
Preface to the second edition

There is no perfect time to write a textbook for a field in its infancy. Act too early and the theory might not have the cohesive structure it will eventually develop, but act too late and you might miss an opportunity to encourage new collaborators to enter and shape the area. The first edition of this book was timed to strike a balance between these two extremes: after a sufficient framework for analytic combinatorics in several variables had been developed, but during a time when fundamental results were still being discovered and incorporated.

As a consequence of this choice, the first edition of the text, while influential and put to use by many others in enumerative combinatorics, was presented in a way that many end users found difficult to follow. Having been given the opportunity to create a second edition of this text, after a decade of further development, we are now able to improve both the content and presentation of the field. We have been conscious in this rewriting of making the book more useful for a variety of readers having different motivations, including making it easier to look up and cite desired asymptotic results.

For the second edition, the original authors welcome our active collaborator Stephen Melczer, whose own introductory book on this topic [Mel21] was published recently, and who has rejuvenated the entire enterprise. In contrast to [Mel21], which skips much of the advanced topological and geometric approach to analytic combinatorics in several variables (ACSV) to focus more on elementary arguments and explicit computation, this text remains dedicated to developing the theory in its most general, and most powerful, form. The field of ACSV has flourished since the publication of the first edition, including numerous workshops, seminars, summer school courses, and many publications exploring applications of the theory [Wil15; dALN15; MM16; Pan17; Vid17; Kov19; Mis19; MW19; RWZ20; GE20; Geo21; GFS21; KLM21; GWW21; Len+23].

Over the last ten years, we have gained an improved understanding of the technical parts of the theory. Perhaps the largest change is to give ACSV a rigorous foundation using stratified Morse theory, whereas in the first edition Morse-theoretic arguments were used to motivate constructions that were then verified with other techniques. In addition to fixing numerous typographical and other errors in the first edition, some the fault of the authors and some of the publisher, the following content changes have been made to improve the book.

- The chapters in Parts I (Combinatorial Enumeration) and II (Mathematical Background) have mainly kept their general structure, however much of their discussion has been rewritten. Of particular note, Section 2.4 has been revised to better explain how ACSV for rational functions extends to algebraic functions via diagonal embeddings, and Section 5.4 has been revised to better explain the proof of Theorem 5.3 (formerly Theorem 5.4.8). Chapter 6 in the first edition has also been moved after the former Chapters 7 and 8 so that Part II now ends in Chapter 6 (which was Chapter 7 in the first edition).
- Part III (Multivariate Enumeration) begins with Chapter 7, which has been completely overhauled and is almost entirely new. The second edition is constructed to put the large majority of the topological and homological arguments in this chapter and the appendices. The main output of the chapter is an expression for coefficient asymptotics as a finite integer sum of saddle-point-like integrals, and those wanting to skip the homological material can simply assume this decomposition in later chapters.
- Chapter 8, which is a complete re-imagining of Chapter 6 in the first edition, discusses how to compute the quantities needed for an asymptotic analysis in a computer algebra system. In contrast to the first edition, we now put additional focus on computing the quantities needed for ACSV – this explains its postponement until Part III.
- Chapters 9–11 have been reworked to begin from the decomposition described in Chapter 7, streamlining their presentation, and to have more explicit results that can be easily cited. Section 11.4 has also been expanded to include a worked example of solving a connection problem via creative telescoping.
- The appendices have been revised and enlarged to be more self-contained, and to give readers a more complete explanation of the constructions they will need for ACSV.
- We have greatly increased the number of exercises and examples, and added many more signposts and guides so that readers with different motivations

can find what they are looking for. Exercises have been split into in-text (shorter and more straightforward, meant to help the reader think over the material) and end-of-chapter (more challenging) problems. We have also listed open problems and ongoing research in Chapter 13.

- Finally, we have created Sage worksheets that cover most of the examples in the book, and some of the exercises.

Supplementary material, including Sage worksheets and a maintained list of errata, are available from the book website

<http://acsvproject.org/acsvbook>

More general resources for the ACSV project are available at

<http://acsvproject.org>

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