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Merging neutron star binaries: equation of state and electrodynamics

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Abstract. Merging neutron star (NS) binaries may be detected by ground-based gravitational wave (GW) interferometers (e.g. LIGO/VIRGO) within this decade and may also generate electromagnetic radiation detectable by wide-field, fast imaging telescopes that are coming online. The GWs can provide new constraint on the NS equation of state (including mass-radius relation and the related nuclear symmetry energy). This paper reviews various hydrodynamical and electrodynamical processes in coalescing NS binaries, with focus on the pre-merger phase.

Keywords. pulsars – neutron stars – gravitational waves – binaries

1. Introduction

I was asked to talk about thermal radiation from isolated neutron stars (NSs). In this meeting, George Pavlov reviewed the X-ray properties of pulsars and thermally emitting NSs (see Kaplan *et al.* 2011), and Wynn Ho discussed central compact objects and their magnetic fields (see Halpern & Gotthelf 2010; Shabaltas & Lai 2012; Vigano & Pons 2012). Recent works on theoretical modelling of NS surface emission can be found in Potekhin *et al.* (2012) (see also Pavlov *et al.* 1995; Harding & Lai 2006 and van Adelsberg & Lai 2006 for reviews). Since these subjects were adequately covered in the meeting, I decided to focus on a different topic that did not receive much attention in this meeting but is likely to become increasingly important in the coming decade.

Merging NS binaries have been studied since 1970s, with major activities in the relativity community since the early 1990s because of their importance as a source of gravitational waves (GWs) (e.g. Cutler *et al.* 1993). They are of great current interest for two reasons: (i) Merging NS/NS or NS/Black-Hole (BH) binaries have been identified as the leading candidate for the central engine of short GRBs (Berger 2011). They are also expected to produce optical and radio transients that may be detected by wide-field, fast imaging telescopes that are coming online (e.g. PTF, LSST) in the next few years (Nissanke *et al.* 2012). (ii) After several decades of promise, gravitational wave astronomy in the Hz-kHz band may finally take off in the next decade. The initial LIGO reached the design sensitivity ($h_c \simeq 10^{-21}$) in 2006, and the enhanced LIGO (with a factor of 2 reduction in h_c) is taking or analysing data. The Advanced LIGO and VIRGO are expected to begin observations in 2015 and reach full sensitivity (a factor of 10 reduction in h_c) in 2018-19 — at which time the detection of GWs from many merging NS binaries seems guaranteed.

The last three minutes of a NS binary's life may be divided into two phases: the inspiral phase, producing quasi-periodic GWs, and the coalescence phase, where physical collision results in "messy" GWs. The recent years, 3D simulations of the final merger in full general relativity (GR) have become possible (see Shibata & Taniguchi 2006; Foucart et at. 2012; Sekiguchi *et al.* 2012). It has long been recognized that the final merger waveforms can provide a useful probe of NS equation of state (EOS; e.g., Cutler *et al.* 1993; Bildsten & Cutler 1992; Lai & Wiseman 1996; Wiggins & Lai 2000). The idea is simple: By measuring the "cut-off" frequency $\propto (GM_t/R^3)^{1/2}$ associated with binary contact or tidal disruption, combined with the precise mass measurement from the inspiral waveform, one can obtain the NS radius (recent numerical simulations can be found in Bauswein *et al.* 2012; Sekiguchi *et al.* 2012; Faber & Rasio 2012).

In the following sections I will focus on the pre-merger phase.

2. Hydrodynamics of merging NS binaries

Prior to binary merger, tidal effects may affect the orbital decay and the GWs. There are two types of tides: *equilibrium tides* and *dynamical tides*. The equilibrium tides correspond to global deformation of the NS, which leads to the interaction potential between the two stars (with the NS mass M and radius R, the companion mass M' – treated as a point mass, and the binary separation a)

$$V(r) = -MM'/a - \mathcal{O}\left(k_2 {M'}^2 R^5/a^6\right), \tag{2.1}$$

where k_2 is the so-called Love number. This would lead to a correction to the number of GW cycles, $dN = dN^{(0)}[1 - \mathcal{O}(k_2 M' R^5/Ma^5)]$. For a Newtonian polytropic NS model, simple analytic expressions can be found in Lai *et al.* (1994). Recent semi-analytic GR calculations of such equilibrium tidal effects (including the more precise determination of the Love number) can be found in numerous papers (e.g., Flanagan & Hinderer 2008; Binnington & Poisson 2009; Damour & Nagar 2009; Penner *et al.* 2012, Ferrari *et al.* 2012). Obviously this effect is only important at small orbital separations (just prior to merger) – there is some prospect of measuring this, thereby constraining the EOS, but it will be challenging (Damour *et al.* 2012). More importantly, at small orbital separations, the quadrupole approximation is not valid; there one must use the numerically computed GR quasi-equilibrium binary sequences to characterize the tidal effect – such sequences have been constructed by several groups since the 1990s (e.g., Baumgarte *et al.* 1998; Uryu *et al.* 2009).

Another aspect of the equilibrium tide concerns tidal dissipation, which leads to a lag of the tidal bulge with respect to the binary axis. It was shown already in the 1990s (Bildsten & Cutler 1992; Kochanek 1992) that because of the rapid GW-driven orbital decay, viscous tidal lag cannot synchronize the NS spin. Thus the NS will be close to irrotational (approximated as a Riemann-S ellipsoid; Lai *et al.* 1994; Wiggins & Lai 2000). Near the final phase of the inspiral, the rapid orbital decay gives rise to a finite lag angle (even with zero viscosity), but this cannot synchronize the NS (Lai & Shapiro 1995; Dall'Osso & Rossi 2012).

The situation is more complicated for **dynamical tides**, which manifest as resonant excitations of internal oscillations of the NS: As two NSs spiral in, the orbit can momentarily come into resonance with the normal modes (frequency ω_α) of the NS:

$$\omega_\alpha = m\Omega_{\rm orb}, \qquad m = 2, 3, \cdots \tag{2.2}$$

By drawing energy from the orbital motion and resonantly exciting the modes, the rate of inspiral is modified, giving rise to a phase shift in the gravitational waveform. This problem was studied by Reisenegger & Goldreich (1994), Lai (1994) and Shibata (1994) in the case of non-rotating NSs, where the only modes that can be resonantly excited are g-modes (with typical mode frequencies $\lesssim 100$ Hz). It was found that the effect is small for typical NS parameters (mass $M = 1.4M_\odot$ and radius $R = 10$ km) because the coupling between the g-mode and the tidal potential is weak. Ho & Lai (1999) studied the effect of NS rotation, and found that the g-mode resonance can be strongly enhanced even by a modest rotation (e.g., the phase shift in the waveform $\Delta\Phi$ reaches up to 0.1 radian for a spin frequency $\nu_s \lesssim 100$ Hz). They also found that for a rapidly rotating NS ($\nu_s \gtrsim 500$ Hz), f-mode resonance becomes possible (since the inertial-frame f-mode frequency can be significantly reduced by rotation) and produces a large phase shift. In addition, NS rotation gives rise to r-mode resonance whose effect is appreciable only for very rapid (near breakup) rotations. Lai & Wu (2006) further studied resonant excitations of other inertial modes (of which r-mode is a member) and found similar effects. Flanagan & Racine (2006) studied the gravitomagnetic resonant excitation of r-modes and and found that the post-Newtonian effect is more important than the Newtonian tidal effect (and that the phase shift reaches 0.1 radian for $\nu_s \sim 100$ Hz). Tsang *et al.* (2012) examined crustal modes and found that the GW phase correction is small/modest and suggested that tidal resonance could shatter the NS crust, giving rise to the pre-cursor of short GRBs. Taken together, these studies suggest that for canonical NS parameters ($R \simeq 10$ km, $\nu_s \lesssim 100$ Hz), tidal resonances have a small effect on the gravitational waveform during binary inspiral. However, it is important to remember that the effect is a strong function of R (e.g., $\Delta\Phi \propto R^4$ for g-modes and $\propto R^{3.5}$ for inertial modes). A larger radius ($R \simeq 15$ km) would make the effect important. In the case of g-modes, the magnitude of the effect depends on the symmetry energy of nuclear matter and could be non-negligible (W. Newton & D. Lai 2013, in prep).

3. Electrodynamics of merging NS binaries

For magnetic NSs, magnetic interactions may play a role. If the binary is embedded in a vacuum, then the interaction potential is $V(r) = -MM'/a - \mathcal{O}(\mu\mu'/a^3)$ (where μ, μ' are the magnetic dipole moments of the two stars). It is easy to check that such magnetic interaction would lead to negligible effect on the GWs unless both NSs have superstrong fields ($\gg 10^{15}$ G) – this is unlikely (e.g., the double pulsars PSR J0737-3039 has 10^{10} G for pulsar A and 2×10^{13} for pulsar B).

Of course, as in the case of isolated pulsars, the circumbinary environment cannot be vacuum. The following discussion is based on Lai (2012). Consider a binary system consisting of a magnetic NS (the "primary", with mass M, radius R, spin Ω_s, and magnetic dipole moment μ) and a non-magnetic companion (mass M_c, radius R_c). The orbital angular frequency is Ω. The magnetic field strength at the surface of the primary is $B_\star = \mu/R^3$. The whole binary system is embedded in a tenuous plasma (magnetosphere). For simplicity, we assume $\mathbf{\Omega}$, $\mathbf{\Omega_s}$ and $\boldsymbol{\mu}$ are all aligned. The motion of the

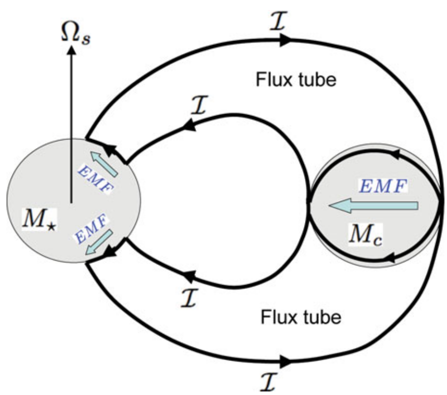

Figure 1. DC circuit model of magnetic interactions in binary systems *a la* Goldreich & Lynden-Bell (1969).

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non-magnetic companion relative to the magnetic field of the primary produces an EMF $\mathcal{E} \simeq 2R_c|E|$, where $\mathbf{E} = \mathbf{v}_{\rm rel} \times \mathbf{B}/c$, with $\mathbf{v}_{\rm rel} = (\Omega - \Omega_s)a\hat{\boldsymbol{\phi}}$ and $\mathbf{B} = (-\mu/a^3)\hat{\mathbf{z}}$. This gives $\mathcal{E} \simeq (2\mu R_c/ca^2)\Delta\Omega$, where $\Delta\Omega = \Omega - \Omega_s$. The EMF drives a current along the magnetic field lines in the magnetosphere, connecting the primary and the companion through two flux tubes. The current in the circuit is given by $\mathcal{I} = \mathcal{E}/(\mathcal{R}_{\rm tot})$, where the total resistance of the circuit is $\mathcal{R}_{\rm tot} = \mathcal{R} + \mathcal{R}_c + 2\mathcal{R}_{\rm mag}$, with $\mathcal{R}$, $\mathcal{R}_c$, $\mathcal{R}_{\rm mag}$ the resistances of the magnetic star, the companion and the magnetosphere, respectively. These resistances depend on the properties of the binary components and the magnetosphere, and can vary widely for different types of systems. The energy dissipation rate of the system is then $\dot{E}_{\rm diss} = 2\mathcal{I}^2 R_{\rm tot} = 2\mathcal{E}^2/\mathcal{R}_{\rm tot}$, where the factor of 2 accounts for both the upper and lower sides of the circuit.

The total magnetic force (in the azimuthal direction) on the companion is $F_\phi \simeq (2R_c)(2\mathcal{I}B_z/c)$, with $B_z = -\mu/a^3$. Thus the torque acting on the binary's orbital angular momentum is $T = \dot{J}_{\rm orb} \simeq (4/c)a\,R_c\mathcal{I}B_z \simeq -(4\mu R_c/ca^2)(\mathcal{E}/\mathcal{R}_{\rm tot})$. The torque on the primary's spin is $I\dot{\Omega}_s = -T$ (where I is the moment of inertia). The orbital energy loss rate associated with T is then $\dot{E}_{\rm orb} = T\Omega$.

The equations above show that the binary interaction torque and energy dissipation associated with the DC circuit increase with decreasing total resistance $\mathcal{R}_{\rm tot}$. Is there a problem for the DC model when $\mathcal{R}_{\rm tot}$ is too small? The answer is yes. The current in the circuit produces a toroidal magnetic field, which has the same magnitude but opposite direction above and below the equatorial plane. The toroidal field just above the companion star (in the upper flux tube) is $B_{\phi+} \simeq -(2\pi/c)\mathcal{J}_r$, where $\mathcal{J}_r \simeq -4\mathcal{I}/(\pi R_c)$ is the (height-integrated) surface current. Thus the azimuthal twist of the flux tube is $\zeta_\phi = -B_{\phi+}/B_z == 16v_{\rm rel}/(c^2\mathcal{R}_{\rm tot})$, where $v_{\rm rel} = a\Delta\Omega = a(\Omega - \Omega_s)$. Clearly, when $\mathcal{R}_{\rm tot}$ is less than $16v_{\rm rel}/c^2$, the flux tube will be highly twisted.

Goldreich & Lynden-Bell (1969) speculated that the DC circuit would break down when the twist is too large. (For the Jupiter-Io system parameters adopted by GL, the twist $|\zeta_\phi| \ll 1$.) Since then, numerous works have confirmed that this is indeed the case. Theoretical studies and numerical simulations, usually carried out in the contexts of solar flares and accretion disks, have shown that as a flux tube is twisted beyond $\zeta_\phi \gtrsim 1$, the magnetic pressure associated with B_ϕ makes the flux tube expand outward and the magnetic fields open up, allowing the system to reach a lower energy state (e.g., Aly 1985; Aly & Kuijpers 1990; Lynden-Bell & Boily 1994; Lovelace *et al.* 1995; Uzdensky *et al.* 2002). Thus, a DC circuit with $\zeta_\phi \gtrsim 1$ cannot be realized: The flux tube will break up, disconnecting the linkage between the two binary components. A binary system with $\mathcal{R}_{\rm tot} \lesssim 16v_{\rm rel}/c^2$ cannot establish a steady-state DC circuit. The electrodynamics is likely rather complex, only a quasi-cyclic circuit may be possible (Lai 2012): (a) The magnetic field from the primary penetrates part of the companion, establishing magnetic linkage between the two stars; (b) The linked fields are twisted by differential rotation, generating toroidal field from the linked poloidal field; (c) As the toroidal magnetic field becomes comparable to the poloidal field, the fields inflate and the flux tube breaks, disrupting the magnetic linkage; (d) Reconnection between the inflated field lines relaxes the shear and restore the linkage. The whole cycle repeats.

In any case, we can use the dimensionless azimuthal twist ζ_ϕ to parameterize the magnetic torque and energy dissipation rate:

$$T = \frac{1}{2}aR_c^2B_zB_{\phi+} = -\zeta_\phi\frac{\mu^2R_c^2}{2a^5}, \quad \dot{E}_{\rm diss} = -T\Delta\Omega = \zeta_\phi\Delta\Omega\frac{\mu^2R_c^2}{2a^5}. \tag{3.1}$$

The maximum torque and dissipation are obtained by setting $\zeta_\phi \sim 1$. If the quasi-cyclic circuit discussed in the last paragraph is established, we would expect ζ_ϕ to vary

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between 0 and ~ 1. Note that in the above, T is negative since we are assuming $\Omega > \Omega_s$. A reasonable extension would let $\zeta_\phi = \zeta(\Delta\Omega)/\Omega$, with $\zeta > 0$.

Gravitational wave (GW) emission drives the orbital decay of the NS binary, with timescale $t_{\rm GW} = a/|\dot{a}| = 0.012\,(a/30\,{\rm km})^4$ s, where we have adopted $M = 1.4M_\odot$ and mass ratio $q = M_c/M = 1$. The magnetic torque tends to spin up the primary when $\Omega{>}\Omega_s$. Spin-orbit synchronization is possible only if the synchronization time $t_{\rm syn} = I\Omega/|T|$ is less than $t_{\rm GW}$ at some orbital radii. With $I = \kappa M R^2$, we find

$$t_{\rm syn} = \frac{2\kappa(1+q)}{\zeta_\phi\Omega}\left(\frac{GM^2}{B_\star^2 R^4}\right)\left(\frac{a}{R_c}\right)^2 \simeq 2\times 10^7 \zeta_\phi^{-1}\left(\frac{B_\star}{10^{13}\,{\rm G}}\right)^{-2}\left(\frac{a}{30\,{\rm km}}\right)^{7/2} {\rm s}, \tag{3.2}$$

where on the right we have adopted $\kappa = 0.4$ and $R = R_c = 10$ km. Clearly, even with magnetar-like field strength ($B_\star \sim 10^{15}$ G) and maximum efficiency ($\zeta_\phi \sim 1$), spin-orbit synchronization cannot be achieved by magnetic torque. For the same reason, the effect of magnetic torque on the number of GW cycles during binary inspiral is small.

The energy dissipation rate is

$$\dot{E}_{\rm diss} = \zeta_\phi\left(\frac{v_{\rm rel}}{c}\right)\frac{B_\star^2 R^6 R_c^2 c}{2a^6} = 7.4\times 10^{44}\zeta_\phi\left(\frac{B_\star}{10^{13}\,{\rm G}}\right)^2\left(\frac{a}{30\,{\rm km}}\right)^{-13/2} {\rm erg\,s^{-1}}, \tag{3.3}$$

where on the right we have used $v_{\rm rel} \simeq a\Omega$ (for $\Omega_s \ll \Omega$) and adopted canonical parameters ($M = M_c = 1.4M_\odot$, $R = R_c = 10$ km). The total energy dissipation per $\ln a$ is

$$\frac{dE_{\rm diss}}{d\ln a} = \dot{E}_{\rm diss} t_{\rm GW} \simeq 8.9\times 10^{42}\zeta_\phi\left(\frac{B_\star}{10^{13}\,{\rm G}}\right)^2\left(\frac{a}{30\,{\rm km}}\right)^{-5/2} {\rm erg}. \tag{3.4}$$

Some fraction of this dissipation will emerge as electromagnetic radiation counterpart of binary inspiral. It is possible that this radiation is detectable at extragalactic distance. But this will depend on the microphysics in the magnetosphere, including particle acceleration and radiation mechanism (e.g., Vietri 1996; Hansen & Lyutikov 2001).

If one assumes that the magnetosphere resistance is given by the impedance of free space, $\mathcal{R}_{\rm mag} = 4\pi/c$, then the corresponding twist is $\zeta_\phi = 2v_{\rm rel}/(\pi c)$, which satisfies our upper limit. The energy dissipation rate is then

$$\dot{E}_{\rm diss} = \left(\frac{v_{\rm rel}}{c}\right)^2\frac{B_\star^2 R^6 R_c^2 c}{\pi a^6} = 1.7\times 10^{44}\left(\frac{B_\star}{10^{13}\,{\rm G}}\right)^2\left(\frac{a}{30\,{\rm km}}\right)^{-7} {\rm erg/s}. \tag{3.5}$$

This is in agreement with the estimate of Lyutikov (2011).

The situation is similar for NS/BH binaries. In the membrane paradigm (Thorne *et al.* 1986), a BH of mass M_H resembles a sphere of radius $R_c = R_H = 2GM_H/c^2$ (neglecting BH spin) and impedance $\mathcal{R}_H = 4\pi/c$. Neglecting the resistances of the magnetosphere and the NS, the azimuthal twist of the flux tube in the DC circuit is $\zeta_\phi = 4v_{\rm rel}/(\pi c)$, which satisfies our upper limit. The energy dissipation rate is (cf. Lyutikov 2011; McWilliams & Levin 2011)

$$\dot{E}_{\rm diss} = \left(\frac{v_{\rm rel}}{c}\right)^2\frac{2B_\star^2 R^6 R_H^2 c}{\pi a^6} \simeq 5.7\times 10^{42}\left(\frac{B_\star}{10^{13}\,{\rm G}}\right)^2\left(\frac{M_H}{10M_\odot}\right)^{-4}\left(\frac{a}{3R_H}\right)^{-7} {\rm erg\,s^{-1}}, \tag{3.6}$$

where we have assumed $M_{BH}/M \gg 1$. Again, it is uncertain whether this radiation can be for binaries at extragalactic distances.

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