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# BOOK REVIEWS

CASSELS, J. W. S. and a Committee appointed by the London Mathematical Society (eds.) *The collected papers of J. E. Littlewood*, 2 volumes (Oxford University Press, 1982), xxxviii + 1675 pp. £120 the set.

These two volumes contain just over one hundred research papers, about half of Littlewood's output; the other half, those of which Hardy was a coauthor, have already been published by the Oxford University Press in *Collected Papers of G. H. Hardy*. Again the papers have been reproduced photographically, so that the original pagination is preserved for reference, and arranged by subject matter. Those on Differential Equations (some 455 pages) and Real Analysis (328) appear in Volume I, while Volume II has sections on the Zeta Function and Number Theory (166), Complex Analysis (347) and Probabilistic Analysis (272), together with a few miscellaneous papers and mathematical notes and an article "A mathematical education" from the book *A mathematician's miscellany*. Each section has been edited by one or more experts who have added a commentary and listed misprints and corrections. Each volume begins with the listed contents of both, followed by J. C. Burkill's note which appeared in *Biographical Memoirs of Fellows of the Royal Society* **28** (1978), and ends with a complete chronological bibliography.

Although the overall picture is distorted by the absence of the Hardy and Littlewood papers, it is still a surprise to find that the twenty-two papers on differential equations make up easily the longest section. Only two of these—one joint with his father, and both on ballistics, in which Littlewood's interest was aroused during his 1914–18 war service—were written before Littlewood's sixtieth birthday, and only a further four before his seventieth! There are half a dozen papers on Van der Pol's equation $\ddot{x}+k(x^2-1)\dot{x}+x=b\mu k\cos(\mu t+\alpha)$ and related equations, five on adiabatic invariance which are mainly concerned with equations of the form $\ddot{x}=-V_x(x,w)$ with $w\equiv w(t)$ a slowly varying function, and three on celestial mechanics which brought their author the London Mathematical Society's Senior Berwick Prize in 1960. The papers on equations of Van der Pol's type resulted from collaborative work with Dame Mary Cartwright, and include the earliest rigorous treatment of problems in large parameter theory.

The section on probabilistic analysis also consists of mature work, containing five joint papers of 1938–48 with A. C. Offord on random algebraic equations and random entire functions, by which is meant functions of the form $\sum(\pm a_n)z^n$ where $\sum a_n z^n$ is entire, and seven others of 1966 or later.

Each of the other three major sections is depleted by the absence of the Hardy papers, but still substantial. The twenty-six papers on real analysis include powerful results on mean values of trigonometric polynomials and power series, the fundamental papers with title "Theorems of Fourier series and power series" by Littlewood and Paley in which the $g$-function first appeared, and of course Littlewood's historic 1911 proof of the first $O$-Tauberian theorem. The editors of Section 3 are able to remark that the many consequences of the Riemann hypothesis which Littlewood derived have stood the test of time so well that one is left with the impression that he proved everything that was within reach. The fifteen papers representing Littlewood's substantial independent contribution to complex analysis include his first five papers, on entire functions, written in 1907–9 when function theory was dominated by special functions. As the introduction points out, this section shows Littlewood characteristically taking an area (such as univalent functions) at or near its beginning, establishing or extending it by proving one or two fundamental results and making striking conjectures, and then leaving the field for others. In such circumstances, the value of the collection is tremendously enhanced by an editorial commentary which indicates subsequent developments, and the contributions to the complex analysis section by Professors Brannan and Hayman are particularly excellent in this respect.

This collection, and in particular the detailed work on differential equations and the pioneering work in function theory and number theory, gives an excellent idea of Littlewood's qualities—on Hardy's estimate he was the man most likely to storm and smash a really deep and formidable problem. These handsomely produced volumes are a fitting memorial to an outstanding mathematician; they will fascinate and stimulate every analyst. We are greatly indebted to the editorial committee and the publishers.

PHILIP HEYWOOD

HUA, L. K. *Introduction to number theory* (translated by P. Shiu) (Springer-Verlag, Berlin-Heidelberg-New York, 1982), xviii + 572 pp. DM 96.

This is the English edition of a book on number theory written for Chinese students and first published in 1957. Its aim is to give a broad introduction to the subject, indicating the close relationship between number theory and mathematics as a whole. Its twenty chapters cover a very wide range of topics and contain much more material than could be dealt with in a single university course. The only existing English textbook with which it compares is the *Introduction to the Theory of Numbers* by Hardy and Wright. My impression is that, although it may not be quite so easy to read, Hua's book goes further into the subject than the earlier work.

As would be expected, the very considerable Chinese contribution to the subject is stressed. Thus the name of Soon Go will be unfamiliar to most western readers, but as his general solution of the Diophantine equation $x^2+y^2=z^2$ appeared much earlier than in the west it is right that he should be credited with his achievement.

In a short review it is not possible to give a full list of all the topics covered, but the following selection indicates the scope of the work. After basic introductory chapters the author discusses the distribution of prime numbers and gives two proofs of the Prime Number Theorem, namely the analytic proof of Wiener and the elementary one of Selberg and Erdös. Classical subjects, such as partition theory and the divisor and circle problems, are discussed, and other topics include trigonometric sums, continued fractions, indeterminate equations, binary quadratic forms, unimodular transformations, integer matrices, $p$-adic numbers, algebraic numbers, Waring's problem, Schnirelmann density and the Geometry of Numbers. Nearly everything required is proved in detail and there are indications of further improvements and more recent work. There are also extensive tables of primitive roots and data associated with quadratic fields.

The translator Peter Shiu has done an excellent job and, with very few exceptions, the text runs smoothly. This is a most excellent textbook and mine of information on the theory of numbers. The volume contains between its covers much work that cannot easily be found in one volume; every number-theorist will hope to be able to afford to place it on his shelves.

As the author asks in his preface to be informed of errors, I mention that, so far as I am aware, the values of the Hermite constants $\gamma_9$ and $\gamma_{10}$, given on p. 543, have not been conclusively established.

R. A. RANKIN

WHITELAW, T. A., *An Introduction to Linear Algrbra*, (Blackie, 1983), ix + 241pp., £7·95 (paper covers).

This book provides a substantial first course in linear algebra, with no previous knowledge of the subject assumed. The necessary terminology and facts about mappings are summarised in an appendix.

The first chapter deals with the geometry of three-dimensional vectors, as far as scalar product, but not discussing the geometrical ideas involved in linear dependence. Then comes an exposition of the elements of matrix algebra, with the basic operations defined but not motivated. A very detailed chapter on elementary row operations leads to the form of the general solution of a system of linear equations. The inevitable chapter on determinants avoids lengthy proofs by confining the details of some proofs to the $3\times3$ case, with an indication of how to attempt the general case.