Abstracts of Australasian PhD theses The complex irreducible characters of Sp(6, q), q even

John Looker

In this thesis the "Green functions" for Sp(6, q), q even, are determined explicitly and used to provide all the complex irreducible characters of these groups. In this context the term "Green functions" is used to denote certain functions defined on unipotent conjugacy classes, each function corresponding to a class of "pseudo-tori" in the associated algebraic group. A pseudo-torus is either a maximal torus or a suitable torus exhibiting a "rank defect". Those functions corresponding to the classes of maximal tori are precisely the Green functions provided by the Deligne-Lusztig theory [1].

To determine the Green functions we proceed as follows. The doublecoset structure of the subgroups $O_+(2n, q)$ and $O_-(2n, q)$ of $\operatorname{Sp}(2n, q)$ is examined and used to deduce the components of certain characters of $\operatorname{Sp}(2n, q)$ induced from these subgroups. The use of these characters together with characters induced from parabolic subgroups enables all the characters which are "unipotent" (in the sense of Deligne and Lusztig) to be identified for $\operatorname{Sp}(6, q)$. The Green functions are then expressed as the restrictions of integral linear combinations of these unipotent characters.

The values of the characters induced from the orthogonal groups are easily obtained from a knowledge of the conjugacy classes of elements in these groups. By means of geometric counting arguments functions g(c; d)

Received 23 September 1977. Thesis submitted to the University of Sydney, March 1977. Degree approved, August 1977. Supervisors: Professor G.E. Wall, Dr L.G. Kovács (Australian National University), Dr G.I. Lehrer.

are evaluated and used to compute values for the characters induced from the parabolic subgroups. Finally the values of the unipotent characters are obtained by using a relation provided by the Deligne-Lusztig theory.

Tables showing the values of the Green functions for Sp(6, q), q even, and other information relevant to the determination of the irreducible characters are set out in the appendix. In particular these tables show that the values of the Green functions are all integral polynomials in q and fall neatly into "dual" pairs.

Reference

[1] P. Deligne and G. Lusztig, "Representations of reductive groups over finite fields", Ann. of Math. (2) 103 (1976), 103-161.