The Hertzsprung Multiple Exposure Technique and its Application to 61 Cygni
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## INTRODUCTION

E. Hertzsprung developed the multiple exposure technique for observing double stars photographically around 1914 at Potsdam (Hertzsprung, 1920). The technique consists in taking one or more rows of 17 to 35 exposures of a small field which normally includes only the double star itself. The total number of exposures on a plate can vary from 17 to 140 , depending primarily on the separation of the double star and the anticipated systematic effects in the observational system. An important feature of the technique is the use of objective gratings to substantially reduce the problem of magnitude error. The large number of exposures serve to reduce the random error, while the objective gratings, together with other innovations introduced by Hertzsprung, help to minimize the systematic errors. In this way Hertzsprung provided us with an observational technique of considerably higher accuracy than the traditional visual micrometer work.

Because there are no reference stars on the plates, the coordinate system must be determined as follows: 1) the position angles are calibrated by exposing on each plate a star trail to define the equator at the epoch of observation; 2) the separations are calibrated by determining the scale value of the telescope as a function of temperature, and this is done by measuring the separations in linear measure on "scale plates" of widely separated ( $\rho \sim 300^{\prime \prime}-1200^{\prime \prime}$ ) pairs of stars of known angular separation.

The Hertzsprung technique is applicable to those systems with separations between about $2^{\prime \prime}$ and $50^{\prime \prime}$. Adjacency effects in the emulsion cause problems at smaller separations; and at the larger separations, errors in the scale and position angle calibrations become important. On the Washington program systematic variations of these calibrations are monitored by our intensive series of observations of $61 \mathrm{Cygni}\left(\rho=29^{\prime \prime}\right)$ and $16 \mathrm{Cygni}\left(\rho=39^{\prime \prime}\right)$.

The multiple exposure technique has been employed at many observatories around the world. The measures of approximately $20,000 \mathrm{plates}$ are entered in the Double Star Observation Catalog. Since the average number of exposures per plate is about 50 , the total number of exposures measured to date is of the order of one million.

THE NAVAL OBSERVATORY PHOTOGRAPHIC DOUBLE STAR PROGRAM
Initiated in 1958 by K. Aa. Strand, the Naval Observatory photographic double star program (as of July 1981) has since amassed about 8500 plates on 1000 double stars with separations between $l^{\prime}: 4$ and $120^{\prime \prime}$, using the $66-\mathrm{cm}$ refractor in Washington. Another 1900 plates were obtained between 1959 and 1965 with the $61-\mathrm{cm}$ refractor of the Lowell Observatory. Of these 10,000 plates, somewhat more than one-third have been measured on the automatic machine (SAMM), while the rest were measured manually. SAMM is incapable of measuring double stars with separations less than about 6". These obser-
vations and some discussion of their errors have been reported in four Naval Observatory publications (Franz, et al., 1963; Kallarakal, et al., 1969; Josties, et al., 1974; Josties, et al., 1978), which will be referred to as DS I, DS II, DS III, and DS IV, respectively.

The median mean error of a single image for the manual measures was $0!37(=3.5 \mu)$, while for the SAMM measures it was $0!05$ ( $=2.5 \mu$ ). The median mean error of a plate mean was $0 \cdot 011$ and $0!.009$ for the manual and automatic measures respectively (the number of exposures was usually larger on a manually measured plate), and the corresponding "external errors", calculated from the interagreement of different plates, were $0!017$ and $0!012$. All of these error quantities have evolved over the years to somewhat smaller values due to various procedural improvements (DS II, III, IV).

The primary sources of external error in these data are the following: 1) systematic measuring error, especially personal equations in the manual measurement of separation on close pairs ( $\rho<4^{\prime \prime}$ ). Remeasurement of these plates on an impersonal machine in the future will therefore eliminate this major source of error (See DS IV). 2) orientation error on wide pairs ( $\rho>10^{\prime \prime}$ ). Our control over the orientation errors has improved considerably since 1970. The simple expedient of taking fewer exposures per plate (only 17 for the widest pairs), and therefore more trails per exposure and more plates per unit telescope time, prevents the orientation error from dominating the other sources of error. A more complete discussion is found in DS IV.

## SYSTEMATIC ERRORS

Because small formal errors are achieved by obtaining many exposures per plate and many plates per year, rather small systematic effects in the data almost inevitably become significant. In his original paper Hertzsprung (1920) introduced procedures for eliminating or evaluating many of the potential sources of systematic error, such as differential atmospheric refraction and dispersion, chromatic lens aberration, temperature dependence of the focal setting and the scale value, magnitude error on the plate and at the measuring machine, systematic personal measuring errors, periodic and progressive errors of the measuring machine, emulsion shifts, and adjacency effects in the emulsion. K. Aa. Strand (1937, 1946, 1954, 1957) and H.M. Jeffers (1951) also have made important contributions to the study of some of these effects. Jeffers' introduction of an automatic double star camera at Lick in 1938 was an important advance. Strand's acquisition of the SAMM automatic measuring machine for the Naval Observatory eliminated the several serious systematic problems associated with manual measurement (albeit, only for pairs with separations larger than $6^{\prime \prime}$ ).

The "external errors" discussed in the previous section are not adequate indicators of the presence of systematic error. They merely represent the interagreement among different plates of the same star taken with the same observational system (telescope, camera, filter, plate, measuring machine, etc.). Errors can, and indeed do, arise which are common to all of the plates on a given star or even to all stars. Such errors may be properly called systematic errors. The identification of
such errors, even if they occur at the $0!01$ level, is a difficult undertaking at best, and is often ambiguous. When differences between two independent observational systems occur in a systematic manner, it is not always possible to decide without ambiguity that one or the other is "correct". In general such ambiguity can be considered resolved only after the emergence at a later time of a much superior (and usually elaborate and expensive) observational technique. An improvement by an order of magnitude in the accuracy of an individual observation may be necessary to effect a practical resolution of such ambiguity. For example, the development of speckle interferometry may provide us with the means of determining the magnitude of systematic errors among visual observers for pairs with separations less than two arc seconds. However, the systematic reliability of speckle observations for the more widely separated photographic binaries is rather more problematical.

THE APPLICATION OF THE HERTZSPRUNG TECHNIQUE TO 61 CYGNI
Sixty-one Cygni is the most extensively observed double star in the sky, in the following sense: there have been 811 plates taken of the system since 1914, with an average of about 42 measured exposures per plate; if we choose to count each exposure as an individual observation, there have been 34,000 photographic observations (not including the plates obtained on parallax series) in addition to the 2000 visual measures. For comparison, 70 Ophiuchi has 15,000 photographic "observations" ( $300 \mathrm{plates} \mathrm{)} \mathrm{and} \mathrm{5000}$ visual measures.

Sixty-one Cygni has been intensively observed on the Washington program for two reasons:

1) to detect possible perturbations in the motion of either component;
2) to detect the presence of, and to monitor the variations in, any systematic effects, especially those which increase with separation. The separation of 61 Cygni is about $30^{\prime \prime}$, whereas the median separation for all other stars on the Washington program is $5^{\prime \prime}$. Therefore, for that important class of systematic error which is proportional to the separation, i.e., scale variations and orientation error, 61 Cygni may be profitably used to place upper limits on the amount of such error for the remainder of the program stars.

In order to distinguish between perturbations and systematic error in the 61 Cygni data, an intensive series of observations has also been obtained on the double star 16 Cygni since 1970. The latter is an ideal astrometric standard star, or comparison star, for 61 Cygni, because it has a somewhat larger separation (39'), crosses the meridian just one hour and 25 minutes before 61 Cygni, has a not too dissimilar declination ( $50^{\circ}$, compared to 61 Cygni's $38^{\circ}$ ), has a similar position angle, and is sufficiently bright that the fine grain metallographic emulsion can also be used for it. As we shall see in the following discussion, the 16 Cygni observations have already been quite useful in interpreting the 61 Cygni data.

THE 61 CYGNI OBSERVATIONAL DATA
Table I lists the visual and photographic normal points used in the analysis, as well as the orbital residuals in position angle (in degrees,



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$P=$ Potsdam 50-cm $\mathrm{P}=$ Potsdam $50-\mathrm{cm} ; \mathrm{S}=\mathrm{Sproul} 61-\mathrm{cm} ; \mathrm{L}=$ Lick $91-\mathrm{cm} ; \mathrm{D}=$ (Sproul 61-cm, Yerkes
Dearborn $46-\mathrm{cm}$, and Lowell $61-\mathrm{cm}$ ) as explained in the text; $\mathrm{V}=$ van Vleck $51-\mathrm{cm} ;$
$\mathrm{W}=$ Washington $66-\mathrm{cm}$ (U.S. Naval Observatory); $\mathrm{F}=$ Lowell $61-\mathrm{cm}$.
as well as in arc length), separation $\rho, X(=\rho \cos \theta)$, and $Y(=\rho \sin \theta)$. The data were obtained from the Burnham Double Star Catalog, the Aitken Double Star Catalog, and the Double Star Observation Catalog. The visual observations were grouped into normal points, taking care to correct for nonlinear effects due to orbital motion. Lower weights were assigned to observers who used smaller aperture instruments, and some discordant observations were omitted entirely. The average number of observations included in a visual normal point was 118, while the average weight was 65. The last column of Table I lists the weight of a visual normal point, or the number of plates in a photographic normal point followed by a letter designation for the observatory. The latter designations are explained at the bottom of the table. Each letter refers to only one observatory except "D", which refers to data combined into yearly normal points from plates obtained at Sproul, Dearborn, Yerkes, and Lowell, as given by Strand (1954, 1957).

All of the normal points in Table I have been corrected, in separation for the perspective effect (Fletcher, 1931) due to the large parallax ( $0 \cdot 294$ ) and radial velocity ( $-62 \mathrm{~km} / \mathrm{s}$ ), and in position angle due to the large proper motion ( $=5!20$ ) in addition to the precession correction. The corrections applied to the data were:

$$
\begin{aligned}
& \Delta \rho=-0.000019 \cdot \rho \cdot(t-2000) \\
& \Delta \theta=+0.0041(t-2000)
\end{aligned}
$$

where $t$ is the epoch of the observation.
The following changes of the published Washington and Lowell data have been incorporated into the data set: 1) the scale values reported in DS II, including the aperture dependence, were applied to the data of DS I; 2) seven Washington plates taken between $8 / 22 / 63$ and $10 / 17 / 73$ were deleted due to filter error (see below) ; 3) the machine \#3 measures (6 plates only) reported in DS I were deleted; 4) twenty-five measures of Washington 1976 plates were deleted due to a period of poor adjustment of SAMM. Remeasures of these plates are included in the data set.

## THE ORBIT OF 61 CYGNI

The observations of 61 Cygni unfortunately cover only a short arc of the orbit, about $56^{\circ}$ in position angle, as can be seen in Figure 1. For this reason the orbit determination is not unique despite the large number of observations, it being possible to trade off a change in one orbital element for a set of changes in the other elements (Harrington, this volume). However, the orbital elements listed in Figure 1, which were derived by an empirical differential correction technique, do represent the observations satisfactorly in the following twofold sense: 1) the sum of the masses calculated from this orbit, using the highly accurate parallax of $0!294$ (van de Kamp, 1973), is $1.13 \mathrm{M}_{\odot}$, which agrees well with the value to be expected from the mass-luminosity relation for these spectral types (K5V, K7V); whereas, the other members of Harrington's possible family of orbits yield smaller values for the sum of the masses. 2) although there is a systematic difference between the visual and photographic observations as can be seen in Figures 2 and 3 , the orbital elements of Figure 1 do minimize the systematic residuals. That is, the systematic offset of the visual normal points is approximately equal to their dispersion, and the same is true of the photographic normal points.

## FIGURE 1.

## ORBIT OF GI CYENI PERIOD 722.01 YEARS

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The orbit may therefore be said to represent the observations fairly. On the other hand, significant changes in the orbit may well be required when more is known about the systematic effects in either the visual or photographic data.

## SYSTEMATIC EFFECTS IN THE 61 CYGNI PHOTOGRAPHIC DATA

Figures 2 and 3 show the orbit residuals in separation and position angle, respectively. Although some systematic effects may be seen in the photographic data we must emphasize that most of these are significant for 61 Cygni only because of its large separation.

The one type of error which does contribute here but which is not in general proportional to the separation is measuring error. Only the plates of 1965-80 have been measured on SAMM. All of the earlier data have somewhat enhanced random and systematic error due to manual measurement.

Three important sources of systematic error to be discussed here are proportional to the separation: 1) filter effects, 2) scale errors, 3) orientation errors.

The largest systematic error in these data is filter error in the 1958-64 Washington data. During this period the filter in the Washington double star camera was positioned several inches above the focal plane. Deviations from flatness of the filter resulted in significant positional errors which are larger for doubles of large separation. For closer pairs the images of the two components at the position of the filter were largely overlapped, and the effect on the relative position was therefore minimal. The 1958-64 Washington data on 61 Cygni should be given low weight. From 1965 until August 1, 1969 the filter remained six inches above the focal plane, but much flatter filters


were used so that the filter error may be negligible. Since August 1, 1969 the filter has been in the plate holder.

Another likely source of error in these data is systematic error in the scale value of the various telescopes, or variations in these values. Such a change in scale value has apparently occurred to the Washington 26 -inch. In October 1978 the tailpiece of the 26 -inch was inadvertently bumped into the floor by contractors working in the dome. There was some minor damage to the focussing mechanism, and subsequent testing demonstrated the need for a change of 0.5 mm . in the focal setting. The net result was an apparent scale change, although this was not realized at the time. Pending a fuller investigation, the best calibration of the scale change is that obtained from the 16 Cygni observations. The observed changed in the separation of 16 Cygni was $\Delta \rho=+0!009 \pm 0 \cdot .001$ based on 198 and 37 plates before and after the change, respectively. The formal error on this figure is spuriously low, but it is estimated to be correct to within $30 \%$. The computed corresponding change in separation for 61 Cygni is $+0 \cup 007 \pm 0 \because 002$, and the data have been adjusted accordingly in Fig. 4. In Figure 2, where the original data are plotted, the 61 Cygni separations can be seen to be slightly too large for 1978-80. The original uncorrected data are given in Table I. It is of course unfortunate that any adjustment to the data of this kind is necessary. However, it is clear that the strength of the calibration with the 16 Cygni observations will increase with time, barring any future accidents.

Orientation errors are also present. The random orientation error has been reduced since 1970 (See DS IV). However, there is apparently a progressive systematic error in position angle in the 1970-80 data, as seen in Figure 3. This run cannot be eliminated by changing the orbital elements by reasonable amounts. That is, it does represent a significant deviation from Keplerian motion, and must be ascribed either to a perturbing body in the system or to systematic error. One approach to distinguishing between these two possibilities is, once again, to examine the 16 Cygni data. The angular velocity deviation, $\Delta \dot{\theta}$, during 1970-80 does seem to occur in 16 Cygni also and is found to be $\Delta \dot{\theta}=+0.0014 /$ year ( $\pm 0^{\circ} .0004 /$ year ), after correcting for the change in $\ddot{\theta}$ to be expected from the law of areas and the observed change in separation. This result should be regarded with some caution because it depends on a small number of photographic measures in 1914 and 1940. An improvement in this determination may become possible using early parallax series measures on 16 Cygni, coupled with recent ones. The error of $\pm 0 \% 0004 / y$ quoted above corresponds to an error of 1 micron $=0.016$ in Hertzsprung's 1914 position angle determination.

The above $\Delta \dot{\theta}$ is to be compared with that of 61 Cygni for 1970-80, $+0.0020 / \mathrm{y} \pm 0.0004 / y$, which is seen in Figure 3 . We conclude that, on the basis of present evidence, there is probably a progressive systematic error in the Washington position angles which is very likely of instrumental origin. However, the effect does appear to be rather smooth and linear and therefore readily modeled, a fact which has two important implications:

1) it should be possible to remove the effect from the data fairly completely, especially if the instrumental source of the problem can be isolated;
2) we can with some confidence simply remove from the 61 Cygni data a linear run in the position angles for the purpose of analyzing that data for the possible presence of perturbations with periods less than 10 years.

## PERTURBATIONS IN THE 61 CYGNI SYSTEM

There have been several astrometric studies in which it was claimed that a perturbation of 61 Cygni was present due to an unseen companion star or planet. Strand (1943) first announced the presence of a companion of mass $0.016 \mathrm{M}_{\odot}$ with a period of 4.9 years and a semiaxis major of the photocenter of $0!320$. Subsequently both Strand (1957) and Deich (1960) confirmed the presence of a perturbation with a period near 5 years, but with the semiaxis major reduced to $0!010$ (Strand) or $0!.014$ (Deich). More recently Deich and Orlova (1977) claimed to have evidence for 3 companions with period of 6,7 , and 12 years. And in his latest paper Deich (1978) has revised this to periods of 6 and 12 years, with semiaxes major of $0!006$ and $0!008$, respectively.

In Figure 4 are plotted the $1970-80$ Washington residuals in $X(=\rho \cos \theta)$ and $Y(=\rho \sin \theta)$, after having linear runs removed from each of these separately. This procedure is justifiable when one is looking for perturbations with periods less than the duration of the data set. The 1978-80 points have been adjusted due to the apparent scale change of 1978. The expected precision for each data point is only slightly less than $\pm 0!002$ on the average.

To whatever extent one accepts the accuracy of the scale adjustment, there is little in Figure 4 which can be regarded as corroborative of the published perturbations mentioned above. Certainly the Strand perturbations can be considered unverified. But an unequivocal refutation of the Deich 6 - and 12-year terms cannot be given at this time. There are uncertainties in the Washington data prior to 1970, and a better calibration of the 1978 scale change is necessary. A few more years of data should clarify the situation.

## ACKNOWLEDGEMENTS

It is a pleasure to thank R.S. Harrington for his collaboration on several aspects of this investigation. Thanks are also due J.W. Christy for helpful discussions, and C.E. Worley for the use of his HP9825 ephemeris program. Jonathan Woodbury created Figure 1 and Table I on the HP9825. Many thanks go to all of the members of the Exploratory Development Staff who have been so tolerant of my 61 Cygni related activities over the last several years.

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FIGURE 4. RESIDUALS IN X, Y, LINEAR TERMS REMOVED


## DISCUSSION

FREDRICK: Could you get an accurate scale by objective grating techniques?
JOSTIES: The precision of that would be low, compared to using stars of known separation, and you would have systematic effects to worry about.

STRAND: Have you thought of remeasuring the earlier material on 61 Cyg taken since 1938 at Sproul, Dearborn and Lowell?

JOSTIES: Yes, it would certainly be worthwhile to remeasure some of the early plate material. At least some of it, of course, has systematic effects in it and therefore would not give us a lot of new information. However, we would get more information on the systematic effects in the data, and there is some, perhaps a good deal, that would be worthwhile remeasuring on SAMM.

WORLEY: One has to be careful in making direct comparisons of scatter between the early visual measures of 61 Cygni and the more recent photographic observations. Not only are the visual measures generally made by less-skilled observers, but they also used telescopes averaging a half or a third the aperture of those used for photographic measures.

I have always been somewhat skeptical of the accuracy of the photographic measures in the milliarcsecond range, because these measures are purely differential and do not include external checks (i. e. comparison stars). Many small, systematic instrumental effects may therefore affect the observations, and these can not be allowed for.

JOSTIES: I am a good deal more well aware of that, I think, than anyone in the audience, and I certainly deal with that problem daily. Of course, to the extent that an intense series of observations is intended to provide information on possible perturbations with periods less than the time span of the observations, constant or simply modelable systematic effects have no bearing on the investigation. For example the quoted precision on 61 Cygni, $\pm!!002(= \pm 0.1$ micron $)$ per yearly normal point, would only be improved were we to discover any correlation between the plate residuals and some observational parameter.

WALKER: We may assume weights were assigned to the visual observations. If so, how did you weight the observations?

JOSTIES: The observers who used very small aperture telescopes, of course, were given lower weight, and some of the primary observers were given a higher weight. If interested, I can give you the exact weights and the residuals for the individual observers.

