10. RESULTS FROM AN ANALYSIS OF THE RADIAL VELOCITY CURVES OF RR LYRAE

By A. VAN HOOF

A modified Baade procedure, applied to Struve and Blaauw's* determinations of radial velocity and of spectral type and to Walraven's† light curve yielded for this star a median radius

$$\overline{R} = 2.0 \times 10^6 \text{ km.} \pm 0.06 \times 10^6 \text{ km.} \text{ (m.e.)}.$$

The median thickness of the atmospheric layer that is limited by the representative level of the $H_{\beta,\gamma,\delta}$ lines at the top and by that of the metallic lines at the bottom was found equal to

$$\delta = 0.14 \times 10^6$$
 km. $\pm 10\%$ (estim. m.e.).

Conclusions

1. The star now appears to obey rather well the empirical relation

$$R = 4 \times 10^6 \times P_{\text{days}} \text{ km.}$$

found to exist for cepheids with P > I day. If this holds for the other cluster variables, the period-luminosity law simply expresses a radius-luminosity law and is in fact equivalent to $I = P^{2T4}$

$$L \sim R^2 T_{\text{eff.}}^4$$

from which it is immediately deduced when the period-spectrum relation is taken into account.

2. With $\overline{R} \sim P$, $P \sqrt{\rho} = \text{const.}$ reduces to $\frac{\mathfrak{M}}{\overline{R}} = \text{const.}$ which makes the period-luminosity

law equivalent to the mass-luminosity law for cepheids and moreover implies the same central temperature for all cepheids. But the validity of $P\sqrt{\rho} = \text{const.}$ becomes questionable.

3. The amplitude of the pulsation in the layer considered amounts to 100% of the average layer thickness; the velocity of propagation of the compression wave in it is supersonic (30 km./sec. against 10 km./sec. for the velocity of sound).

4. It is tempting to think of the pulsations as originating, not at the centre of the star but in a spherical stratum far from the centre. This might explain the occurrence of secondary maxima beat periods and perhaps even the gaps in the frequency function of the periods.

II. SPECTROSCOPIC CHARACTERISTICS OF THE CLUSTER-TYPE VARIABLES

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The cluster-type variables deserve particular attention since they are connected with two problems of fundamental importance: the phenomenon of pulsating stars and the existence of different subsystems or populations of stars in our own and other galaxies. On the other hand there are some obscure points in our understanding of the physical and dynamical properties of this group of stars as, for instance, their failure to give a period-spectrum relation or to obey the proper period-density relation. With this in view Dr Struve suggested to me during my stay at the Yerkes and McDonald Observatories to undertake a comparative spectrophotometric investigation of a series of these stars. The spectra of nineteen galactic cluster-type variables in different phases and those of twenty standard non-variable stars of early spectral class have been photographed by

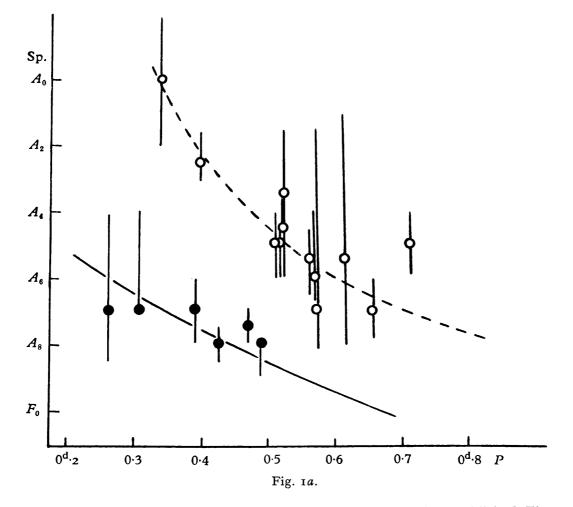
* Ap. J. 108, 60, 1948.

† B.A.N. **9**, 17, 1949.

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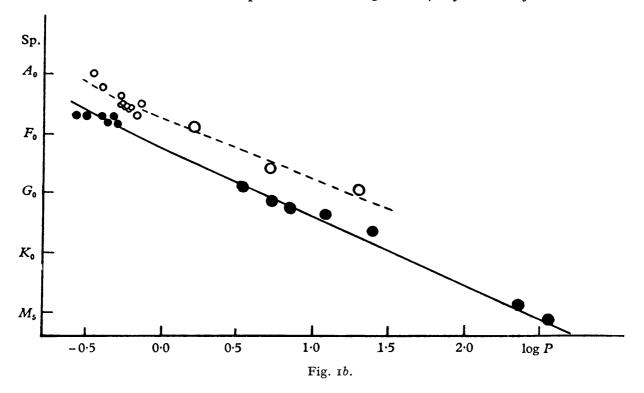
Dr Struve and myself in 1949 with the two-quartz-prism spectrograph and short-focus Schmidt camera attached to the McDonald reflector. This combination gives a rather small linear dispersion of 170 A./mm. on the film but a good resolving power and a wide wave-length range reaching well beyond the Balmer limit in the ultra-violet. The exposures have been calibrated photometrically with the aid of a sensitometer.

The spectra have been registered on the recording microphotometer of the Yerkes Observatory and the registrograms were taken to Toruń for further treatment. The depths of about sixty spectral lines have been evaluated in magnitude scale on each spectrogram and the observed contours of the three Balmer lines: γ , δ , η and the K-line of Ca⁺ have been obtained.



For a comparative study a common system of reference must be established. The most suitable one would be the temperature scale. Lacking this, the spectral class as temperature equivalent is often used. Yet there is no common classification system for stars belonging to different populations. Münch and Terrazas have determined the colour indices of some cluster-type variables and found that a classification based on the lines of metals follows approximately the colour-temperature scale. In the present investigation a system of classification based on the measured intensity of the K-line of Ca⁺ is established, assuming that the cluster variables have the same intensity in the K-line as the non-variable giants of the same spectral class. The individual exposures have been classified according to this principle, then the most probable median spectrum for each variable derived and plotted against its period. It was noted then that there exists a period-spectrum relation for the cluster-type variables and that this relation is double (Fig. 1a).

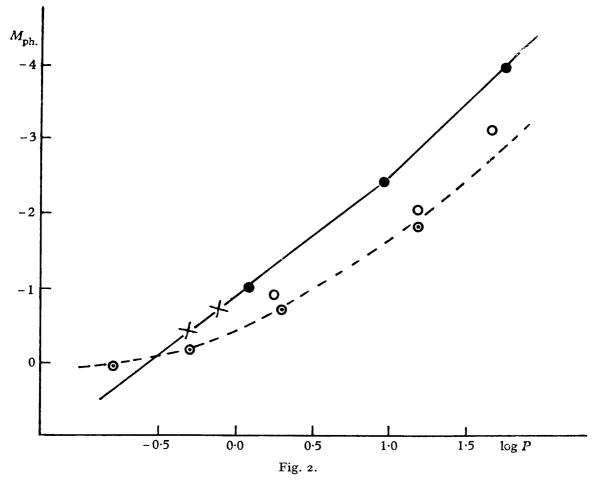
The majority of cluster-type variables are concentrated along the branch running through earlier spectral classes; six of nineteen stars, presumably of shorter periods (BC Eri, UZ Leo, TZ Aur, AR Per, RY Com, BB Pup), form a second nearly parallel branch, shifted by about 0.3-0.4 of one spectral class interval towards later subdivisions. The distinction proves to be undoubtedly real when allowance is made for the uncertainty limits of the median spectrum determinations. The long suspected presence among the spherical subsystem of cluster-type variables of an admixture of population I stars finds a confirmation in the double period-spectrum relation as well as a means for discrimination. The six stars with one exception (RY Com) are situated close to the galactic plane and have moderate radial velocities also with one exception (BB Pup). There are also differences in line intensities between the two groups of stars, which will be referred to below. The fact that this admixture is presumably populated by shorter periods explains the velocity-period relation recently found by O. Struve as well as the strong galactic concentration of variables with periods between 0.3 and 0.4 days noted by B. Kukarkin.



It occurs further that the two branches of the period-spectrum relation form a straight extension of the corresponding relations existing for variables of longer periods, the population I branch fitting well to the period-spectrum relation of classical cepheids, the other forming with the variables recently investigated by Joy in globular clusters a second nearly parallel line (Fig. 1b).

Since the existence of the double period-spectrum relation over the entire range of periods of pulsating stars is a fact, it seems quite natural that there exist also two different period-luminosity curves corresponding to different populations. The generally adopted period-luminosity curve is then a symbiosis of the population I branch based chiefly on cepheids of the Small Magellanic Cloud and the population II branch formed by variables in globular clusters. Trying to guess the complete picture of the double period-luminosity relation with the help of the recently published data by Dartayet and Dessy for the Small Magellanic Cloud and by Joy for the globular clusters, I see two smooth curves intersecting at the critical point log P = -0.5, $M = 0^{\text{m}}$, the steeper curve representing the population I (Fig. 2). One may also trace two branches for the pulsating stars of the two populations on the Hertzsprung-Russell diagram (Fig. 3).

The period-density relation needs also a revision from this point of view. One of our young collaborators, Mrs Lubiénska-Iwaniszewska of Toruń has attempted to calculate the mean density for pulsating stars using the double period-luminosity curve, double period-spectrum relation and double mass-luminosity relation, the last based on the diagram given by P Parenago. She got for each population separately tolerable constancy of the product $P_{\sqrt{\rho}}$, the constant for population II being larger.



The measured depths of all spectral lines and blends, which can be considered in the actual dispersion as the total absorption equivalents for the weak lines, have been plotted against the spectral class for variables as well as for standard stars. Consideration of these diagrams leads to the following conclusions:

The Balmer lines and continuum are very much weakened in population II variables and a little weakened in population I variables as compared with standard stars.

The weak lines of most metals, neutral and ionized, are strengthened in variables, especially in those of population I.

The lines of Sc, V, Ti, rare earths (Eu, Pr) and perhaps some others are more strengthened in population II variables. I call this group 'rare elements' since they are deficient in the Sun's atmosphere.

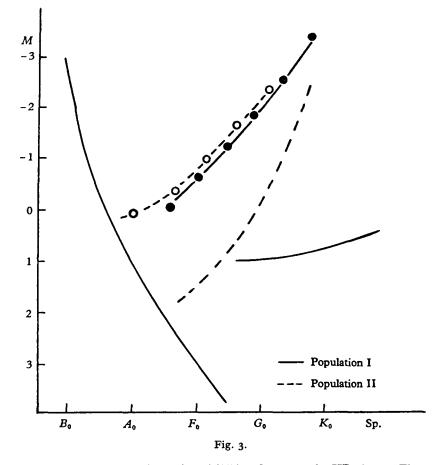
It appears that the spectral behaviour of the population I cluster-type variables is normal for their position as an extension of the classical cepheids towards earlier spectral classes. As to the proper or population II cluster-type variables, they show some common spectral features with the non-variable high-velocity stars, allowance being made for the difference in spectral class and luminosity. Here and there the hydrogen lines and continuum are very much weakened; the lines of 'rare elements' strengthened. The slight strengthening of other metallic lines (Fe, Si, Mn, etc.) in variables not present

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in non-variable high-velocity stars may be the effect of interception of the responsibility for the opacity from the negative hydrogen ion by the hydrogen atom corresponding to the higher temperature in the variables.

As to the high-velocity stars whose spectra I have investigated from Saltsjöbaden, I should like to mention on this occasion that an attempt has recently been made by our young collaborator Miss Tomasik of Warsaw to establish a colour temperature scale for these stars using objective-prism spectra taken with the Draper telescope in Toruń. According to her results earlier spectral classes should be assigned to the high-velocity



stars than those based on the intensity of CH band or even the HD classes. The classification based on the intensity of the Fe or Ca lines would agree with the colours. In this new frame of classification the spectral characteristics of the high-velocity stars are expressed by the very weak hydrogen lines, very strong CH band and the rare elements, the other lines with Fe, Ca, Sr⁺ and even CN rather normal for their class and luminosity.

The observed contours of the hydrogen and Ca⁺ lines which have been obtained for cluster-type variables and standard stars, can reveal only rough differential effects with the dispersion used. Thus it can be seen that in general the lines in population II variables have narrower cores and relatively wider wings than those of population I. The variables of population I show similar or a little broader line contours than standard giants. One of these variables, BC Eri, has exceptionally broad lines: tentatively interpreted in terms of axial rotation it would involve a rotational speed of about 70 km./sec.