

HOLOMORPHIC FLEXIBILITY PROPERTIES OF SPACES OF ELLIPTIC FUNCTIONS

DAVID BOWMAN

(Received 17 December 2016; first published online 2 March 2017)

2010 *Mathematics subject classification*: primary 32H02; secondary 32E10, 58B12.

Keywords and phrases: Oka manifold, mapping space, elliptic function, complex space, several complex variables.

Let X be an elliptic curve and \mathbb{P} the Riemann sphere. Since X is compact, it is a deep theorem of Douady that the set $\mathcal{O}(X, \mathbb{P})$ consisting of holomorphic maps $X \rightarrow \mathbb{P}$ admits a complex structure. If R_n denotes the set of maps of degree n , then Namba has shown for $n \geq 2$ that R_n is a $2n$ -dimensional complex manifold. We study holomorphic flexibility properties of the spaces R_2 and R_3 .

Firstly, we show that R_2 is homogeneous and hence an Oka manifold. Secondly, we present our main theorem, that there is a six-sheeted branched covering space of R_3 that is an Oka manifold. It follows that R_3 is \mathbb{C} -connected and dominable. We show that R_3 is Oka if and only if $\mathbb{P}_2 \setminus C$ is Oka, where C is a cubic curve that is the image of a certain embedding of X into \mathbb{P}_2 .

We investigate the strong dominability of R_3 and show that if X is not biholomorphic to \mathbb{C}/Γ_0 , where Γ_0 is the hexagonal lattice, then R_3 is strongly dominable.

As a Lie group, X acts freely on R_3 by precomposition by translations. We show that R_3 is holomorphically convex and that the quotient space R_3/X is a Stein manifold.

We construct an alternative six-sheeted Oka branched covering space of R_3 and prove that it is isomorphic to our first construction in a natural way. This alternative construction gives us an easier way of interpreting the fibres of the branched covering map.

The full thesis is available at <https://arxiv.org/abs/1609.07184>.

DAVID BOWMAN, School of Mathematical Sciences,
University of Adelaide, Adelaide, South Australia 5005, Australia
e-mail: davidbowman@netspace.net.au

Thesis submitted to the University of Adelaide in September 2016; degree approved on 6 December 2016, principal supervisor Finnur Larusson and associate supervisor Nicholas Buchdahl.

© 2017 Australian Mathematical Publishing Association Inc. 0004-9727/2017 \$16.00