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ABSTRACT

The highly inhomogeneous nature of the solar atmosphere leads us to suggest that solitons may occur in magnetic structures such as coronal loops or photospheric flux tubes. The theory is outlined for the simple case of a magnetic slab in a field-free atmosphere and shown to lead to the Benjamin-Ono equation.

1. INTRODUCTION

A soliton is a nonlinear solitary wave of permanent form that behaves in many respects like a particle. Solitary waves were first observed by John Scott Russell, in 1834, who, in a famous horseback ride, pursued one such wave as it propagated along a canal near Edinburgh. This observational discovery conflicted with the theoretical understandings of the day, but eventually inspired, in 1895, the celebrated equation of Korteweg and de Vries. Numerical solutions of the KdV equation, exhibiting no permanent loss of identity when two solitary waves interact, led Zabusky and Kruskal (1965) to refer to such waves as solitons. For a recent review of the literature on water waves, see Miles (1980).

Here we consider the possibility that MHD solitons may occur on the sun. Recall that solitons can arise when both nonlinearity (wave breaking, shock forming) and dispersion occur in a physical system. The importance of nonlinearities on wave propagation in the solar atmosphere, where the gas density changes by many orders of magnitude, is well-known and needs no emphasis here. Dispersive effects, however, have tended to be ignored.

Dispersion typically arises whenever an atmosphere is structured, so that the system possesses a natural length-scale. In ideal MHD, with gravity neglected, dispersion of waves does not occur. But in a structured field the magnetoacoustic modes are dispersive, their phase-speeds depending upon the wavenumber. Thus, since the solar atmosphere abounds

with magnetic structure — intense tubes, sunspots, coronal loops, magnetic canopies, for example — we may anticipate that solitons will also occur.

Indeed, we expect solitons to occur whenever magnetic structuring and nonlinearity are important. The coronal loop and the photospheric flux tube are two obvious examples. Here we will consider one simple example of a magnetically structured atmosphere, the magnetic flux tube, ignoring the effect of gravity. This is the example discussed by Roberts and Mangeney (1982).

2. LINEAR AND NONLINEAR WAVES

Consider a magnetic slab of field B , with width $2a$, embedded in a field-free atmosphere. The undisturbed pressure and density of the gas within the magnetic field are p and ρ , their external values being p_e and ρ_e .

The linear free modes of vibration for such a configuration have been studied in detail in Roberts (1981) and Edwin and Roberts (1982). Figure 1 shows the types of wave modes allowed.

Note that geometrically we can have either symmetric (sausage) modes or antisymmetric (kink) modes of vibration. Additionally, a mode may be either a surface wave (for which the amplitude declines away from the slab boundaries) or a body wave (in which the motions penetrate into the slab interior). The dispersion diagrams for these modes are given in Roberts (1981) and Edwin and Roberts (1982).

Our interest here is in the behaviour of the surface sausage mode in the long wavelength, thin tube approximation ($ka \ll 1$). In this approximation, the sausage mode is a longitudinal sound wave guided along

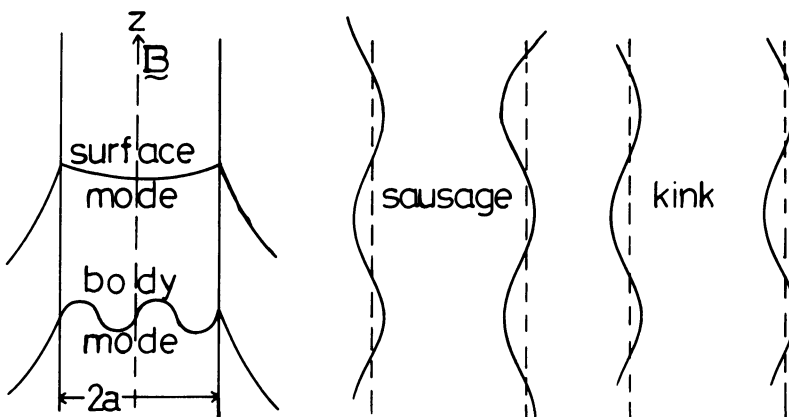


Figure 1. Magnetoacoustic modes of a slab.

the magnetic slab; for a mode travelling along the slab, its frequency ω and longitudinal wavenumber k are approximately related by (Roberts, 1981)

$$\omega = kc_T - \alpha k|k| \quad (1)$$

Here c_T is the tube speed, given in terms of the sound speed $c_s = (\gamma p/\rho)^{1/2}$ and Alfvén speed $v_A = B/(\mu\rho)^{1/2}$ by

$$c_T = c_s v_A / (c_s^2 + v_A^2)^{1/2} \quad (2)$$

and α , given by

$$\alpha = \frac{1}{2} (\rho_e/\rho) a \left(\frac{c_T}{v_A} \right)^3 c_T \quad (3)$$

is a measure of the inertial properties of the slab's environment. It is the presence of an environment that gives the tube (magnetoacoustic) mode its dispersive character and, ultimately, permits a soliton to occur.

The dispersion relation (1) is quite different in form from the well-known relation ($\omega = k - k^3$) associated with the KdV equation. This difference arises here as an immediate consequence of magnetic structuring: the fact that there is a length-scale (namely, $2a$) for disturbances within the tube, but none for the disturbance outside the tube. The form of (1) arises in the propagation of water waves on an internal density inhomogeneity, and there leads to the Benjamin-Ono equation when nonlinearities are allowed for (see Benjamin, 1967; Ono, 1975).

Much the same kind of analysis may be made for the magnetic slab, with the result (Roberts and Mangeney, 1982):

$$\frac{\partial v}{\partial t} + c_T \frac{\partial v}{\partial z} + \beta v \frac{\partial v}{\partial z} + \frac{\alpha}{\pi} \frac{\partial^2}{\partial z^2} \int_{-\infty}^{\infty} \frac{v(z', t)}{z' - z} dz' = 0 \quad (4)$$

This is the Benjamin-Ono equation for the longitudinal velocity $v(z, t)$ of gas within the magnetic slab. The nonlinearity coefficient β is given by

$$\beta = \frac{1}{2} v_A^2 [3c_s^2 + (\gamma + 1)v_A^2] / (c_s^2 + v_A^2)^2 \quad (5)$$

The derivation of (4) is lengthy and so will not be given here; suffice it to remark that it follows from a multiple-scale analysis of the thin flux tube equations (with gravity ignored) with the external pressure terms fully incorporated.

Equation (4) describes the behaviour of weakly nonlinear long waves in a magnetic slab. It is known to possess soliton solutions. In particular, the one-soliton solution is (Benjamin, 1967)

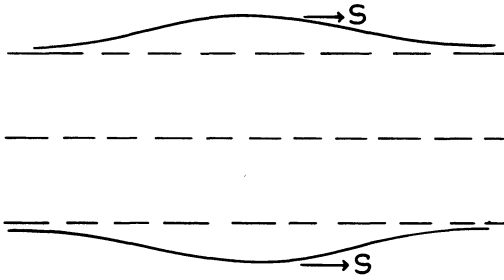


Figure 2. The soliton in a magnetic slab.

$$v(z, t) = \frac{a}{1 + \left(\frac{z - st}{\ell}\right)^2}, \quad (6)$$

where a is the velocity amplitude, and s and ℓ are the speed and characteristic scale of the soliton:

$$s = c_T + \frac{1}{4} \beta a, \quad \ell = \frac{4a}{a\beta}. \quad (7)$$

Figure 2 illustrates the propagation of the soliton. It resembles a symmetrical bulge, like a knee cap, propagating at a speed slightly above the speed c_T . Under photospheric conditions, this gives a soliton speed of about 7 km s^{-1} .

3. DISCUSSION

What conclusions can be drawn from the above analysis? Firstly, it is evident that solitons are likely, at least on theoretical grounds, to propagate in the solar atmosphere. Our example gives a general illustration of this possibility. Of course, we have made a number of simplifying assumptions in our analysis, and so it would be premature to claim that a soliton of precisely the form calculated can arise. Nonetheless, the indications that solitons can arise are clear enough. Such solitons may occur, for example, in coronal loops — though here the influence of an external magnetic field becomes important (Edwin and Roberts, 1983) — or in intense flux tubes. The difficulty in applying the present soliton theory to an intense tube is the neglect of gravity, and it is clear from the study of linear waves that gravitational effects are particularly important (e.g., Spruit, 1983; Roberts and Webb, 1978). However, ignoring such effects allows us to speculate that the tube soliton may become manifest in the chromosphere as a spicule. Indeed, many of the properties of spicules are reminiscent of those of a soliton.

In conclusion, we have demonstrated (following Roberts and Mangeney, 1982) that magnetoacoustic solitons may arise in ideal MHD if the effects

of magnetic structuring are allowed for. This suggests, since the solar atmosphere is highly structured, that the sun may exhibit the soliton phenomenon. However, the theory is not yet sufficiently advanced as to permit direct applications to the solar atmosphere with any confidence.

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DISCUSSION

GOKHALE: Since the slender fluxtube approximation is used I wonder how the result could be applied to magnetic fields in the solar atmosphere, for which the fluxtubes diverge very rapidly with height.

ROBERTS: As I stressed in my talk, stratification will have important consequences, and I suspect the soliton that I have described will be of limited relevance to that problem. That is an area currently under investigation. But my general aim is to demonstrate the theoretical existence of an MHD soliton in a structured field, and to do this I have taken the simplest example of a structured field I could think of. Of course, the neglect of gravity will be valid for coronal disturbances, so coronal solitons are entirely possible and may be similar to the type described here.

IONSON: You mentioned that thin fluxtubes can support MHD solitons. Are these solitary wave solutions of the KdV equation or nonlinear Schrödinger equation, and are they two or three dimensional?

ROBERTS: The solitons are given by the Benjamin-Ono equation, which is quite distinct from the KdV or nonlinear Schrödinger equations. The soliton I give is two dimensional. It may be in a slab (as described in my talk) or in a cylindrical fluxtube.

STIX: Does the occurrence of solitons on the sun require special conditions of excitation (i.e. special initial conditions)?

ROBERTS: I don't think so. If we look at the laboratory results for internal waves in a fluid (reported by Benjamin (1967), Davis and Acrivos (1967), and very recently by a group in California, all published in the *J. Fluid Mechanics*), we find that solitons are very readily generated. A wavemaker at one end of a tank generates the disturbance. As soon as the transients die out — which is very quickly — we see solitons formed and reflecting in a preserved form from the ends of the tank. In the sun I would guess that granular buffeting, for example, could generate waves in the fluxtube, which would then quickly result in solitons. But for photospheric tubes we must not forget the effect of stratification, which has yet to be included in our description of soliton dynamics.

FRISCH: Is there any evidence for the existence of *torsional* solitons as predicted for slender vortex tubes?

ROBERTS: None that I can think of, but it is interesting to note that vortex motions of a non-magnetic fluid in a cylindrical geometry may give rise to solitons of a type not very different from the magnetic soliton described here. There are some extensions of the theory that can be made, which may give rise to other soliton solutions, e.g. to consider the kink ($m = 1$) mode in a cylindrical fluxtube.

ROXBURGH: A colleague of mine, Dr. Chris Oviden, has obtained soliton solutions for waves propagating in the solar wind and shown that scattering of these solitary waves can produce transverse temperatures greater than longitudinal temperatures within the solar wind plasma. This is hardly direct evidence that solitary waves are generated, but it is at least suggestive. Perhaps these solitons are actually produced in the solar surface layers.

ROBERTS: Presumably these are Alfvén solitons, depending upon plasma effects for dispersion rather than the ideal MHD case I have described here.