Self, includes the sphere of the totality of being, the exteriority of the Other, of the non-Euclidean, is a presence according to the modality of non-being. The new state of consciousness which opens the era of the non-Euclidean, in the specific sense of the term, is born as a consequence of the re-interiorization (*Er-innerung*) of the re-recollection of the exterior. Consequently, it is not the being which is called from oblivion to recollection, it is the non-being which is released from its status of externality and integrated in the being. The Self integrates the Other, recollecting itself, taking it into itself as belonging to itself. Thus from the outset – in the original meaning of the verb ἐξίστασθαι – the ex-sistence of the other, the non-Euclidean, is revealed as having been an authentic *existentia*, as the end of Book II of Plato's *Republic* hints in a fascinating metaphor: as if by divine magic, somebody departs from, ἐξίσταιτο, their own form, ἰδέα, and either changes themselves by means of their own selves, αὐτὸ ὑφ' ἑαυτοῦ, or by means of something outside themselves, ὑπ'ἄλλου.

The newly won knowledge raises the subject from the condition of alienation and prompts its return to itself: the subject becomes aware of the fact that knowledge of the Other and the Alien is also knowledge of its own Self.

This return also signifies the reconciliation of the subject with itself.

The subject thereby raises itself to the state of self-consciousness. It becomes aware that knowledge of the Other, in the hypostasis of the Alien, is in fact an act of self-alienation. It therefore also becomes aware that it is only by simultaneously choosing both – the Euclidean and the non-Euclidean – that only then does it remain close to itself, for 'the Other' – whether represented by the Euclidean or the non-Euclidean – 'is always immediately present in itself', as we can read in Hegel's *The Phenomenology of Mind*.

It is this absolute freedom of the subject which has become conscious of itself which became visible in its reality and its truth by means of the foundation of non-Euclidean geometry.

The freedom of the subject is radically opposed to the arbitrary.

Choosing one or the other, attributing truth *either* to the Euclidean *or* to the non-Euclidean, is always an arbitrary act, an act which precipitates the consciousness into unhappiness and into that unendurable state where it is ceaselessly exposed to the caprices of perpetual and constantly reversible oscillations between the two poles of the contradiction.

When the subject decides *either* in favour of the truth of the Euclidean *or* of the truth of the non-Euclidean, it knows itself to be true as regards its Euclidean self, but at the same time it recognizes its simultaneous consubstantiality with the Other and the Alien, the non-Euclidean. The Other henceforth appears to it like the other Self. And the decision to recognize its own Self in both at the same time – the Euclidean and the non-Euclidean – to accept them simultaneously, to recognize them as the inalienable property of its essence, is specifically what opens knowledge to the subject and gives it the certainty of its authentic freedom.

But freedom also involved the irreversible certainty of 'self-identity', of the absolute consubstantiality of the subject with its two hypostases, the Self and the Other.

The Other, the non-Euclidean commentary on Euclid, becomes the Work. The slave becomes aware of himself as a sovereign being, and he becomes his own master. The commentary is raised to the status of work and takes an autonomous position – like another Self – in the universe of the *épistémè*.

The limit of freedom is not necessity, but the arbitrary. Knowing this, first stated in Spinoza's *Ethics*, opened the path towards a higher and radically new level of consciousness.

The choice between two opposites which are both in the field of the purely *possible* is always the work of the arbitrary. And the works of the arbitrary are reversible.

Freedom opposed to arbitrariness is characterized by a specific dimension of necessity. The works of freedom are to be found in the realm of the *necessary*. '*L'esprit libre aime ce qui est nécessaire*', wrote Albert Camus in *L'Homme révolté*.

The works of freedom are irreversible.

'*Mentis Amor intellectualis erga deum, sive Libertas*' – the truth and reality of freedom consist in the decision of the subject in favour of necessity, in the presence of a constraint opposed to its deployment. This is why it is not possible to identify freedom with the preferential choice, προαίρεσις in the absence of constraint, of Aristotle. For such a choice does not exclude the arbitrary. Just as true peace is not *belli privatio*, true freedom is not the absence of constraint, but '*virtus est, quod ex fortitudine animi oritur*', as Spinoza states – a force which emanates from the interior of the subject and is opposed to the existing constraint.

The decision of the subject, having in its freedom become conscious of itself, simultaneously to attribute truth to the Euclidean axiom and to the non-Euclidean axiom appeared as a necessity which opposed the persistent constraint of discursive logic, that of the axiom of the excluded contradiction. Thus the new era of the – unprecedented – deployment of the totality of mathematical thought, both theoretical and historical. Thus the new mathematical world was born, a set of universes structured in an enantiomorphic fashion in relation to one another. And the existence of this non-Euclidean meta-galaxy is irreversible.

'Man is free', Hegel wrote, 'but he does not know it. Therefore he is not free.' Which means that the freedom of the subject only attains the ontic modality of actual reality when the subject becomes aware of its freedom. The consciousness of freedom determines its freedom of being. This freedom of being is a conscious being. Freedom *is* when it is present as self-knowledge. Being free means recognizing oneself in one's freedom. The actual reality of freedom is the presence of self-awareness.

It was only with the institution of the non-Euclidean that geometry was raised to the condition of consciousness of its freedom. Which amounts to saying that it was only with non-Euclidean geometry that freedom became an effective reality for mathematics as a whole.

In the historical context of events, the hermeneutics of the non-Euclidean event decodes freedom's hidden manifesto.

Through *Letter VII* and *The Republic*, but most acutely in the *Epinomis*, we know how Plato despised the ridiculous name 'geo-metry' – surveying – by which was designated the science, freed from all earthly soiling, of pure forms and eternal truths. But even if the forms are pure, the pages of the book where its diachronic history is written are soiled with the visible traces of its base earthly descent. In fact, according to a famous account by Herodotus, its birthplace was the Nile delta. But neither on the earth nor in the heavens was there a Nile or an *agri-mensor*, whether Egyptian or non-Egyptian, which could have measured the square cited by Aristotle in his *Eudemian Ethics*, a square the sum of whose angles was equal to eight right angles, whose diagonal was equal to two sides, and whose perimeter represented a single straight line closed upon itself. It was not only its heteroclite forms and its truth; it was also the non-Euclidean history which was absolutely pure, free from all trace of earthly origin.

The non-Euclidean genesis was a parthogenesis.

Only the non-Euclidean conception was immaculate.

Kant's question 'How is *pure* geometry possible?' concerned Euclidean geometry, it therefore meant, truth to tell: how that which is, is possible?

But the only geometry to be *pure* in all respects is the non-Euclidean. In these circumstances Kant's question must be reformulated as follows: how that which is not, which is not possible, how is the impossible possible?

The hermeneutic circle closes: Beltrami and the Italian interpretation of non-Euclidean geometry

Eugenio Beltrami's commentary on the text of the geometry closed one era while at the same time inaugurating a new one. A new era, since, for the first time in the history of mathematics, a professional work included the word 'interpretation' in its title.

'Interpretation', a mathematical *terminus technicus*? Until Beltrami's *Interpretazione*, this word had only been found in the vocabulary of hermeneutics, a branch of metaphysics!

What Beltrami achieved in his *Interpretazione della geometria non euclidea* was the construction of what he himself called a 'model' of the non-Euclidean world within the Euclidean world. It was actually a case of a geographical map, a *mappa mundi* of the non-Euclidean universe, immediately followed by simpler models of the same kind, eliminating certain distinctive characteristics and limitations of Beltrami's pseudosphere, constructed by Félix Klein and Henri Poincaré. Poincaré's open circular disc, for example, represented the world map of an infinite non-Euclidean universe in its integrity. Given that the disc was open, the orthogonal arcs of the circle were equally open at their two extremities: these arcs were open at their extremities, they were closed by neither a first nor a last point, their terminal points did not belong to them, they belonged – at the same time as the periphery – to the complementary part of the space of immersion of the Euclidean plane. The Euclidean periphery of the circle represents the Absolute of the non-Euclidean plane, an infinite line closed upon itself which, strangely, is a straight line closed on itself possessing the same topological structure as an ordinary circle. The periphery of the Euclidean circle is not part of the map of the world, but it continues to enjoy a well-founded Euclidean reality, and it marks the complementary part of the plane. But the Absolute which corresponds to it in the non-Euclidean world is found beyond the ontic domain of the non-Euclidean universe, its locus is situated in non-being, in the complementary part of the whole being which encompasses all that is. The Euclidean world is also limited by an Absolute whose topological structure is, however, much more complicated, even extremely paradoxical in relation to the Absolute of the non-Euclidean world. The Absolute – both Euclidean and non-Euclidean – is always a categorical non-being in relation to one or other of the opposing worlds. The term 'Absolute' – an expression of purely metaphysical resonance – has been applied to it by one of the most positive mathematical minds of the nineteenth century, Arthur Cayley. And with good reason: for it is the Absolute – therefore a geometrical non-being – which is the ultimate foundation of the geometrical being; it is the topological structure of this non-being, the Absolute, which necessarily determines the geometric structure of the being, the specific nature of the world of real geometrical objects.

Every first-year student in maths, physics, or technical sciences can display – as an exercise – the internal geometry of the circular Euclidean disc. And they will soon arrive at the result that three arcs of the orthogonal circle which intersect within the circular disk form a triangular configuration the sum of whose three angles is less than two right angles. Again very easily, they will discover the existence of configurations whose three sides – orthogonal at the periphery – osculates mutually each other, the reason why the sum of the angles of this trilateral equals zero.

But what does all this have to do with non-Euclidean geometry?

Nothing indeed, nothing.

For what is advanced as evidence here is the internal geometry of a finite Euclidean circle, and nothing else. And the composition of this little geometrical night music takes place with the immanent necessity of a chain of logical consequences in Euclid's context of axioms and theorems. Non-Euclidean knowledge is not necessary for it. And the internal geometry of the circular disc can be studied just as well if one categorically denies the existence of a non-Euclidean world.

But to establish a relation of correspondence between the open but finite circular disk of the Euclidean plane, and the plane, also open but infinite, of another completely foreign world, a non-Euclidean world, the presence of a subject is imperatively required. This subject, it seems clear to me, cannot be a surveyor, it cannot be an internal subject whose role is limited to the cognition of the immanent Euclidean geometry of the finite circular disk. This subject must be familiar with both, has to be as familiar with the Euclidean circular disc as with the totality of the non-Euclidean world, and must also be capable of making comparisons between them, mediate between the two, to establish whether there is any similitude, any correspondence, between them.

This subject must therefore be amphibious.

It is essential for this subject to be amphibious, a citizen of two opposing and irreconcilable worlds: it has to be immersed simultaneously in the two worlds; but it must besides, at the same time, have a place situated outside the two worlds and occupy it effectively. For, in order to make comparisons between them, to mediate between them, these worlds must be viewed from the outside, as given totalities.

A cognitive subject, the subject which recognizes the *mappa mundi* of an infinite non-Euclidean world in the internal geometry of the Euclidean disk, can only be a poet.

Yes, a poet.

For it is the task of the poet to recognize in the orthogonal arcs of a circle on the periphery of the Euclidean plane – which is the natural environment in which he lives – a metaphor for a non-Euclidean straight line, and to recognize in the banal play of its orthogonal arcs of the circle an exotic and exciting spectacle, the allegorical representation of a foreign world, the theatrical representation, a θεώρημα, of the non-Euclidean beyond on the circular stage of a Euclidean amphitheatre. 'Black milk of the dawn, we drink the night . . .' – the poet assigns to the word, which has a well-established significance in his native language, previously unknown and foreign meanings, meanings which contradict

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those which are known. He abuses the word in order to give it a radically new interpretation, and by means of this paronomasia he offers his own world of feelings to the person who welcomes within himself the new meaning of the word.

The amphibology of words is the vital element of poetry.

With Beltrami, it appeared that in the exact science of geometry the amphibology of language also played an essential role, that here, too, artistic paronomasia was extremely fruitful.

Mathesis and poïesis: it is only in the two realms of the mind that the amphibology of language reveals itself as the source and vehicle of new meanings of knowledge.

And geometry is in this sense different only by its extraordinary simplicity from the incomparable sophistication of poetry.

The paronomasia which lay at the basis of Beltrami's *Interpretazione* is also characterized by its prodigious simplicity, and the Italian geometrician himself provided the key to decode it.

The interpretation of Poincaré's metaphor is simpler still. The original expression of the Euclidean mother tongue, 'orthogonal arc of circle', means 'non-Euclidean straight line'. All at once the primitive term 'straight line' becomes ambiguous. For one and the same subject, the term designates a rectilinear object in the non-Euclidean world at the same time as a circular object in the Euclidean world. The banal expression in the Euclidean language, 'finite and open disc', an expression which designates a finite circular disc of the plane, comes to signify by paronomasia the 'infinite universe' of an invisible non-Euclidean world, situated beyond the real Euclidean universe. But this world is open in all directions, and the circular disk can only be the metaphor for it if its periphery is taken away, if, as with a nut, its shell is removed.

'O God, I could be bounded in a nut-shell and count myself a king of infinite space!' Hamlet who as king of this infinite non-Euclidean world was also its internal subject: and as such, appeared on the circular stage of the Euclidean plane playing in reality the role of the poet Shakespeare – who wrote and staged the θεώρημα. But to reassure his friends Rosencrantz and Guildenstern, who undoubtedly only saw within this nut-shell the banal configurations of the Euclidean world which they inhabited, the Prince of Denmark added at once, 'were it not that I have bad dreams'. And is the non-Euclidean world not quite simply a nightmare of the Euclidean subject? A nightmare represented in the well-known engraving of Maurits Cornelis Escher, *Circle Limit IV*, with the form of an endless dance of white angels and black devils within an open Euclidean disk, limited by a 'circle limit' located outside it. Poincaré's circle only represents, within the Euclidean universe, the *mappa mundi* of another, non-Euclidean, world provided that it is transformed into a hermeneutic circle. But such a transformation can only be the result of the work of *poïesis*.

Who, then, has only observed that the metaphors of the technical language of geometry have invaded rhetoric for two thousand years? No one has been surprised at the paronomasia which has plundered these terms of circle, ellipse, parabola, and hyperbole from their geometrical usage to make them designate figures of style, purely rhetorical conic sections. Beltrami's rhetoric, which stimulated prodigious interest in its day, founded a branch of mathematics – of meta-mathematics in fact – that was until then quite unknown, unexpected. A branch of mathematics that nobody would even have held for possible: model theory. One could also have called it the theory of mathematical metaphor. But the term, *Interpretazione*, which Beltrami used to describe his work was not a metaphor, was not a figure of rhetoric. Its meaning was that of the veritable and known signification of the term. For with Beltrami's *Interpretazione*, hermeneutics penetrated mathematics. And another distinctive trait of mathematics amongst all the sciences is that hermeneutics has been elevated there to the status of a professional branch.

The first significant result of this mathematical hermeneutics was Beltrami's theorem, which caused a sensation at the time, and according to which the logical absurdity of the non-Euclidean world necessarily implied the absurdity of the Euclidean world, since a contradiction in the non-Euclidean language would necessarily be translated by a logical contradiction in its Euclidean metaphor. The self-annihilation of the non-Euclidean world thus necessarily involves the self-destruction of its map situated in the interior of the Euclidean world, and involves at the same time the 'Big Crash' of the Euclidean world which contains the map as an authentic and organic finite part of it. Just as with MAD ('Mutual Assured Destruction', the insane nuclear strategy of the Cold War), if the Hot War, Father Saccheri's *diuturnum proelium*, had really taken place between the two opposing geometrical worlds, they would have been reciprocally and simultaneously precipitated into cosmic catastrophe. Saccheri's defeat was his victory. For if the conclusive scholion of his work had been revealed as true, if he had in effect completed *hujus Libri Thorematum unicus scopus*, if he had succeeded in effectively demonstrating the inconsistency of the non-Euclidean *inimica hypothesis – sibi ipsi repugnantem ostendam*, as he wrote – if he had succeeded in eradicating it totally – *a primis usque radicibus revulsam* – then he would have been forced to be present at the spectacle of the self-destruction of the Euclidean world which he had believed established on solid foundations, at the same time as the destruction of the non-Euclidean monster.

But the Italian alliance between Saccheri and Beltrami also founded an optimistic logical solidarity between the two opposing worlds. For if the Euclidean world was exempt from logical contradictions, then the non-Euclidean world could also subsist and live without logical danger.

Like Tristan and Isolda, only together could the Euclidean world and the non-Euclidean world live and die.

The Evil Demiurge and its perverted geometry in the interpretation of Plato and Aristotle

At the origin of all non-Euclidean history is to be found a commentary: Aristotle wrote a commentary on some strange heterodox results which were already emanating the diabolical smell of a geometrical heresy.

These propositions were non-Euclidean assertions which could not possibly have been born as the solution of specifically geometrical problems. They could not, and still cannot, even today, have been motivated by the requirements of everyday life, nor by technical problems internal to geometry. They were manifestly the result of purely speculative work which had as its established aim the submission to critical commentary of the text of Euclid's *Elements*, as it then existed.

In his *Posterior Analytics*, Aristotle referred to these propositions by an oxymoron as fine as it is expressive: these are non-geometrical geometrical propositions, γεωμετρικόν πως καὶ ἀγεωμέτρητον, in the sense that one would speak of non-musical music, non-poetic poetry. The comparison between geometry and art is certainly very original, but it

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is also strange. It is by relying on the parallel with art that Aristotle makes a commentary on the geometrical event, and he explains the presence of non-geometrical geometry by the fact that its heterodox propositions were undoubtedly the consequences of the geometrical ἀρχαί, axioms which would contain geometricity in a degenerate and perverse mode – φαύλως.

As Vittorio Hösle demonstrated in a brilliant essay published some fifteen years ago, in his *Kratylos* Plato dedicated a commentary remarkable in all respects to the results of the geometricians working at the Academy or gravitating round it. This geometrical perversion can only be the work of an Evil Demiurge – Δημιουργὸς κακός – which deliberately placed a false hypothesis – τοῦ πρώτου ψευδοῦς – as the first assertion, at the foundation of geometrical language, and which subsequently, from this first false hypothesis, derived a large number of theorems whose logical relation nevertheless remained mutually coherent: ξυνφωνεῖν, ὁμολογεῖν ἀλλήλοις.

Here Plato called upon an event which evidently occurred off stage, in the existential field of geometrical research, in order to refute Kratylos' thesis, according to which logical consistency would be – in itself – the supreme criterion and unique guarantee of the truth: Μέγιστον τεκμήριον τῆς ἀληθείας.

Here we most stop for a moment to take stock of and admire the categorical character, the stylistic acuity, and the intellectual density of the concision of this assertion. It is not so much its modernity – already surprising – which is remarkable, as the high level of its professional sophistication. It is scarcely conceivable that the dialectic confrontations or the exercises in judicial or political rhetoric should have lead to an experience which made it possible to state the thesis of the *criterion of consistency* of the truth with the calm certainty with which Kratylos utters it. It is only in geometrical rhetoric that, already in Plato's time, one can deduce from a given hypothesis a great number – πάμπολλα – of consequences respecting the strict criterion of rigorous logical consistency. Kratylos undoubtedly articulated a thought which had matured in the circles of the geometricians.

To the work of the Evil Demiurge who had evidently led the geometricians to develop a degenerate geometry, Plato added the remarkable observation that it was not the internal consistency of the context, but solely the knowledge of eternal realities of pure Forms which could protect geometry from the danger of compromising itself with a theory that was admittedly consistent, but false.

And this commentary which Plato added to the text, admittedly consistent but also disquieting, about non-geometrical geometry, of a false text, by which, therefore, an Evil Demiurge misled the geometricians of his day – we know this text as one of the fundamental theses of Plato's philosophy.

Text, hermeneutics, interpretation, and exegesis. Commentary and meta-commentary, super-commentary, hyper-commentary: non-Euclidean geometry was born as the final result of the uninterrupted work of commentary on the text of Euclid.

Philosophy

The dialogue between the work and the commentary took place in temporal space. Its beginning is unknown to us. Divided into two halves by the present point in time, this space is open to the future.

The substance of the time which fills this noetic space is a-temporal. It is not subject to the natural law of physical time. As human beings, locutors are crucified in a point of physical time. It is impossible for them to escape from the moment of their present into the future or the past.

But *verba volant*. Words flow with time, and in time. For discourse is free. And it is also free to move sometimes in the direction of the future, sometimes even – one suspects – in the direction of the past. The young Aristotle's suspicion that he could sometimes hear Plato in his *Parmenides* speaking to his future pupil, Aristotle the Stageira, was not entirely groundless. And one even asks oneself whether the future voices of Cantor and Dedekind were not also heard in the points and counterpoints of the arguments and counter-arguments that were carried on by many voices in the Academy. And in fact, in the whole score of the Aristotelian *corpus*, some chords are in harmony with the voices of the geometricians who were to work in the two millennia to come in the progressive development of non-Euclidean geometry.

The system of spiritual reference for the past is the present. 'There is a key for the anatomy of the monkey in the anatomy of man', wrote the young Marx in Paris, at a time when he was undoubtedly a Young Hegelian, but not yet a Marxist.

The uninterrupted dialogue of the texts

The isolated voices of the totality of intellectual space cause interference one with another, their theses interpenetrate, texts overlap, the *continuum* of the discourse is interrupted by other discourses, the medium of writing acquires a new dimension with the subtly interlinked thread of new writings. The context also reveals local discontinuities and branching cleavages. The fabric of propositions is composed of knots, thoughts are affected by osmosis, and the ideas of one system are surreptitiously imported into the opposing system.

Words play with words – texts speak to each other.

Various texts polemicize each other, they are engaged in a constant exchange of views. The verbal ways which lead from one place to another are tangled, often hidden. And yet, the global context remains a connection, its often very different constituent parts remaining inseparably bound together by the thread of contradiction.

In his metaphysics, Aristotle repeats Empedocles' fine phrase: 'Discord unites' – τὸ νεῖκος συνκρίνει. The intimate bond of self-negation is the unbreakable thread which binds the mind into a single whole.

It is difficult to orientate oneself in this space. Its structure is complex and marked by subtle differentiations. Totally unknown to the geometricians, the topology of the polyphonic text appears exotic, even mysterious. Everything happens as if a multidimensional polyhedron had been projected onto the surface plane of the writing.

Chaos and order of the mind

At first glance, the Brownian movement of verbal molecules cannot fail to give the spectator the impression of an anarchic chaos of words. They say everything that comes into their head, they constantly contradict one another, they argue and gesticulate, get excited, behave badly towards each other. One never sees the end, never any clear and unambiguous conclusion. One never sees any way out from the disconcerting *consensus omnium contra omnes*. 'To tell the truth, there is virtually no question of truth in these encounters', one reads in Leibniz's *New Essays*, 'also opposing theses are supported at different times from the same chair. Casaubon was shown the *salle* at the Sorbonne and told: Here is a place where they have disputed for centuries; he replied: What did they conclude?'

With time, however, it seems that the whole space of aleatoric discourses obeys a law of negative entropy which is in opposition to the positive entropy of the physical environment: in the space of the mind, the quantity of information is constantly increasing, despite – or because – of these perpetual disagreements.

It is difficult to escape the impression that in all this context a hidden *logos* is at work moving the verbal mass in a certain direction, and that somewhere on the horizon, in the transfinite, there was really a *telos* that the thought which seeks to think itself seeks to discern. Quite obviously, there were indemonstrable matters there, perhaps only a mystical belief. But as frequently happened with things that are based on nothing else as belief, this idea – if one really wants to follow it – could, perhaps, also prove useful even to those who did not believe in it.

In this context, it will none the less be recalled that the great physicist Ludwig Boltzmann had recognized that the causal imperative of mechanics was incapable of explaining the development of the universe, that is why he replaced mechanical determinism by the idea of a growing positive entropy which introduced into the cosmic chaos a teleological necessity and thus determined the irreversible flux of positive time.

The chaotic cosmos of the mind also seemed to obey a necessity, not of the mechanical causality type, but rather teleologically, a negative entropy which appears to be inherent to it. It is precisely this latter, the negative entropy, which grounds the constant increase in the quantity of information, and what thus determines the irreversible flux of a strange historical time (specific to the development of the mind), also negative, like its entropy.

The positive entropy progresses from lower to higher probabilities; it dissolves the least probable structures and pushes the anorganic world towards growing and irresistible disorder. The chaotic disorder tends towards the absolute stability of an amorphous mixture, and the probability of its realization is greater and always increasing.

Orientated in the opposite direction, the path of negative entropy went from greater probabilities to increasingly small probabilities. Its chances of winning the great prize of realization grow ever smaller. Yet its world exists: it is the global sphere of everything which lives, everything which thinks.

The world of the mind is a realm of the improbable.

'Casaubon was shown the *salle* in the Sorbonne, and they said to him: Here is a place where they disputed for centuries.' Casaubon replied with the question: 'What did they conclude?' The reply is now: 'non-Euclidean geometry'. And one could undoubtedly assert that its appearance was entirely improbable. For the probability of its appearance can, without the least hesitation, be evaluated as equal to 'zero'. Fundamentally, it was not a question of the aleatoric transition from the 'possible' to the actual reality – that the ergodic theorem at any rate guarantees – but of this surprising, wonderful, and even absurd event of which Plato spoke in his *Parmenides*, of this instantaneous, Τὸ ἐξαίφνης, this a-temporal reversal of non-being into being. Non-Euclidean geometry realized the sudden transition from the impossible to the real.

Palimpsest

Whether we like it or not, we all find ourselves at the centre of a world whose spherical envelope surrounds us.

In the firmament we now see a sun which is in an eight-minute-long past, immediately beside and simultaneously with it we see a star which is situated in an old past of a thousand years, or of several million years. The sky is spangled with luminous points.

Beneath our eyes is the heavenly spectacle of different pasts which all – stacked one on top of the other on the same visible surface – appears to us in a simultaneous presence.

The palimpsest of the spiritual firmament

The firmament of the mind is a palimpsest. And as it is said in Autolycus, its *epiphaneia* is constituted from a multitude of all the signs, σημεῖα, of the sphere of the universe: πάντα τὰ ἐπὶ τῆς ἐπιφανείας τῆς σφαίρας σημεῖα – the luminous signs of various pasts. And it is no small matter to decode this cryptic writing, to identify in these signs the Bear, the Dragon, the Southern Cross, and Coma Berenices, to decipher the propagation of helium there, the flux of neutrons and photons, and the black holes in the galaxies come from distant pasts.

The centre of this firmament is everywhere where there is a subject which reads its text. The subject contemplates the firmament of the texts within the spiritual sphere. Its *epiphaneia,* the surface on which the writing appears beneath his eyes has a negative curvature. It is undoubtedly the sphere of the world of which Hermes Trismagistus spoke in his *Pimandre.*

Reading the text means remembering the past. By this means what is outside, the text, is projected into the interior of the subject. All reading accomplishes a transformation of the text which exists objectively and by itself outside the subject, a transformation which geometry knows by the name of 'inversion around a sphere'. Everything which is on the outside, in the space which surrounds the sphere, is reproduced in its interior, and simultaneously the interior is projected outside. The inversive sphere is fixed and, as an autonomous whole, it is immobile: transformation inverts it into itself.

Every subject is a sphere of inversion of the objective mind. The centre of the spiritual sphere is the locus of the subject which inverts the exterior into its interior, which simultaneously memorializes diverse pasts. What has happened has not disappeared, does not simply dissolve into the empty nothingness of oblivion. Memory is a specific ontic state of the being of the past in the present. What has been becomes present, and the interior space of the self, its memory, thus fills with a meaning. The subject reads the signs of the external text on the inside, whose curvature is negative, of the *epiphaneia* of its spiritual firmament.

The exterior is the past, everything which has been and is no longer. The past is the non-being, and through the fact that it is memorialized it is raised to the status of being (of the actual being of its own present) by the subject. 'Nothing is more present than the past', Sarah Smith said to me once.

Omnis lectio est selectio. And all choice is itself an interpretation.

In my work, *Palimpseste: Propos avant un triangle,*[3] I attempted to propose to the reader an intertextual configuration which united in a unique configuration certain texts which I pulled out of the amorphous mass of those which exist. They were not ordered following the alphabetic taxonomy of an encyclopaedia, nor the chronological sequence which a history book requires, nor by the systematic structure of the disciplines. Gathered into an a-temporal simultaneousness, Aristotle and Lobachevsky appeared there, together with Euclid and Bolyai, Plato and Dedekind with Eudoxus and the Cardinal of Cusa, Zeno, and Saccheri with Hegel and Cantor, Plotinus and Mallarmé with Hilbert; Descartes and Valéry with Franz Rosenzweig and Thomas Aquinas; simultaneous pasts, they were all gathered together, *hic et nunc*, in the present of a sole totality whose connection was commentary.

The decoding of written signs, first and foremost scattered and unconnected, seemed to enable me – require even – to assemble them into one sole and indissoluble sign of the zodiac of the galactic flow of thought. This is the star which guided the totality of the diachronic development of the non-Euclidean idea.

Commentary: dialogue between texts

It is not only commentary and interpretation, but also decoding which is necessarily a dialogue. The result is in fact a first commentary, which re-covers part of the existing palimpsest with a new layer of signs.

Commentary is a dialogue with what has been.

The time assigned to it is the internal time of the subject. And this internal time of the subject is itself also inverted. The time of the *Historia* – which looks directly at the spectacle of the past and observes it retrospectively – is negative; it is orientated from the present in the direction of the past. It contrasts with natural time not only in its inverted orientation but also, and most particularly, by the fact that it does not pass, and cannot pass. For its dimension – its geometrical extent – is nil. The time of the *Historia* is artificial, it is an artefact of the subject – of the cognitive subject of the past – it is an expressly paraphysical time. For just as the total constant mass, energy, and radiation of the physical cosmos of the future was condensed into a single point of the four-dimensional diverseness of space and time at the moment of the Big Bang (as testified today by the cosmologists, who are its historians), so the total mass of the past of the spiritual cosmos of thought is concentrated at the a-temporal point of the consciousness of its cognitive subject. It is the *topos* of the eternal *now,* situated outside space and time, the place without a home-land where the subject of the *Historia* has its seat.

The subject does not abandon its place. It is Zeno's arrow, '*cette flèche ailée, qui vibre, qui vole et ne vole pas*', as Paul Valéry put it so beautifully. Its substance is composed of what has been. The totality of negative time is focused in this indivisible place. There the accumulated richness of the past is preserved to be transported further forwards, enriched by the values of the present, higher up its historical trajectory, in the direction of the future.

New words pierce the contextual fabric of the *corpus* of the author long disappeared, whose language has been declared a 'dead language'. But solely as a result of the fact that his or her text is spoken of, their language continues to speak in the eternal present of the intertextual milieu.

Reden ist Leben, schweigen ist Tod – speaking is living, silence is death.

And because of the fact that yesterday's text nevertheless continues to be read and commented on today, the Other adopts it in its thought, makes it its own and raises it to the status of a present-being. 'You think me, therefore I am.'

In its own meta-language, the commentary speaks of the text as an object of autonomous knowledge. Thus one necessarily attributes to the past a place in the interior of the present. The commentary interiorizes (*er-innert*) the past, so that it thinks it by means of its thought. It integrates the text of the past in the interior of the living consciousness of the present by the fact of speaking of it and about it.

Through this integration into its own consciousness, the commentary confers on the past an actual present in the world of being. The commentary gives life to the past and 'fills it with days'.

The firmament of texts, the *philosophia perennis*, is the imperishable palimpsest of the mind.

Imre Toth
Paris
(translated from the German by Denis Trierweiler;
translated from the French by Juliet Vale)

Notes

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1. I refer the reader to my book (1994), *I paradossi di Zenone nel Parmenide di Platone* (Naples: Istituto Italiano per gli Studi Filosofici).
2. The author makes play here on the German word, *Zweifel*, 'doubt': literally, 'split in two' (translator's note).
3. Imre Toth, *Palimpseste: Propos avant un triangle* 2000 (Paris: Presses Universitaires de France).