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# SHORT NOTE

# EQUATIONS FOR DETERMINING THE BRINE VOLUME OF SEA ICE FROM $-0{\cdot}5^{\circ}$ TO $-22{\cdot}9^{\circ}$C.

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ABSTRACT. Brine volume of sea ice is a function of the salinity and temperature of the ice, and it is related to its strength. This paper gives three equations which can be used to compute the brine volume for three temperature ranges from $-0{\cdot}5^{\circ}$ to $-22{\cdot}9^{\circ}$C. A less accurate equation covering the total range is also presented.

RÉSUMÉ. *Equations pour déterminer le volume de saumure de la glace de mer entre $-0{,}5^{\circ}$ et $-22{,}9^{\circ}$C.* Le volume de saumure de la glace de mer est une fonction de la salinité et de la température de la glace, et il est lié à sa solidité. Cette communication donne trois équations qui peuvent être utilisées pour calculer le volume de saumure pour trois domaines de température entre $-0{,}5^{\circ}$ et $-22{,}9^{\circ}$C. Une équation moins précise est aussi donnée pour tout le domaine de température.

ZUSAMMENFASSUNG. *Gleichungen zur Bestimmung des Salzsole-Volumens in Meereis bei Temperaturen von $-0{,}5^{\circ}$ bis $-22{,}9^{\circ}$C.* Das Salzsole-Volumen in Meereis ist eine Funktion des Salzgehaltes und der Temperatur des Eises und steht mit seiner Festigkeit in Beziehung. In dieser Arbeit werden 3 Gleichungen zur Berechnung des Sole-Volumens für 3 Temperaturbereiche von $-0{,}5^{\circ}$ bis $-22{,}9^{\circ}$C. angegeben. Auch eine weniger genaue Gleichung für den Gesamtbereich wird vorgelegt.

## INTRODUCTION

The relative volume of brine in standard sea ice is a function of the salinity and temperature of the ice and it is related to its strength. The results of a large number of tests by many investigators have shown that the strength of the ice decreases with increasing brine volume. Also, other properties are related to brine.

Probably the most widely used method for determining brine volume is to use Assur's (1960) brine-volume table. This paper derives three equations, based on the values from this table, which can be used to compute the brine volume from $-0{\cdot}5$ to $-22{\cdot}9^{\circ}$C.

## ANALYSIS AND RESULTS

Assur constructed a table which gives the relative volume of brine ($\nu$) in standard sea ice of salinity of 1‰ (parts per thousand) depending on temperature (°C.). He computed the table from the phase relations in standard sea ice. The values in the table are the basis for the three equations developed here.

Assur computed the volume of brine, $\nu$, from

$$\nu = b_{\mathrm{r}} \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{b}}} \tag{1a}$$

where $b_{\mathrm{r}}$ is the brine content by weight, $\rho_{\mathrm{i}}$ is the theoretical density of sea ice and $\rho_{\mathrm{b}}$ is the density of the brine. He also noted that

$$b_{\mathrm{r}} = S/S_{\mathrm{b}} \tag{1b}$$

without precipitation of salts, where $S$ is the salinity of the ice and $S_{\mathrm{b}}$ the salinity of the brine. On the other hand,

$$\frac{1}{S_{\mathrm{b}}} = 1 - \frac{54{\cdot}11}{\theta} \tag{1c}$$

with $\theta$ the ice temperature.

Substituting Equation (1b) and then (1c) into Equation (1a) gives

$$\nu = S\left(1 - \frac{54{\cdot}11}{\theta}\right)\frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{b}}}. \tag{1d}$$

We see that the relation can be linearized in terms of $\nu$ versus $1/\theta$. This holds in wide temperature ranges as we shall see below.

The values at $-1{\cdot}0°$ and $-2{\cdot}0°$C. were used to compute a new equation, namely

$$\nu = S\left(\frac{52{\cdot}56}{\theta} - 2{\cdot}28\right). \tag{2}$$

The minimum limit of Equation (2) was determined by computing the deviation of Equation (2) from Equation (1d) between $-1{\cdot}0°$ and $-2{\cdot}0°$C. and between 0° and $-1{\cdot}0°$C. The point between 0° and $-1{\cdot}0°$C. where the percentage deviation equaled the maximum percentage deviation between $-1{\cdot}0°$ and $-2{\cdot}0°$C. was used as the minimum limit of Equation (2). This limit occurred at $-0{\cdot}5°$C.

The brine-volume values from the table were plotted for the temperature range between $-2{\cdot}0°$ and $-8{\cdot}2°$, and between $-8{\cdot}2°$ and $-22{\cdot}9°$C. The limit of the two plots at $-8{\cdot}2°$C. is the result of $Na_2SO_4 \cdot 10H_2O$ beginning to precipitate at this temperature. A least-square fit of the two plots, which were forced to have the same value of 6·53 at $-8{\cdot}2°$C., gives the following equations:

$$\nu = S\left(\frac{45{\cdot}917}{\theta} + 0{\cdot}930\right) \tag{3}$$

for the range between $-2{\cdot}0°$ and $-8{\cdot}2°$C. and

$$\nu = S\left(\frac{43{\cdot}795}{\theta} + 1{\cdot}189\right) \tag{4}$$

for the range between $-8{\cdot}2°$ and $-22{\cdot}9°$C. Equation (3) has a standard error of 0·00224 and a correlation coefficient of 0·99994, while Equation (4) has a standard error of 0·00059 and a correlation coefficient of 0·99965. This indicates that the equations can be used with good accuracy.

Equations (2) and (3) intercepted at $-2{\cdot}06°$C., which is then the maximum limit of applicability for Equation (2) and the minimum limit of Equation (3). The maximum limit of Equation (3) and minimum limit of Equation (4) was fixed at $-8{\cdot}2°$C. and the maximum limit of Equation (4) was fixed at $-22{\cdot}9°$C. The final equations with the limits are

$$\nu = S\left(\frac{52{\cdot}56}{\theta} - 2{\cdot}28\right) \quad \text{for} \quad -0{\cdot}5° \leqslant \theta \leqslant -2{\cdot}06°,$$

$$\nu = S\left(\frac{45{\cdot}917}{\theta} + 0{\cdot}930\right) \quad \text{for} \quad -2{\cdot}06° \leqslant \theta \leqslant -8{\cdot}2°,$$

$$\nu = S\left(\frac{43{\cdot}795}{\theta} + 1{\cdot}189\right) \quad \text{for} \quad -8{\cdot}2° \leqslant \theta \leqslant -22{\cdot}9°,$$

where $\nu$ is the brine volume in parts per thousand, $S$ is the salinity of the ice and $\theta$ is the absolute value of the ice temperature in °C.

In cases where a simple but less accurate equation is desired, the following can be used from $-0{\cdot}5°$ to $-22{\cdot}9°$C.:

$$\nu = S\left(\frac{49{\cdot}185}{\theta} + 0{\cdot}532\right). \tag{5}$$

The correlation coefficient is 0·99951 and the standard error is 0·15448.

The volume of brine for ice temperatures between $-0{\cdot}1°$ and $-0{\cdot}4°$C., taken directly from the table, is:

| $\theta$ | $-0{\cdot}1$ | $-0{\cdot}2$ | $-0{\cdot}3$ | $-0{\cdot}4$ |
|---|---|---|---|---|
| $\nu$ | 500·9 | 250·5 | 167·1 | 125·4 |

For temperatures below $-22{\cdot}9°$C. one should refer to the table.

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## REFERENCE

Assur, A. 1960. Composition of sea ice and its tensile strength. *U.S. Snow, Ice and Permafrost Research Establishment. Research Report* 44.