PERMEABILITY, BRINE CONTENT AND TEMPERATURE
OF TEMPERATE ICE

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ABSTRACT. A temperate glacier is defined as a glacier containing liquid inclusions in which the concentration of salts is not too high. Nevertheless these salts suffice to produce a depression in temperature comparable with that due to the pressure, and much greater than that due to interfacial energies. Because of this a large part of the liquid water present in the ice is not mobile, contrary to the theory of Nye and Frank. Deformation and recrystallization is bound to close off capillary intergranular channels, for glacier ice is usually impermeable. An explanation is given of why firn, at a depth which the annual cold wave does not reach, is nevertheless transformed relatively suddenly into practically impermeable ice. Saline inclusions will migrate with a velocity inversely proportional to the potential temperature (difference from the melting point of pure ice at the pressure in question) and proportional to the gradient of this potential temperature. This velocity, the salinity, the liquid water content, and the ice temperature, parameters which are all functions of the depth, are calculated for a steady state in a stagnant or moving glacier. Under the action of anisotropic stresses, isolated inclusions perpendicular to the maximum compressive stress will enlarge at the expense of their neighbours. If however the two inclusions are connected by a capillary channel, no enlargement occurs, but instead the salt content decreases, evacuated to the other inclusion. This process would constitute an objection to the theory of glacier sliding by melting and refreezing around small obstacles, unless new subglacial mechanisms were to occur.

RÉSUMÉ. *Perméabilité, teneur en saumure et température de la glace tempérée*. Une glace tempérée est définie comme une glace renfermant des inclusions liquides dont la concentration en sels n'est pas trop forte. Ces sels suffisent néanmoins pour provoquer un abaissement de température aussi important que celui dû à la pression, beaucoup plus que celui dû aux énergies interfaciales. De ce fait une grande partie de l'eau liquide présente dans la glace n'est pas vadose, contrairement à la theorie de Nye et Frank. Déformation et recristallisation doivent obturer les canaux capillaires entre les grains, car la glace de glacier est généralement imperméable. Les inclusions salines migrent à une vitesse inversement proportionnelle à la température potentielle (différence avec la température de fusion de la glace pure sous la pression considérée) et proportionnelle au gradient de cette température potentielle. Cette vitesse, la salinité, la teneur en eau liquide et la température de la glace, paramètres tous fonction de la profondeur, sont calculés pour un état de régime, dans un glacier stagnant ou en mouvement. Sous l'effet des contraintes anisotropes les inclusions isolées perpendiculaires à la compression maximale grossissent aux dépens de leurs voisines. Lorsque deux inclusions communiquent par un canal capillaire il n'y a pas grossissement, mais disparition des sels, évacués vers l'autre inclusion. Ce processus constituerait une objection à la théorie du glissement par fonte et regel contre les petits obstacles, si on ne faisait pas intervenir de nouveaux mécanismes sous-glaciaires.

ZUSAMMENFASSUNG. *Permeabilität, Salzgehalt und Temperatur von temperiertem Eis*. Ein temperierter Gletscher ist definiert als Gletscher mit flüssigen Einschlüssen, in denen die Salzkonzentration nicht allzu hoch ist. Trotzdem genügen die Salze zu einer Absenkung der Temperatur, die mit der druckabhängigen Absenkung vergleichbar, jedoch viel grösser als jene ist, die durch Energieaustausch zwischen den Ebenen hervorgerufen wird. Aus diesem Grunde ist im Gegensatz zur Theorie von Nye and Frank ein grosser Teil des im Eis vorhandenen Wassers nicht beweglich. Deformation und Rekristallisation müssen die kapillaren Kanäle zwischen den Körnern verschliessen, da Gletschereis normalerweise undurchlässig ist. Eine Erklärung dafür wird gegeben, warum Firn in einer Tiefe, die von der jährlichen Kältewelle nicht erreicht wird, trotzdem verhältnismässig schnell in praktisch undurchlässiges Eis umgewandelt wird. Salzige Einschlüsse wandern mit einer Geschwindigkeit, die umgekehrt proportional zur potentiellen Temperatur (Differenz zum Schmelzpunkt reinen Eises unter entsprechendem Druck) und proportional zum Gradienten dieser potentiellen Temperatur ist. Diese Geschwindigkeit, die Salzhaltigkeit, der Gehalt an flüssigem Wasser und die Eistemperatur,—Parameter, die alle von der Tiefe abhängen—werden für einen stagnierenden oder bewegten Gletscher in stationärem Zustand berechnet. Unter der Einwirkung von anisotropen Spannungen vergrössern sich isolierte Einschlüsse rechtwinklig zum Maximum der Druckspannung auf Kosten ihrer Nachbarn. Wenn jedoch die beiden Einschlüsse durch einen kapillaren Kanal verbunden sind, tritt keine Vergrösserung auf, sondern stattdessen nimmt der Salzgehalt durch Abwanderung in den anderen Einschluss ab. Dieser Prozess würde gegen die Theorie des Gletschergleitens durch Schmelzen und Wiedergefrieren an kleinen Hindernissen sprechen, es sei denn, dass neue subglaziale Mechanismen auftreten würden.

DEFINITION OF TEMPERATE ICE

It is usual to define *temperate ice* as ice at "its" melting point. Melting can occur at the boundary of the solid (external surface, interface with a liquid inclusion) or at a defect in the solid. The melting temperature which occurs will depend if we are rigorous on the stresses, the interfacial free energies, the salt content of the liquid phase, and on the nature of the defect. It has therefore not been fully defined, and the definition above is not rigorous.

Another definition could be: temperate ice is ice in thermal equilibrium with a liquid phase, but still ought one not to specify on what scale there is thermal equilibrium? Is pure ice entirely at −0.1°C through a glacier 155 m deep and in equilibrium with pure water at its lower surface to be described as temperate ice?

A definition which seems to be better is: *Temperate ice is ice which contains within it a liquid phase* (whether the liquid inclusions communicate or not) *and which is in local equilibrium with it.* It is in effect the existence of these heat sources (or sinks of cold) dispersed through the ice which the liquid inclusions in the course of freezing provide, which gives temperate ice its peculiar character.

Such a definition excludes the example given above, but includes on the other hand sea ice usually regarded as cold, since it contains brine inclusions. One could also call this temperate ice, at least while the temperature is above about −8°C, and a third phase has not appeared, i.e. a crystalline deposit of salts within the inclusion. In fact most of the considerations which follow apply equally to saline ice which is moderately cold, at least qualitatively. But we shall be more restrictive, and will add the further condition that for ice to be called temperate the concentration of salts in the liquid phase should not be too high. One can therefore calculate the melting point using the classical formula valid for dilute solutions.

WATER CONTENT IN THE ICE OF TEMPERATE GLACIERS

Ice in Alpine glaciers and glaciers in similar climates is temperate. In their accumulation areas the firn in summer is soaked by liquid water. Because of the water which capillarity retains in the firn, the cold wave has a definite front. It does not penetrate more than 15 m (at 3 550 m a.s.l. in the French Alps). At the base of the glacier geothermal heat melts about 1 cm of ice per year. Inside the glacier heat is generated by the deformation. It is thus normal for all the glacier to be temperate, and this characteristic persists down-glacier because the winter snow cover is thick and the mean air temperature is positive there.

Annual variations in the liquid water content are to be expected. This has been verified for me by Joubert (1963). A calculating error was corrected by Lliboutry (1964–65, Tom. 1, p. 108). More precise measurements were made by Dupuy (unpublished). It varies from 0 to 0.6% in the accumulation area, 0 to 1.7% in the ablation area. Further, in the same accumulation area studied by Joubert, Vallon (unpublished) found a correlating variation in crystal size, the water content being higher where the crystals were smaller.

We have known since Renaud's measurements of conductivity (Renaud, [1952], 1958) that glacier ice contains saline inclusions, and that these are concentrated around the edges of the grains. The concentration of ions in falling snow is very variable as seen in Table I.

For snow in the Soviet Union a total of 1.29 milliequivalents per litre was found, which is doubtless a maximum value (1 equivalent = 1 mole per valence of the ion). Gorham (1958) measured global contents of 0.07 to 0.25 milliequivalents per litre, fluctuating with depth, in Nordaustlandet (Svalbard). It is probable that there are annual variations in this content, because the hard frost (rime) which is preferentially deposited on the firn at certain periods in the year is very rich in salts.

TABLE I. REPORTED VALUES OF SALT CONCENTRATION IN SNOW PRECIPITATION

Location	Na^+ $\mu g\ kg^{-1}$		Cl^- $\mu g\ kg^{-1}$		SO_4^{--} $\mu g\ kg^{-1}$	Reference
Greenland	17.7–63.5		84–271		420–720	Federer (unpublished)
Glacier de St Sorlin (Alps)	512–557		2 690–3 030			Dupuy (unpublished)
U.S.S.R.	5 120		5 460		9 170	Chilingar (1956)

Location	CO_3H	NO_3^-	Ca^{++}	Mg^{++}	NH_4^+	Reference
U.S.S.R.	18 200	1 700	4 820	1 740	210	Chilingar (1956)

LOCALIZATION OF THE LIQUID PHASE

When local equilibrium is achieved, it has been accepted since the work of Steinemann (1958) that liquid inclusions are situated at four-grain intersections. This has recently been denied by Nye and Frank (in press). It would be true if the contact angle between ice and water ϕ were greater than 60°. The inclusions would then have the form of tetrahedra with spherical faces, convex on the outside if $\phi > 70°$ 32′, concave on the outside if $60° < \phi < 70°$ 32′. But since $\phi = 10°$ to 30° (Ketcham and Hobbs, 1969), the faces must be concave non-spherical surfaces and the vertices of the tetrahedra extend into capillary channels along three-grain intersections joining neighbouring inclusions. Because of this the ice should be permeable.

This reasoning is based on the variation of the freezing point because of solid-liquid interfacial energy γ_{SL}. If r_1 and r_2 are the principal radii of curvature of the surface (positive for a convex surface), ρ_i the density of ice, T the absolute temperature and L the latent heat of fusion,

$$\Delta T = \frac{T\gamma_{SL}}{L\rho_i}\left(\frac{1}{r_1}+\frac{1}{r_2}\right). \tag{1}$$

The most stable inclusion will be that for which ΔT is a minimum. If a second inclusion is present in its neighbourhood, ΔT will be higher there and so a flux of heat will occur. This will correspond to melting around the first inclusion and freezing around the second.

We note that, as the liquid occupies less volume than the solid from which it comes, a vapour bubble will form in the first inclusion, and at its surface a deviatoric stress will appear close in magnitude to the pressure in the ice p. On the other hand in the second inclusion the increase in volume will produce a much higher deviatoric stress. These very local stresses will change the freezing point in a sense which opposes the process which produced the stresses. The speed of temperature equalization will therefore be controlled not only by the thermal conductivity of ice but also by the stress relaxation.
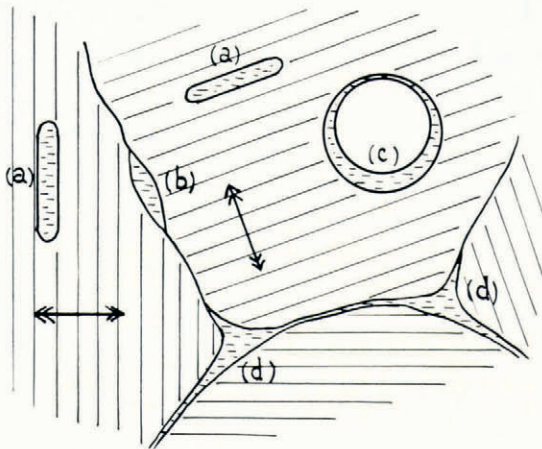

Fig. 1. *Different types of liquid inclusions in polycrystalline ice.*

In order of increasing stability, we shall therefore find (Fig. 1):

(a) *Intracrystalline inclusions* with convex surfaces, for which $\Delta T > 0$. As γ_{SL} is lower when the interface lies in the crystallographic plane (0001) than when it is in a plane parallel to the optic axis ((11$\bar{2}$0) for example), intracrystalline inclusions will have the form of discs flattened in the basal plane. These are *Tyndall flowers*.

The value of γ_{SL} is in effect connected to the solid-solid surface energy by the relation (Nye and Frank, in press)

$$\gamma_{SL} = \gamma_{SS}/2 \cos (\phi/2) \approx 0.51\gamma_{SS} \qquad (2)$$

where γ_{SS} is equal to the difference of the free energies at the crystal boundaries calculated by Yosida (1967). It is about 2.5×10^{-2} J m^{-2} for the $(0001)/(11\bar{2}0)$ interface. Based on Jacobi's nucleation experiments, Dufour and Defay (1963) found a mean value for the surface energy $\gamma_{SL} = 2.38 \times 10^{-2}$ J m^{-2} at $0°$C. From crystal growth experiments Hardy and Coriell (1968) found a mean value for faces parallel to the c-axis of $\gamma_{SL} = 1.6 \times 10^{-2}$ J m^{-2}. From this one deduces for the basal plane $\gamma_{SL} = 3.94 \times 10^{-2}$ J m^{-2}.

(b) *Very flat inclusions at the surface separating two grains.* The temperature rise will be very slight in this case because of their large curvature. The two interfaces join each other along a dihedral angle ϕ.

(c) *Air bubbles*, with very small curvature compared with that of the inclusions, covered with a water film. $\Delta T \approx 0$.

(d) *Inclusions at four-grain and three-grain intersections* as described by Nye and Frank.

PERMEABILITY OF TEMPERATE ICE

The permeability of ice due to capillary channels existing along three-grain junctions was much debated in the last century. It was called upon to explain the growth of crystals or the deformation of ice, at a period when the deformation and recrystallization of solids was not known. A survey of this discussion was given by Forel (1887) and summarized by Lliboutry (1964–65, Tom. 1, p. 112–13).

Agassiz (1847), the Schlagintweit brothers (Schlagintweit and Schlagintweit, 1850), Bertin (1866), Grad (1867) and Grad and Dupré (1869) observed that dye could penetrate ice, particularly blue ice (very low in air bubbles) in the top few metres of a glacier and in crevasse fields (ice in a state of tension). Hugi (1843), Huxley (1857), Hagenbach-Bischoff (1882 p.348) and Forel (1887) found that it did not in regions of compression or in the walls of a natural cave at the bottom of the Glacier d'Arolla.

More recently Carol (1947) made observations right at the bedrock, in front of a bedrock protuberance (where ice deformation is intense and there is therefore a significant release of heat internally in the ice) and found water being squeezed from the compressed ice. According to H. Oeschger (personal communication), the composition of air bubbles at the front of the Grosser Aletschgletscher can be explained if it is assumed that the ice has been in contact, since its formation, with its volume of water, which implies a certain circulation of water within the glacier.

As has been stated above, Nye and Frank (in press) conclude that glacier ice is permeable. Assuming that all liquid inclusions are of type (d), the liquid water content f should be related to the mean radius of the capillary channels R and the mean grain diameter a by the following equation, deduced by Frank (1968) using a model in which the crystals are semi-regular truncated octahedra (bounded by 8 hexagons and 6 squares, sides all equal to $2^{\frac{1}{2}}a/4$)

$$f = 6\pi 2^{\frac{1}{2}}(R/a)^2,$$

so that $R \approx 200$ μm when $f = 10^{-2}$ and $R \approx 60$ μm when $f = 10^{-3}$. $\qquad (3)$

Vertically there are four channels inclined at $45°$ crossing an area a^2 parallel to the square faces. The water flows under the effect of a pressure gradient $dp/dz = (\rho_w - \rho)g$ (where ρ_w is the density of water and ρ is the overall density of the glacier ice), in accordance with Poiseuille's law, so that the outflow per unit area in height of water per unit time

$$q = \frac{\pi R^4}{2^{3/2}a^2} (\rho_w - \rho_i)g = \frac{(\rho_w - \rho_i)g f^2 a^2}{640\eta}. \qquad (4)$$

For pure water at $0°C$, $\eta = 0.0179\,P = 0.565 \times 10^{-15}$ bar year, and $(\rho_w - \rho)g = 10^{-2}$ bar m^{-1}. Taking $a = 10^{-2}$ m, we find a vertical outflow of percolation water of 2.8 m year^{-1} if $f = 10^{-3}$ and 280 m year^{-1} if f is 10^{-2}.

Such high values for the water flowing through the glacier seem highly improbable; one should hardly ever see melt-water streams at the glacier surface. All the summer melt water when it is formed could flow directly to the rock bed through intergranular pores.

If this picture were true, in a very short time the glacier would be riddled with vertical holes throughout its thickness under the effect of Newtonian energy liberated by this percolation water. The heat released in a time dt per unit length (in mechanical units) is $\rho_w g Q dt$, and would increase the radius by dR where

$$\rho_i L 2\pi R\, dR = \rho_w (\rho_w - \rho) g^2 \pi R^2\, dt/8\eta, \qquad (5)$$

or

$$dR/dt = R^3/\alpha. \qquad (6)$$

where

$$\alpha = \frac{16 \rho_i \eta L}{\rho_w (\rho_w - \rho) g^2} = 0.87 \text{ m}^2 \text{ s}. \qquad (7)$$

If R_0 is the radius at time $t = 0$,

$$\frac{1}{2R_0^2} - \frac{1}{2R^2} = \frac{t}{\alpha}. \qquad (8)$$

The radius becomes infinite, and the whole glacier would be melted after a time $\alpha/2R_0^2$ or, for $R_0 = 60\mu m$, after 4 years! The ice at the snout of the Grosser Aletschgletscher, according to H. Röthlisberger (personal communication) is 500 to 1 000 years old. For this process to remain negligible $\alpha/2R_0^2$ would have to be > 500 years, or $R_0 < 5\mu m$ and $f < 7 \times 10^{-6}$.

It is thus clear that some important factor must have been omitted by Nye and Frank in their analysis. The fact that the capillary channels are not circular but triangular cannot change the order of magnitude of the results.

One can therefore think of obstacles created by air bubbles, as suggested by the old observations. Glacier ice contains at least $e = 1$ to 2% of air (by volume). As a result of recrystallization, the position of the capillary channels, situated at three-grain boundaries, has no longer any correlation with the location of air bubbles. On average in 1 m of length, the channel will traverse e metres of air. If $\bar{D}$ is the arithmetic mean of bubble diameters, the intersection of one bubble will have an average a length of $2\bar{D}/3$, and the column is therefore cut $3e/2\bar{D}$ times. The length of the water columns is therefore on average $2\bar{D}/3e$. Everywhere where the water column is split up in this way, air-water menisci appear, more concave at their upper extremity than at their lower (Fig. 2a). The capillary pressure difference can in general compensate the pressure difference $(\rho_w - \rho)g\,\Delta z$. To put it another way, capillary forces can suspend the column of water.

However, the water may be able to find another path downwards (Fig. 2b). For this it is sufficient that $2\bar{D}/3e$ be significantly larger than the mean length of crystal edges. That is certainly the case if $\bar{D}$ is of the order of 1 mm. Air bubbles do not therefore constitute the obstacle to water flow that we are looking for.

I believe that we have to appeal to the deformation and continuous recrystallization of the ice for a blockage of the channels. The deformation of temperate ice, despite the geometrical incompatibility of the deformations within the individual grains, occurs thanks to melting and refreezing at grain boundaries caused by very local but very high stresses which occur there. (This seems to be established even for mean ice temperatures significantly negative (Barnes and Tabor, 1968; Mellor and Testa, 1969[a], [b]). At the crystal boundary we should not think of there being, properly speaking, a free water film flowing according to Poiseuille's law, but rather a liquid-like layer analogous to that which has been suggested for the surface

of isolated crystals (Drost–Hansen, 1967; Jellinek, 1967). It is here that the dislocations moving out of the crystal during plastic deformation disappear. Although the outflow from this liquid-like layer is probably completely negligible compared with that of a capillary channel, it could be sufficient to lower the pressure in certain places along the channel without the channel diameter having to increase. Thus refreezing could cause blockage of the channel.

In other places, the diameter of the channel having become, as a result of deformation less than the thickness of this liquid-like layer of high viscosity, it would find itself practically obstructed. One can even talk of sintering having formed a bung of ice.

Finally in other places it is recrystallization which, by making certain crystals disappear, can transform capillary channels between three crystals into inclusions between two crystals, and the latter into inclusions within a single crystal.
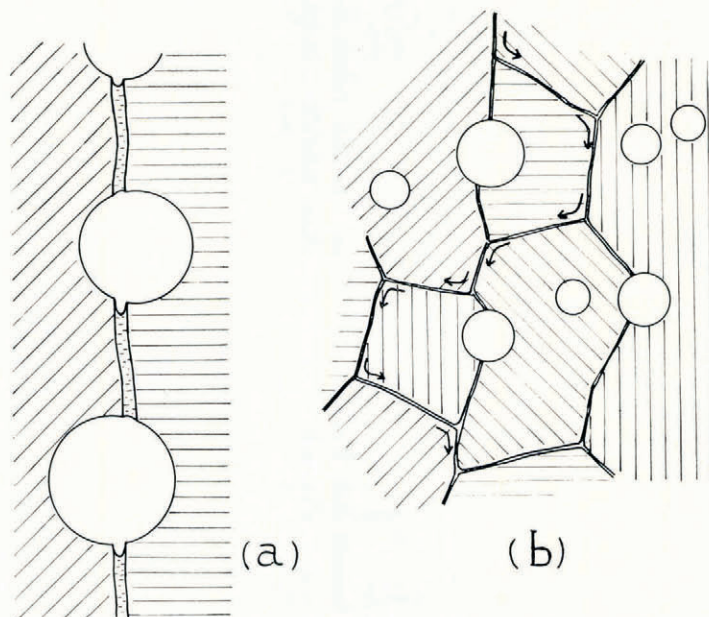

Fig. 2. *Separation of capillary channels by air bubbles.*

These three processes (and possibly others not mentioned here) deserve experimental study. This will be difficult because it is necessary to superimpose a hydrostatic pressure on the specimen. Without this deformation produces microcracks (Gold, 1963). These micro-cracks will acquire liquid water, and this will modify the processes which normally exist deep within a glacier.

Ice is thus permeable at any given time for a distance of several grains, but over the whole glacier its permeability is negligible. It cannot be modelled by a porous medium to which Darcy's law can be applied. Accepting that $R = 5 \mu$m (which is, as we have seen, a maximum), the water percolating through the glacier is no more than 0.012 cm year^{-1}. As time proceeds some channels close, others open. It is the deformation of the ice at the scale of the individual crystals which controls the downward movement of water.

On the other hand, there is doubtless a lot of water included in the ice other than at three- or four-grain boundaries. We shall return to this point later.

TRANSFORMATION OF FIRN INTO IMPERMEABLE ICE

In cold glaciers in the percolation zone melt water refreezes at the contact with cold ice giving superimposed ice that is completely impermeable unless it is exposed at the surface. Nothing of this sort happens in a temperate glacier. We were therefore very surprised when coring firn in the Mont Blanc Massif, at about 3550 m a.s.l., to observe at about 30 m depth a sudden change from firn to layers of ice separated by layers of firn saturated with water for about 1 m followed by ice which was compact and apparently impermeable. As the accumulation was on average 2.7 m of water equivalent per year and the mean density of the firn about 0.63 Mg m^{-3}, this horizon was no more than about 7 years old. It always occurred at the same depth. The winter cold wave, as we have said, never penetrated beyond 15 m.

It is not freezing of water which obstructs the capillary channels which are very abundant in the firn, it is the flow of ice. The firn grains are subjected to the pressure of 30 m of firn above them, whereas the interstitial water is at atmospheric pressure.

Nye (1953) has given the theory of the closure of a circular hole under the action of a hydrostatic pressure p acting on the ice. Assuming Glen's law between the effective shear stress τ and the effective shear strain-rate $\dot{\gamma}$

$$\dot{\gamma}/2 = (\tau/A)^n$$

he found that the radius R decreased at a rate $\dot{R}$ given by

$$\dot{R}/R = -(p/nA)^n.$$

If b is the specific net balance in metres of water per year and s the area of ice in a unit section of firn, the pressure which is applied to the firn t years after its deposition is $p \approx 0.1bt/s$. To a first approximation we can take $s = 1$, $n = 3$, $A = 0.8$ bar year$^{-1/3}$ and

$$\dot{R}/R = -(0.1\, bt/2.4)^3, \tag{9}$$

$$\ln(R/R_0) = -(6/b)(bt/24)^4. \tag{10}$$

The obstruction will thus be very rapid when $bt \approx 24$ m. If in the firn originally $R_0 = 0.3$ mm (i.e. s of the order of 5% and a porosity of the order of 15%), the following results are obtained

Age	year	5	6	7
Depth	m	22.7	26.4	30
R	μm	51	7.3	1.3

At that point the ice has become very poorly permeable, and can no longer pass the abundant melt water, which has to flow off down-glacier, staying at the base of the firn.

INFLUENCE OF STRESS AND SALINITY

The equilibrium temperature of an ice–water interface is

$$\theta = -\frac{T}{L}\left(\frac{1}{\rho_i}-\frac{1}{\rho_w}\right)p - \frac{RT^2C}{L} + \frac{T\gamma_{SL}}{L\rho_i}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$= -Ap - A'C + A''\left(\frac{1}{r_1}+\frac{1}{r_2}\right) \tag{11}$$

where p is the pressure in the liquid phase, equal to the stress in the solid phase perpendicular to the interface (we shall return to this point), C is the number of saline ions per kilogramme of water. It must be small for this formula to hold. Using the values $T = 273.2 + \theta \approx 273$ deg, $L = 3.34 \times 10^5$ J kg^{-1}, ρ_i, the density of pure ice, $= 0.9168$ Mg m^{-3}, ρ_w, the density of the liquid phase ≈ 1 Mg m^{-3}, $R = 8.32$ J deg^{-1}, $\gamma_{SL} = 0.016$ to 0.0394 J m^{-2}, we get $A = 0.0074$ deg bar^{-1}, $A' = 1.86$ deg mole^{-1} kg, $A'' = 2.12 \times 10^{-8}$ deg m.

In a mountain glacier, p is of the order of 0 to 30 bar. We have said that the salt content of the ice is of the order of 10^{-4} ions per kg, and its water content of the order of 10^{-2} to 10^{-3}. Thus $C = 10^{-2}$ to 10^{-1}. The temperature depressions due to the hydrostatic pressure in the ice and to the salinity can both reach some tenths of a degree. For the third term to be of the same order of magnitude, there would have to be radii of curvature of the surface of the order of 0.1 μm, a value which we have said is impossible because of sintering phenomena. It is thus always a very small term compared with the other two.

Everything which we have said above concerning the relative stability of different kinds of inclusions is therefore only valid if the salt concentrations are the same in all inclusions. *Intracrystalline inclusions can therefore exist in a perfectly stable state*, it is sufficient if their salt concentration is slightly greater than that of the water at the junctions of three or four grains.

We shall proceed to study the behaviour of isolated inclusions without considering the interfacial energies as dominant.

ISOLATED LIQUID INCLUSIONS IN STAGNANT ICE

In a mass of stationary ice, the stresses are isotropic and equal to $p = \rho gz$ in all directions. Brine inclusions must have the same salt concentration unless their temperature is to be different. There would then be melting around the more saline inclusions, refreezing around the less saline, by the processes described above. The temperature will therefore become

$$\theta = -Ap - A'C. \tag{12}$$

As long as there are salts present, the quantity $(\theta + Ap)$, which I propose to call *potential temperature* must be negative.

But this refers to a *local equilibrium*. On a larger scale, the inclusions are subject to a pressure gradient (1 bar/11.5 m), and to a temperature gradient. This temperature gradient is not necessarily that which would be observed in the absence of salts, 1 deg/1 550 m, and we must calculate it.

In a significant temperature gradient, inclusions will migrate towards the warmer region. This process has been studied in sea ice by Whitman (1926), Kingery and Goodnow (1963) and Hoekstra and others (1965), theoretically and experimentally with good agreement. We shall make a calculation taking account of the pressure gradient, and show that what enters is not the temperature θ but the potential temperature $\theta + Ap = \theta'$.

The warm side of the inclusion melts, the cool side refreezes. The resulting concentration gradient which results within the inclusion is compensated by a flux of salt diffusing from the cold to the warm side. Let us suppose that the temperature gradient is vertical and take the z-axis vertical downwards. The number of equivalents of salt diffusing per unit time is

$$\frac{dn}{dt} = -D \frac{dC}{dz}. \tag{13}$$

For the salt ions of sea ice $D = 0.0765$ m² year⁻¹.

During time dt, over unit area, a volume dV of ice transforms into brine, by melting and acquiring $dn = C\,dV$ equivalents of salt. The speed of the inclusion is

$$v = \frac{dV}{dt} = \frac{1}{C}\frac{dn}{dt} = -\frac{D}{C}\frac{dC}{dz}. \tag{14}$$

From Equation (12) one deduces, introducing the potential temperature, that

$$\frac{dC}{C} = \frac{d\theta'}{\theta}. \tag{15}$$

Since $dp/dz = \rho g$,

$$v = -\frac{D}{\theta'}\frac{d\theta'}{dz}. \tag{16}$$

The velocity of migration is thus independent of the size of the inclusions; all the liquid phase moves *en bloc* with respect to the ice.

When there is local equilibrium, the concentration C of the inclusions is equal to the number of equivalents per kilogramme of ice n divided by the water content of this ice (kg of water per kg of ice). We shall call this latter quantity w to distinguish it from the mobile water considered by Nye and Frank. Equation (12) can then be written

$$\theta' = -A'n/w. \tag{17}$$

Solution of this problem for the complete thickness of the glacier involves using equations expressing conservation of heat and conservation of salts. We shall only write them for the steady state. In unit time across a section perpendicular to the z-axis of unit area there passes a flux of salts vn and a heat flux $-K\,d\theta/dz + vw\rho(c_w - c_i)\theta$, where c_w is the specific heat of the saline water, c_i the specific heat of ice. These fluxes must be conservative, so

$$d(vn)/dz = 0 \tag{18}$$

$$-K\frac{d^2\theta}{dz^2} + \rho(c_w - c_i)\frac{d(vw\theta)}{dz}. \tag{19}$$

If k and n are two constants

$$vn = k \tag{20}$$

$$-K\frac{d\theta}{dz} + \rho(c_w - c_i)vw\theta = \text{constant}. \tag{21}$$

Eliminating v between Equations (16) and (20)

$$n = -\frac{k\theta'}{D(d\theta'/dz)}. \tag{22}$$

This expression substituted in Equation (17) gives

$$w = -\frac{A'n}{\theta'} = \frac{A'k}{D(d\theta'/dz)}. \tag{23}$$

Note that n and w must be positive and θ' negative; k and $d\theta'/dz$ must have the same sign.

Substituting the values of v and w in Equation (21) finally gives the equation for θ'. The constant of integration, which becomes the value of $d\theta'/dz$ for $z = 0$, can be expressed in terms of the liquid water content for $z = 0$, or w_0, using Equation (23). We obtain

$$\frac{d\theta'}{dz} = k\epsilon\frac{z}{\theta'} + \frac{kA'}{Dw_0} \tag{24}$$

where

$$\epsilon = A\rho^2 gA'(c_w - c_i)/K$$
$$= 3.1 \times 10^{-5} \text{ deg}^2 \text{ year kg m}^{-3} \text{ mole}^{-1} \tag{25}$$

(The numerical value has been calculated assuming $g = 1$ bar/11.5 m, $c_i/c_w = 0.492$ and $K/\rho c_i = 40$ m^2 year^{-1}).

Equation (24) can be written

$$\frac{d\theta'}{dz} = \frac{kA'}{Dw_0}\left[1 + \frac{D\epsilon wz}{A'\theta'}\right], \tag{26}$$

$D\epsilon/A' \approx 1.3 \times 10^{-6}$ deg m^{-1}. w_0 being more than a few per cent and z some hundreds of metres, then it must be that $\theta' < 10^{-4}$ deg in order for it not to be true that

$$\frac{d\theta'}{dz} \approx \frac{kA'}{Dw_0} \tag{27}$$

which would imply that $w \approx w_0$ (constant liquid water content) and

$$\theta' \approx \theta'_0 + \frac{kA'z}{Dw_0} \qquad (\theta'_0 < 0) \tag{28}$$

$$n = -\frac{\theta'_0 w_0}{A'} - \frac{kz}{D} \tag{29}$$

$$v = \frac{kA'D}{-\theta'_0 w_0 D - kA'z}. \tag{30}$$

$-\theta'_0$ must be of the order of $kA'H/Dw_0$, H being the thickness of the glacier. We thus deduce that k is of the order of 10^{-6} and v of the order of a centimetre per year.

Because of this migration, most of the intracrystalline inclusions (type a) will before long be captured by grain boundaries, becoming inclusions of the more stable type b. This fact explains Renaud's observations.

The equations correspond to a *steady state throughout a stagnant glacier*. The value of k depends on the conditions at the lower boundary. If $k < 0$, the salinity of the ice increases with depth, and inclusions migrate upwards, less and less quickly as the depth z increases. If $k > 0$ the reverse is true.

It is possible to find a rigorous solution to Equation (24). Putting $\theta' = zu$ the variables separate

$$\frac{\mathrm{d}z}{z} = \frac{-u\,\mathrm{d}u}{u^2 - kA'u/Dw_0 - k\epsilon}. \tag{31}$$

As $\epsilon \ll A'/Dw_0$, the roots of the denominator are

$$u_1 \approx -\epsilon Dw_0/A', \qquad u_2 \approx kA'/Dw_0. \tag{32}$$

Putting

$$\alpha = \frac{-u_1}{u_2 - u_1} \approx \frac{\epsilon D^2 w_0^2}{kA'^2} \ll 1, \qquad \beta = \frac{u_2}{u_2 - u_1}, \tag{33}$$

Equation (31) can be written

$$\frac{\mathrm{d}z}{z} = -\left[\frac{\alpha}{u - u_1} + \frac{\beta}{u - u_2}\right]\mathrm{d}u. \tag{34}$$

The exact solution can be written, returning to the original variables and taking account of the fact that $\alpha + \beta = 1$,

$$|\theta' - u_1 z|^\alpha \cdot |\theta' - u_2 z|^\beta = \Theta, \qquad \text{a constant.} \tag{35}$$

ISOLATED INCLUSIONS IN MOVING ICE

In moving ice there are anisotropic stresses which do not relax, but which re-form as the deformation tends to remove them. In this case if local equilibrium is reached, the concentration of salts in flattened inclusions depends on their orientation.

The temperature is effectively given by Equation (11) with a pressure in the liquid phase p which is higher for inclusions perpendicular to the maximum compression ($p = \sigma_3$) than for those perpendicular to the minimum compression ($p = \sigma_1$). The former, having a lower temperature, receive heat and dilute by melting, while the latter give out heat and concentrate by refreezing. If $\sigma_3 - \sigma_1$ is of the order of a bar, variations of volume and of concentration of a few per cent are sufficient to restore equilibrium.

This phenomenon perhaps explains why sometimes in a thin section of ice certain crystals, having their optic axes in about the same direction, develop numerous visible Tyndall flowers, while those with their optic axes approximately perpendicular do not (observations in progress by Rado at the Laboratoire de Glaciologie du C.N.R.S.).

However, the calculation made above concerning the steady state has to be repeated taking account of the ice movement and the heat released by its deformation. This time we shall ignore the term in $(c_w - c_i)$ which is of secondary importance, and will replace z by Z, the distance from the rock bed. Equation (16) becomes, the velocity of migration being taken as positive upwards,

$$v = -\frac{D}{\theta'}\frac{\mathrm{d}\theta'}{\mathrm{d}Z}. \tag{36}$$

We shall assume Nye flow, i.e. a thickness H of the glacier and horizontal velocities U and V independent of X and Y, and a vertical strain-rate independent of Z.

The mass balance of the glacier being supposed stationary, with a specific balance b constant in the zone under consideration, the vertical velocity of the ice is

$$W = -bZ/\rho H. \tag{37}$$

We shall assume that the new parameters n, w, v, θ' are also independent of X and Y.

The flux of salt through unit area in the three directions X, Y and Z will be respectively Un, Vn and $(W+v)n$. Conservation of mass of salt can be written for the steady state

$$\frac{\mathrm{d}(Un)}{\mathrm{d}X} + \frac{\mathrm{d}(Vn)}{\mathrm{d}Y} + \frac{\mathrm{d}(Wn)}{\mathrm{d}Z} + \frac{\mathrm{d}(vn)}{\mathrm{d}Z} = 0, \tag{38}$$

or, taking account of the incompressibility of ice $(\mathrm{d}U/\mathrm{d}X + \mathrm{d}V/\mathrm{d}Y + \mathrm{d}W/\mathrm{d}Z = 0)$

$$\frac{\mathrm{d}(vn)}{\mathrm{d}Z} - \frac{bZ}{H}\frac{\mathrm{d}n}{\mathrm{d}Z} = 0. \tag{39}$$

Neglecting terms in $(c_w - c_i)$, and putting c for the specific heat of the whole system, the heat flux components are $c\rho\theta H - K\,\mathrm{d}\theta/\mathrm{d}X$, $c\rho\theta V - K\,\mathrm{d}\theta/\mathrm{d}Y$, $c\rho\theta W - K\,\mathrm{d}\theta/\mathrm{d}Z$. Heat generated per unit volume is equal to $\tau\dot{\gamma} = B\tau^{n+1}$, if we take for the law of deformation of ice

$$\dot{\gamma} = B\tau^n. \tag{40}$$

The heat equation can therefore be written

$$K\frac{\mathrm{d}^2\theta}{\mathrm{d}Z^2} + \frac{cbZ}{H}\frac{\mathrm{d}\theta}{\mathrm{d}Z} + B\tau^{n+1} = 0 \tag{41}$$

θ' being small, B can be considered constant; n being large, τ can be replaced by $\tau_{xz} = \rho g(H-Z)\sin\alpha$ (α being the surface slope). Equation (41) can be written, putting $cb/2KH = a$,

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}Z^2} + 2aZ\frac{\mathrm{d}\theta}{\mathrm{d}Z} + \frac{B}{K}(\rho g\sin\alpha)^{n+1}(H-Z)^{n+1} = 0. \tag{42}$$

From this we can find $\mathrm{d}\theta/\mathrm{d}Z$, and $\mathrm{d}\theta'/\mathrm{d}Z = \mathrm{d}\theta/\mathrm{d}Z + A\rho g$, so

$$\frac{\mathrm{d}\theta'}{\mathrm{d}Z} = \frac{B}{K}(\rho g H\sin\alpha)^{n+1}\exp(-aZ^2)\int_0^z\left(1-\frac{Z}{H}\right)^{n+1}\exp(aZ^2)\,\mathrm{d}Z - G\exp(-aZ^2) + A\rho g \tag{43}$$

G being a constant which in the steady state of the bedrock is at maximum equal to the geo-thermal heat flux. $\mathrm{d}\theta'/\mathrm{d}Z$ being calculated, and hence θ', one can calculate $v(Z)$ with the help of Equation (36). Equation (39) takes the simple form

$$\phi(Z)\frac{\mathrm{d}n}{\mathrm{d}z} + \psi(Z)n = 0,$$

from which $n(Z)$ and $w = -A'n/\theta'$ can be found. The integrations can only be done numerically. The constants of integration can be deduced from the fact that a film of water

circulates between ice and bedrock, carrying with it all the saline impurities. Thus for $\mathcal{Z} = 0$, n vanishes, and since w is certainly not zero, θ' must also vanish. The velocity of migration v therefore tends to infinity towards the bottom as one approaches $\mathcal{Z} = 0$.

In practice in the course of rapid migration the inclusions will all end up by combining into positions (d) between 3 or 4 grains and the ice will become permeable.

REMARK ON THE EQUILIBRIUM BETWEEN WATER AND ICE UNDER AN ANISOTROPIC STRESS

The question of the equilibrium temperature at the interface between a solid under an anisotropic stress and water has been debated for more than a century (LaChapelle, 1968). Many of the arguments are either erroneous as a result of confusing local equilibrium, at the interface itself, and the equilibrium of the whole system, or else because they envisage mechanisms which cannot be realised in practice which compress the ice without compressing the interposed water film (Lliboutry, 1964[a], 1964[b], p. 34–36). On the other hand Thomson's (1862) original argument seems to me irrefutable.

According to Thomson, the ice which appears by refreezing will not enlarge existing crystals, but will instead form separate crystals within the liquid phase. This is not necessary: if (with the help of numerous dislocations) the atomic layers abut the existing crystals, they will not initially be stressed, but will rapidly become so as a result of creep of the whole mass of ice. This creep tends to make the stresses more homogeneous.

Thomson's hypothesis is still without experimental proof; the spicules of ice observed by LaChapelle in subglacial cavities are not, as I see it, those predicted by Thomson. They appear where water under pressure exudes from permeable ice. I do not think they can be considered as being formed from water already present in the cavity.

BRINE INCLUSIONS IN VARIOUS ORIENTATIONS WHICH INTERCONNECT

Let us consider again two liquid inclusions, one (3) perpendicular to the maximum compressive stress σ_3, the other (1) to the minimum compressive stress σ_1, but now let us suppose they are connected by a capillary channel. This will be possible if they are situated at grain boundaries. There will therefore be a flow of liquid from the cavity where the pressure is greater and equal to σ_3 to that where it is lower and equal to σ_1, at a mean velocity (x being measured along the capillary channel)

$$\bar{u} = -\frac{\delta^2}{\eta}\frac{d\sigma}{dx}. \tag{45}$$

For a circular channel of radius r, $\delta^2 = r^2/8$. For a very thin film of thickness d, $\delta^2 = d^2/12$.

This flow will carry salts from (3) to (1). But the salts want to diffuse against the flow. If C is the brine concentration, in ions per unit volume, in the steady state

$$D\frac{d^2C}{dx^2} = \bar{u}\frac{dC}{dx}. \tag{46}$$

Assuming that δ is independent of x, one can integrate the relation obtained by eliminating $\bar{u}$ between Equations (45) and (46), to obtain

$$D\ln\frac{C_1}{C_3} = \frac{\delta^2}{\eta}(\sigma_3 - \sigma_1). \tag{47}$$

With the values of D and η previously given, $D\eta = 0.43 \times 10^{-16}$ bar m².

Even if $(\sigma_3 - \sigma_1)$ is exceptionally high as a result of local stresses and reaches some tens of bars, it would be necessary for δ not to exceed some tens of Ångstrøms for C_3 not to be extremely small compared with C_1. (For such thin channels Poiseuille's law of viscous flow and Fick's law for diffusion would no longer be applicable).

The temperature in the steady state is $\theta(x)$

$$\theta = -A\sigma - A'C, \tag{48}$$

whence, eliminating σ, and since $C_3 \ll C_1$,

$$\theta_1 - \theta_3 = \frac{AD\eta}{\delta^2} \ln \frac{C_1}{C_3} - A'C_1. \tag{49}$$

$\theta_1 > \theta_3$ is necessary for there to be melting around inclusion (3), refreezing around inclusion (1), and flow from (3) to (1) allowing $C_3 \ll C_1$. A final equation can be written introducing the conductivity of ice K, which brings the heat from (1) to (3). If the size of the inclusions is small compared with their separation, the temperature in the steady state at a point r_1 from inclusion (1) and r_3 from inclusion (3) will be

$$\theta = \theta_0 + \frac{Q}{4\pi K}\left[\frac{1}{r_1} - \frac{1}{r_3}\right], \tag{50}$$

Q being the total heat absorbed by melting at (3) and released by refreezing at (1). If r is a mean radius of inclusions,

$$\theta_1 - \theta_3 = Q/2\pi Kr. \tag{51}$$

Objection to current theories of sliding

Equations (45) to (49) remain valid when one considers, on a larger scale, the melting of ice against a bedrock protuberance. In Weertman's theory (1957) in its first and most simple form, one considers cubic protuberances of side a and mean separation λ. If f is the friction per unit area on the bedrock, the pressure difference between the up-glacier face, where there is melting, and the down-glacier face, where there is refreezing, is

$$\sigma_3 - \sigma_1 = f\lambda^2/a^2. \tag{52}$$

The amount of heat absorbed by the melting will be, if U is the velocity of sliding due to this process, $Q = Ua^2$. Putting K_b for the thermal conductivity of the protuberance,

$$\theta_1 - \theta_3 = Q/Ka = LUa/K. \tag{53}$$

In the classical theory one assumes

$$\theta_1 - \theta_3 = A(\sigma_3 - \sigma_1) \tag{54}$$

so that

$$Ua/K = Af\lambda^2/a^2 \tag{55}$$

and U is proportional to the friction. In reality, taking account of the saline inclusions, Equation (54) must be replaced by Equations (47) and (49)

$$\theta_1 - \theta_3 = A(\sigma_3 - \sigma_1) - A'C_1 \tag{56}$$

so that

$$Ua/K = Af\lambda^2/a^2 - A'C_1. \tag{57}$$

As this process proceeds, saline inclusions contained in the ice will be concentrated in the cavity down-glacier, reducing its temperature. Rapidly $\theta_1 = \theta_3$, and the process will be stopped. This result even occurs if one considers a water pocket down-glacier from the protuberance. This is a serious objection to the theory, since the process of melting and refreezing around small obstacles ($a < 1$ cm) is essential for there to be any sliding.

To resolve this paradox, two phenomena must be taken into account: (a) As a result of strong local deformations, the water content will increase near the bottom of the glacier. The glacier will become permeable there. Water will exude from the ice compressed above

the protuberance, and its production will be greater than LUa, that is to say to that which refreezes down-glacier. Melting by the terrestrial heat flux also increases this effect. The down-glacier cavity therefore must have a drain; water does not remain there indefinitely.

(b) Refreezing occurs against the down-glacier face of the protuberance, the water pocket downstream will find itself separated from the protuberance, included in the ice, and must be periodically carried away by the moving glacier (Lliboutry, 1968).

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