COSMOLOGICAL SYNTHESIS OF THE ELEMENTS*

ROBERT V. WAGONER

Dept. of Astronomy and Center for Radiophysics and Space Research, Cornell University, Ithaca, N.Y. 14850, U.S.A.

and

*Institute of Theoretical Physics, Dept. of Physics, Stanford University, Stanford, Calif. 94305, U.S.A.***

Abstract. The results of improved calculations of the abundances of the nuclei produced in big-bang models of the early universe are presented. In addition to the standard model, other possible universes are considered, including the recent statistical bootstrap theory of Carlitz, Frautschi, and Nahm. Some conclusions which can be drawn about the nature of the early universe, depending upon whether the observed deuterium and helium are of galactic or cosmological origin, are presented.

1. Introduction

As we have heard in the preceding talks by Drs Blair, Partridge, and Boynton, both the spectrum and isotropy of the 2.7 K background radiation provide impressive evidence that the Universe has emerged from a state of much higher temperature and density. At present, perhaps the most powerful method of obtaining information about the physical conditions in such a big-bang universe at redshifts $Z \gtrsim 10^9$ is through an analysis of element production. In this lecture, I will compare the results of an improved calculation of nucleosynthesis in such models with recent abundance determinations.

The first detailed calculations of this sort were carried out by Peebles (1966a, b). At about the same time, William Fowler and Fred Hoyle realized the potential importance of this type of confrontation of cosmological theory with observation, and initiated (Wagoner *et al.*, 1967) a series of investigations of somewhat broader scope. The reader is referred to the most recent publication (Wagoner, 1973) for general background and a more detailed discussion of much of the work reported here.

2. Nature of the Early Universe

We shall make two fundamental assumptions regarding our description of the Universe. They are:

(1) Gravitation is described by a metric theory (i.e., one in which special relativity is locally valid for freely-falling observers). All presently viable theories of gravity are in this class (Will, 1973).

(2) That portion of the Universe of interest (e.g., the Galaxy, the Local Group, etc.)

* Supported in part by the National Science Foundation (GP-26068) at Cornell University and (GP-39178) at Stanford University.

** Present address.

M. S. Longair (ed.), Confrontation of Cosmological Theories with Observational Data, 195–210.

was reasonably homogeneous and isotropic during the epoch of nucleosynthesis. The lack of significant anisotropy ($\leqslant 0.1\%$) of the 2.7 K background radiation, which, however, reflects conditions at redshifts $z \lesssim 10^3$, provides support for this assumption, at least on the observed large scales. The geometry of the Universe is then described by the Robertson-Walker metric.

In contrast to these fundamental assumptions (which define the general class of models investigated), the following assumptions (which define the 'standard' model of the Universe) are not on as firm an observational footing, and so effects of their violation will be considered as well.

(1) The temperature was once high enough for statistical equilibrium among all particles present.

(2) The net baryon number is positive.

(3) Only known particles were present (and magnetic fields were negligible).

(4) All particles were non-degenerate.

(5) General relativity is valid.

The evolution of the standard model of the universe is discussed in most recent books on cosmology (e.g., Peebles, 1971).

3. Process of Nucleosynthesis

Those nuclear reactions which have been explicitly included in the present computer program are indicated in Figure 1. Fortunately, most of the cross sections of importance have been experimentally determined, so that the estimated uncertainty in the calculated final abundances is less than two percent for ^{4}He, and less than a factor of two for other nuclei of mass number $A \leqslant 7$.

The evolution of the nuclear abundances and baryon mass density in a typical standard model is shown in Figure 2. At temperatures T_9 (in units of 10^9 K) $\gtrsim 10$, the neutron/proton ratio is held at its equilibrium value through the weak reactions indicated in Figure 1. At lower temperatures, the neutron decay rate is slow compared to the expansion rate

$$V^{-1}\,\mathrm{d}V/\mathrm{d}t = \sqrt{24\pi G\varrho} \tag{1}$$

of any comoving volume element V. Since virtually all of the neutrons are used to make ^{4}He, its final abundance is determined most strongly by the precise temperature at which the nucleon weak reactions 'freeze out' of equilibrium, which in turn is determined by the equality of their rate and the expansion rate.

On the other hand, the final abundances of the other nuclei depend upon the baryon density ϱ_b at the temperature $T_9 \sim 1$ when they can be synthesized, or more conveniently, upon the parameter

$$h = \varrho_b T_9^{-3} \cong \text{const}, \tag{2}$$

which is inversely proportional to the entropy per baryon. Its (constant) value h_0

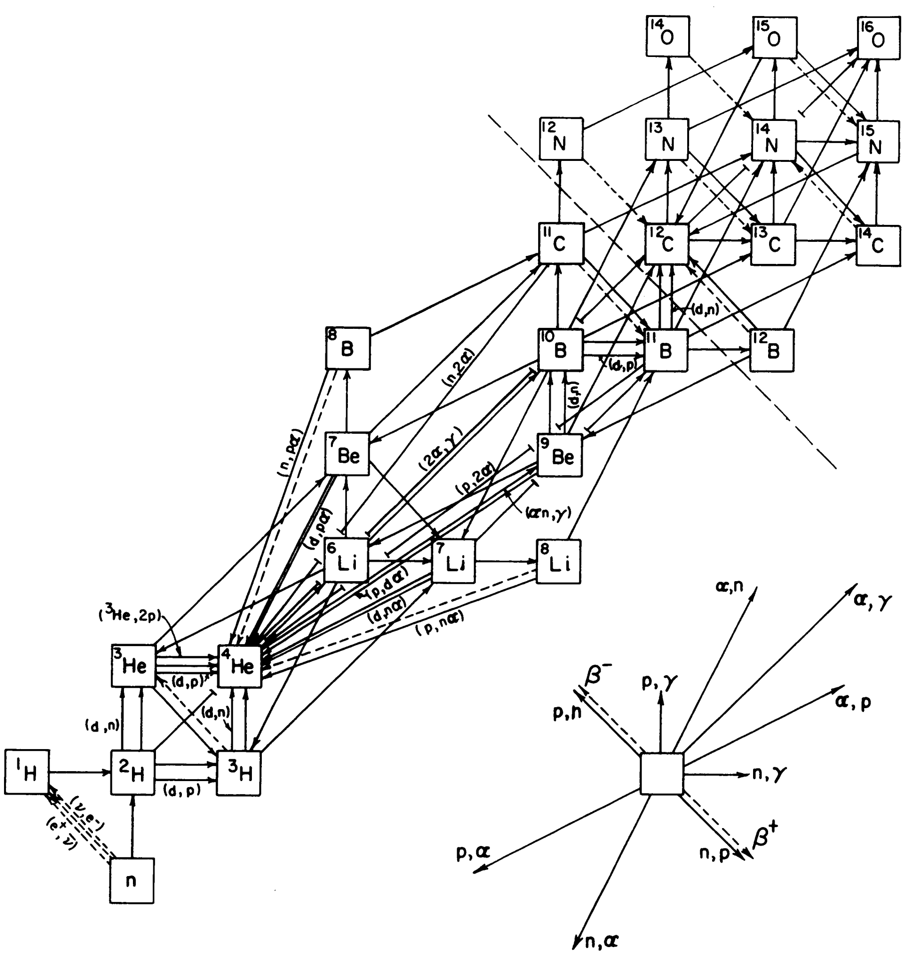

Fig. 1. Diagram of all nuclear reactions included in the computer program. The exoergic directions are indicated by the arrows.

before pair annihilation is related to the present baryon density by

$$\varrho_b(T=2.7\,\mathrm{K})=7.15\times10^{-27}\ h_0\quad \mathrm{g\,cm^{-3}}. \tag{3}$$

The qualitative behavior of models which do not differ too greatly from the standard model will be the same as that indicated above.

4. Observed Abundances

Since fairly complete discussions of the relevant abundance data have been recently given by Reeves *et al.* (1973) and by Wagoner (1973), we will merely summarize the results and discuss the more recent observations.

In general, abundance determinations for ^{4}He give mass fractions in the range

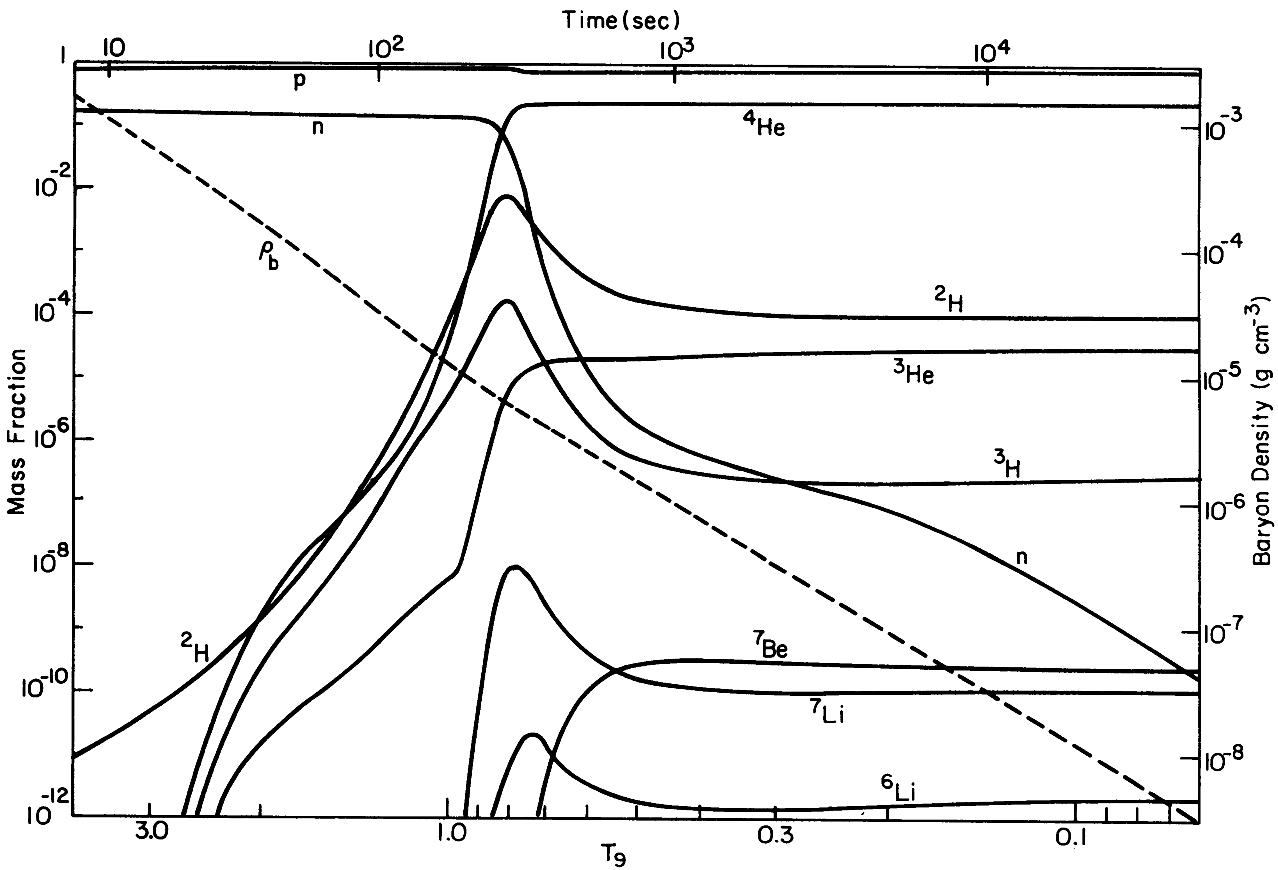

Fig. 2. Evolution of the nuclear abundances and baryon density (dotted line) during the expansion of a typical standard big-bang model (present baryon density $= 2.3 \times 10^{-31}$ g cm^{-3}).

$0.22 \leqslant X(^4\text{He}) \leqslant 0.34$ for young stars in our Galaxy and the interstellar medium in our Galaxy and in other nearby galaxies. Especially interesting are the dwarf blue galaxies investigated by Searle and Sargent (1972). They are bright but low mass objects in which the abundance of ^{4}He is normal, while those of ^{16}O and ^{20}Ne are only $\sim 10\%$ of their normal values. These properties at least suggest that these are young galaxies in which stars have produced fewer heavy elements, while the helium is of primordial origin. Of course, the helium could be due to a previous generation of massive stars, but one then wonders why its abundance is equal to the 'universal' value.

Low helium abundances have been indicated in some blue halo stars and H II regions in the center of our Galaxy, and in a few quasars. However, detailed analyses of the blue halo stars indicate that they can no longer be regarded as evidence for a low pregalactic helium abundance (Baschek *et al.*, 1972). In addition, the physical conditions in the H II regions in the galactic center and in quasars are not yet understood well enough to draw firm conclusions regarding their helium abundance.

Most models of our Galaxy indicate that the production of ^{4}He by stars only contributed a mass fraction of 0.01–0.04. Thus, if the ^{4}He is universal, we would expect its pregalactic abundance to lie in the range $0.22 \leqslant X(^4\text{He}) \leqslant 0.32$.

The situation regarding deuterium has changed greatly within the past two years. It now appears that the terrestrial and meteoritic value $X(^2\text{H}) = 2.3 \times 10^{-4}$ represents the effects of fractionation, since the Solar wind abundance of ^{3}He provides an upper limit of $X(^2\text{H}) \lesssim 1 \times 10^{-4}$ (Geiss and Reeves, 1972; Black, 1972) to the proto-solar value. The first direct observation of interstellar deuterium has been made by Jefferts *et al.* (1973) and Wilson *et al.* (1973) of DCN in a cloud within the Orion Nebula. The quoted abundance of $\text{DCN/HCN} = 6 \times 10^{-3}$, but fractionation processes occurring in the dense cloud (Solomon and Woolf, 1973; Watson, 1973) indicate that a total deuterium abundance $X(^2\text{H}) \sim 10^{-5}$–$10^{-4}$ is likely. The molecule HD has also been detected in front of several bright stars by the *Copernicus* satellite (Spitzer *et al.*, 1973). After correction for differential shielding in the interstellar clouds, abundance ratios (by number) of $2 \times 10^{-3} \leqslant \text{HD/H}_2 \leqslant 2 \times 10^{-2}$ were obtained. However, fractionation effects during molecular formation should again make a lower total deuterium abundance more likely.

These complicated corrections for interstellar chemistry are avoided in the case of interstellar atomic deuterium. Cesarsky *et al.* (1973) have possibly detected the 91.6 cm hyperfine line in the direction of the Galactic center. If the feature is real, the indicated abundance is $4 \times 10^{-5} \leqslant X(^2\text{H}) \leqslant 7 \times 10^{-4}$. Very recently, it has been reported that the *Copernicus* satellite has also detected absorption in the Lyman lines of deuterium, giving $X(^2\text{H}) = 2 \times 10^{-5}$, the best value to date.

In summary, then, all these observations may be at least consistent with a present interstellar abundance of $X(^2\text{H}) = 2 \times 10^{-5}$. If this deuterium is of cosmological origin, then its pregalactic abundance would have been higher due to subsequent stellar destruction, but the factor is difficult to estimate reliably.

As we shall see, although the abundances of ^{2}H and ^{4}He are potentially the most important carriers of information about the 'primeval fireball', many big-bang models synthesize interesting amounts of ^{3}He, ^{6}Li, ^{7}Li, and ^{11}B as well. Table I summarizes estimates of the present-day abundances of the light elements in the interstellar medium.

TABLE I

Observed abundances

Element	Mass fraction
^{2}H	2×10^{-5}
^{3}He	3×10^{-5}
^{4}He	0.22–0.34
^{6}Li	4×10^{-10} (^{7}Li/^{6}Li = 14.6)
^{7}Li	6×10^{-9}
^{9}Be	1×10^{-10}
^{10}B	5×10^{-10}–5×10^{-9} (^{11}B/^{10}B = 4)
^{11}B	2×10^{-9} –2×10^{-8}

5. Calculated Abundances

We shall first consider the final abundances produced in standard models of the Universe, indicated in Figure 3 and Table II. The results are only a function of the present average baryon mass density in the Universe. The observed amount of matter in galaxies (Shapiro, 1971) constrains this parameter to the range $\varrho_b(T=2.7\text{ K})\geqslant5\times\times10^{-32}(H_0/50)^2$. The present 'favored' value of the Hubble constant is $H_0=55\pm7$ km s^{-1} Mpc^{-1} (Tammann, 1974).

We first note that the ^{4}He abundance is relatively insensitive to this parameter (as the discussion in Section 3 indicated), and compares well with the lower values of the observed abundance for $5\times10^{-32}\lesssim\varrho_b\lesssim10^{-28}$ g cm^{-3}. Secondly, a universe with $\varrho_b\leqslant6\times10^{-31}$ g cm^{-3} can also produce the required pregalactic deuterium abundance $X(^2\text{H})\geqslant2\times10^{-5}$. In addition, such models ($5\times10^{-32}\leqslant\varrho_b\leqslant5\times10^{-31}$) appear capable of producing the required amount of ^{3}He (which also depends upon estimates of galactic production and destruction), but fall short for the other elements (with the possible exception of ^{7}Li).

Let us now consider element production in other big-bang models, in which we relax the assumptions listed in Section 2. One class of models will have an expansion rate differing from that given by Equation (1), due to a different theory of gravity, the presence of other particles or a strong magnetic field, etc. Since nucleosynthesis occurs over a fairly narrow range of temperature, we can simply generalize Equation (1) to

$$V^{-1}\,\mathrm{d}V/\mathrm{d}t=\xi\sqrt{24\pi G\varrho}, \tag{4}$$

and vary the parameter ξ. This we have done, and the results are shown in Figures 4 and 5. Although the effects of varying ξ can be great for the other elements, as seen in Figure 4, the effect on ^{4}He provides the most information. Note that a change in

TABLE II

Element production in 'standard' Big Bang

$\log h_0$	$\varrho_b(T=2.7\text{K})$ (g cm^{-3})	$X(^2\text{H})$	$X(^3\text{He})$	$X(^4\text{He})$	$X(^6\text{Li})$	$X(^7\text{Li})$	$X(^{11}\text{B})$	$X(A\geq 12)$
−6.00	7.15×10^{-33}	8.5×10^{-3}	3.6×10^{-4}	0.089	2.6×10^{-11}	2.0×10^{-9}		
−5.75	1.27×10^{-32}	5.5×10^{-3}	2.8×10^{-4}	0.131	3.7×10^{-11}	3.0×10^{-9}		
−5.50	2.26×10^{-32}	3.1×10^{-3}	1.9×10^{-4}	0.171	3.6×10^{-11}	2.8×10^{-9}		
−5.25	4.02×10^{-32}	1.4×10^{-3}	1.1×10^{-4}	0.200	2.3×10^{-11}	1.5×10^{-9}		
−5.00	7.15×10^{-32}	5.8×10^{-4}	6.7×10^{-5}	0.217	1.1×10^{-11}	5.0×10^{-10}		
−4.75	1.27×10^{-31}	2.2×10^{-4}	4.3×10^{-5}	0.227	4.5×10^{-12}	2.2×10^{-10}		
−4.50	2.26×10^{-31}	8.9×10^{-5}	2.8×10^{-5}	0.234	2.0×10^{-12}	3.4×10^{-10}		
−4.25	4.02×10^{-31}	3.6×10^{-5}	1.8×10^{-5}	0.240		1.2×10^{-9}		
−4.00	7.15×10^{-31}	1.3×10^{-5}	1.2×10^{-5}	0.246		3.5×10^{-9}		
−3.75	1.27×10^{-30}	3.3×10^{-6}	8.5×10^{-6}	0.251		7.2×10^{-9}		
−3.50	2.26×10^{-30}	3.9×10^{-7}	5.8×10^{-6}	0.255		1.2×10^{-8}		
−3.25	4.02×10^{-30}	9.8×10^{-9}	4.1×10^{-6}	0.260		1.7×10^{-8}		
−3.00	7.15×10^{-30}	1.2×10^{-11}	3.3×10^{-6}	0.265		2.5×10^{-8}		
−2.75	1.27×10^{-29}		2.7×10^{-6}	0.270		3.8×10^{-8}	1.0×10^{-12}	2.4×10^{-12}
−2.50	2.26×10^{-29}		2.4×10^{-6}	0.275		6.0×10^{-8}	1.7×10^{-12}	1.0×10^{-11}
−2.25	4.02×10^{-29}		2.1×10^{-6}	0.280		9.4×10^{-8}	2.7×10^{-12}	5.0×10^{-11}
−2.00	7.15×10^{-29}		1.8×10^{-6}	0.284		1.5×10^{-7}	4.0×10^{-12}	2.5×10^{-10}
−1.75	1.27×10^{-28}		1.5×10^{-6}	0.289		2.2×10^{-7}	5.4×10^{-12}	1.2×10^{-9}
−1.50	2.26×10^{-28}		1.1×10^{-6}	0.294		3.0×10^{-7}	6.4×10^{-12}	5.4×10^{-9}
−1.25	4.02×10^{-28}		7.8×10^{-7}	0.299		3.7×10^{-7}	6.2×10^{-12}	2.1×10^{-8}
−1.00	7.15×10^{-28}		4.3×10^{-7}	0.304		3.7×10^{-7}	4.6×10^{-12}	6.5×10^{-8}

Note: No entry indicates $X<10^{-12}$.

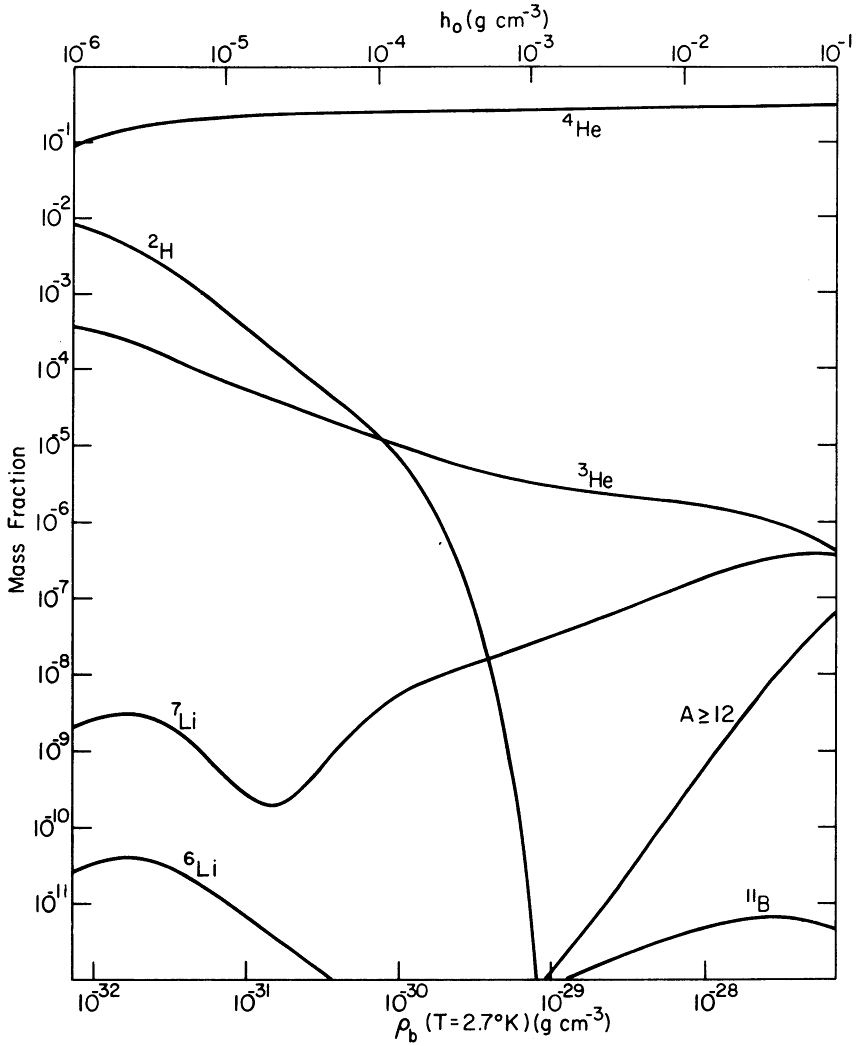

Fig. 3. Final abundances produced by standard big-bang models.

the expansion rate by only a factor of two moves the helium abundance outside the observed range. This is solely due to the effect of the expansion rate on the freeze-out temperature of the neutron-proton weak reactions.

As was shown by Wagoner *et al.* (1967), neutrino degeneracy also effects element production strongly, due to the shift in the neutron-proton equilibrium ratio as well as the increased expansion rate due to the higher total density $\varrho \cong \varrho_\gamma + \varrho_\nu + \varrho_e$. For the present purposes, it will be sufficient to point out that as the ratio of electron-lepton number to baryon number is increased above $L_e/B \sim 10^4 h_0^{-1}$, less and less ^{4}He (as well as the other elements) is produced. On the other hand, for $L_e/B \lesssim -10^4 h_0^{-1}$, too much ^{4}He or ^{2}H is produced.

https://doi.org/10.1017/S0074180900235535 Published online by Cambridge University Press

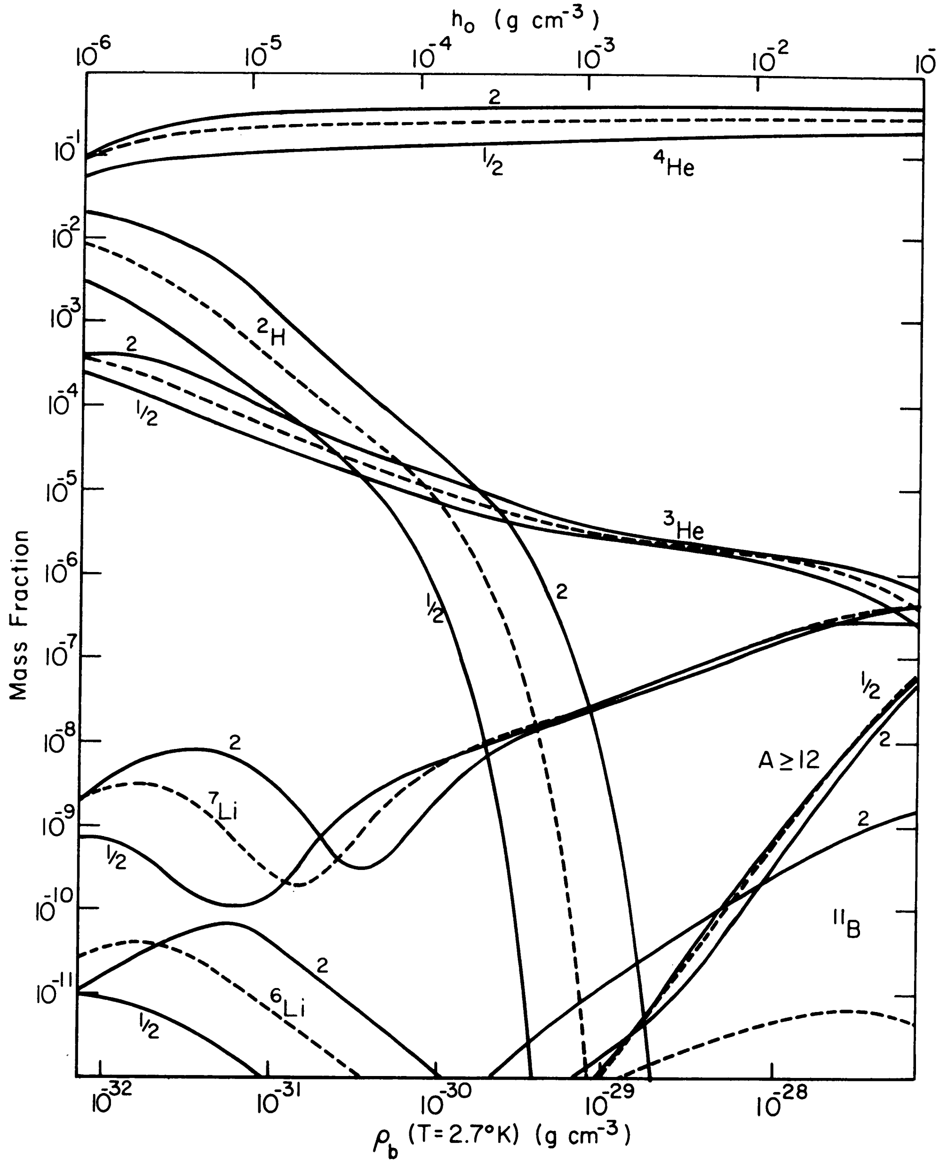

Fig. 4. Final abundances produced by models with $\xi = \frac{1}{2}$, 2, compared with abundances produced by the standard model (dashed curves).

Models in which $B = 0$ (e.g., Omnes, 1971) are not yet sufficiently well-developed to be able to predict element production accurately. In addition, they may be in conflict with observation (Steigman, 1974). Nevertheless, it appears that virtually no ^{4}He will be produced in such universes.

Turning to the remaining standard-model assumption, if the temperature never

https://doi.org/10.1017/S0074180900235535 Published online by Cambridge University Press

exceeded $T \sim 10^{11}$ K, no neutrons would become available through the weak reactions, and so no helium could have been synthesized.

The effects of inhomogeneity or anisotropy will not be considered explicitly here, but some aspects are discussed by Wagoner (1967, 1973). We adopt the general point of view that a generic universe tends to be unstable against the growth of irregularities, so that the approximate uniformity of the Universe in the recent past (as inferred from the isotropy of the background radiation) implies that the Universe must have

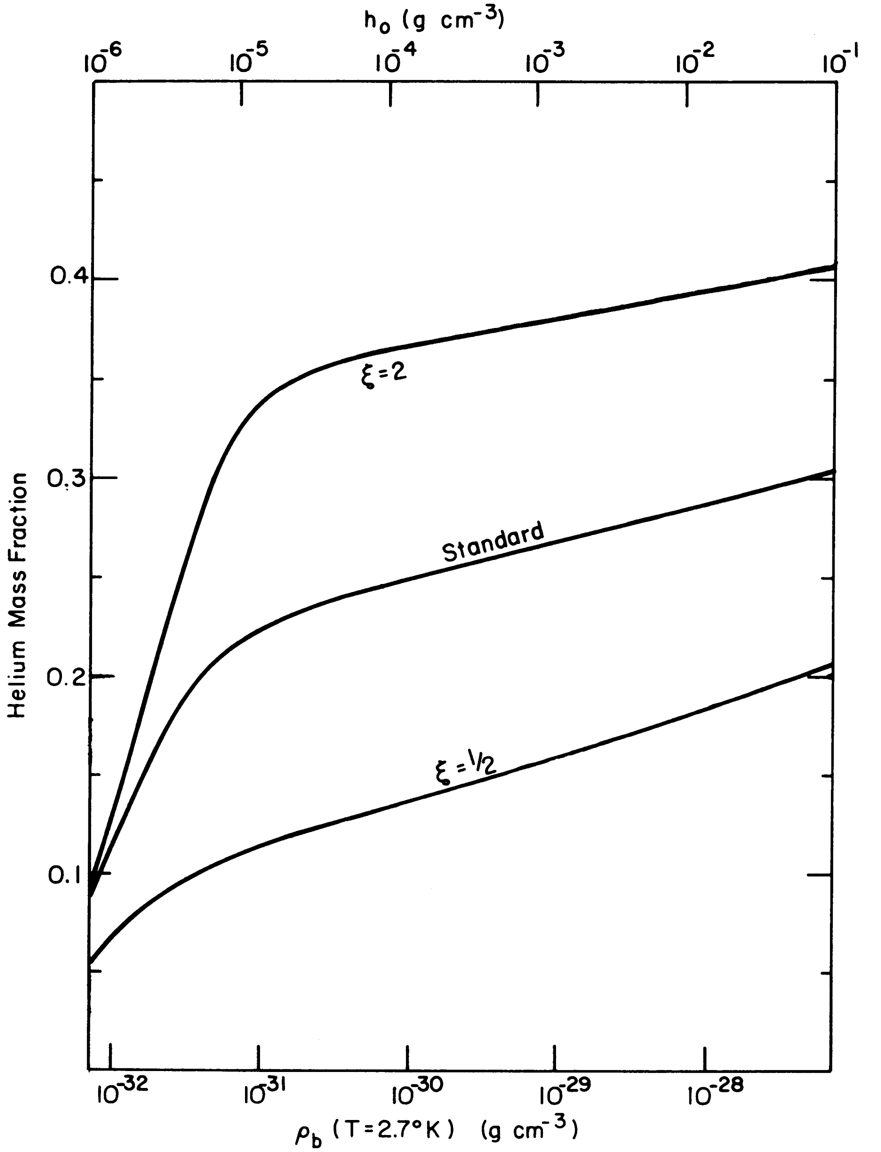

Fig. 5. Comparison of ^{4}He production by the models referred to in Figure 4.

been even more uniform in the distant past (Peebles, 1972). However, the presence of small-scale inhomogeneities is harder to argue against.

Finally, we consider a particular model involving unobserved particles, the statistical bootstrap model of Carlitz *et al.* (1972). In this theory of hadrons (Frautschi, 1971), the mass of the Universe condenses into single particles of mass $m_H \sim \varrho(ct)^3 \sim$ $\sim (c^3/6\pi G)\, t \sim 10^{38} m_\pi$ when the horizon size reaches $ct \sim \lambda_\pi$. Baryon conservation requires that these 'particles' have $B \gg 1$ if $B \neq 0$, although their radius remains $\sim \lambda_\pi$. A criticism of this model is that no such 'superbaryons' have been observed in accelerator experiments.

These superbaryons decay slowly through the emission of particles of average mass

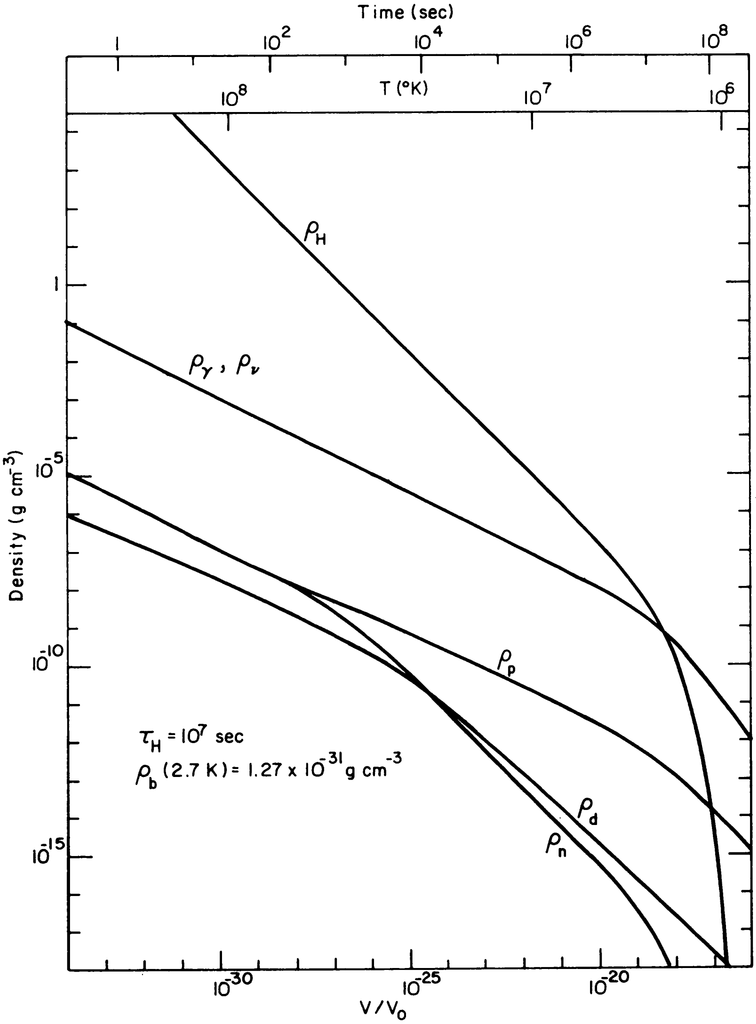

Fig. 6. Evolution of the various components of the density in a typical statistical bootstrap cosmological model. H, γ, ν, p, n, d refer to superbaryon, photon, neutrino, proton, neutron, and deuteron, respectively.

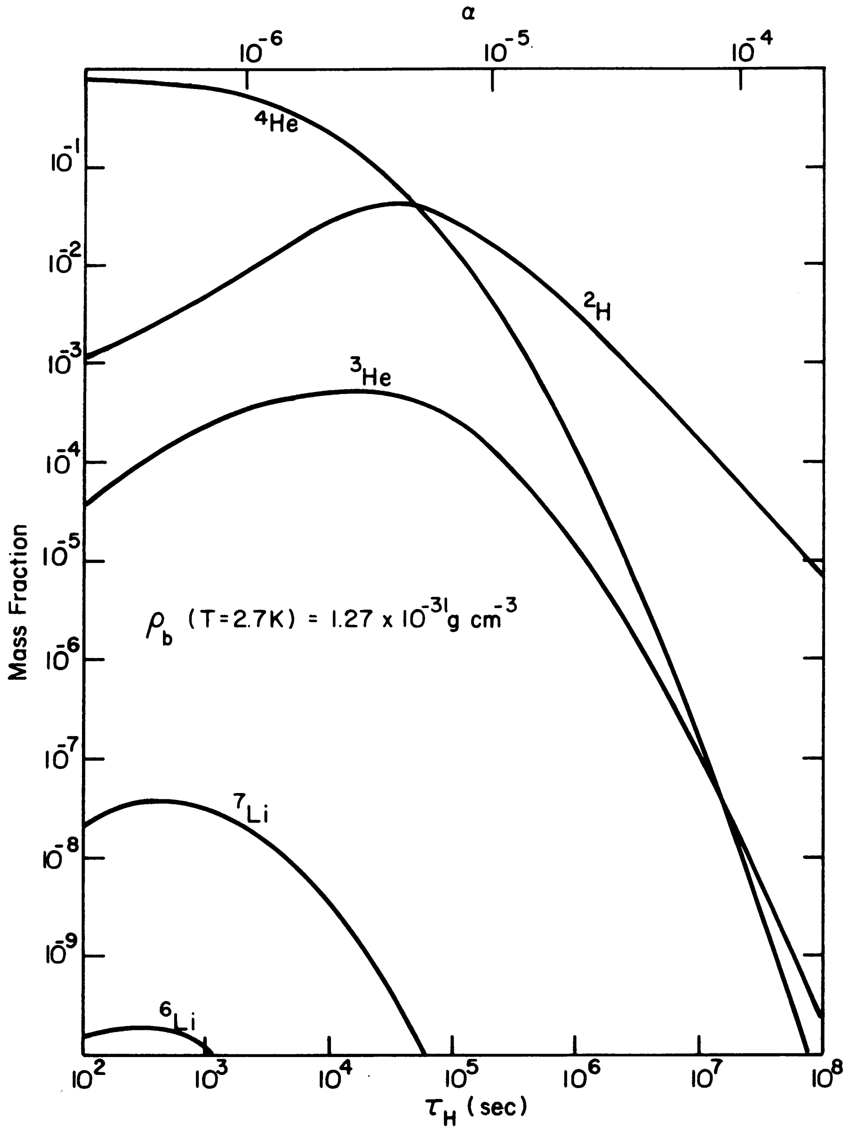

Fig. 7. Final abundances produced in various statistical bootstrap cosmological models having a present baryon density of 1.27×10^{-31} g cm^{-3}.

$(m_H m_\pi)^{1/2}$, with the overall lifetime in the range

$$10^{-4}\,\mathrm{s} \lesssim \tau_H \lesssim 10^{15}\,\mathrm{s}. \tag{5}$$

The second unknown parameter in this theory is the fraction (denoted by α) of the energy of the decays which finally appears in the form of nucleons. The bulk of the energy emerges roughly equally in the form of neutrinos and photons with typical energy $m_\pi c^2$. The superbaryons and nucleons are non-relativistic. Thus, the decays generate the observed entropy of the Universe, and so the branching ratio can be related to the present baryon density by

$$\alpha = 1.40 \times 10^{-8} \, \tau_H^{1/2} [\varrho_b (T = 2.7 \,\mathrm{K})/10^{-31}]. \tag{6}$$

The evolution of a typical model is shown in Figure 6. The Universe remains matter dominated until $t \sim \tau_H$, with $\varrho_p \cong \alpha\varrho_\gamma \cong \alpha\varrho_\nu$. For $\tau_n(926\,\mathrm{s}) \lesssim t \lesssim \tau_H$, the neutron abundance is no longer equal to the proton abundance due to their equal branching ratios, but is determined by the equilibrium which is reached between neutron pro-

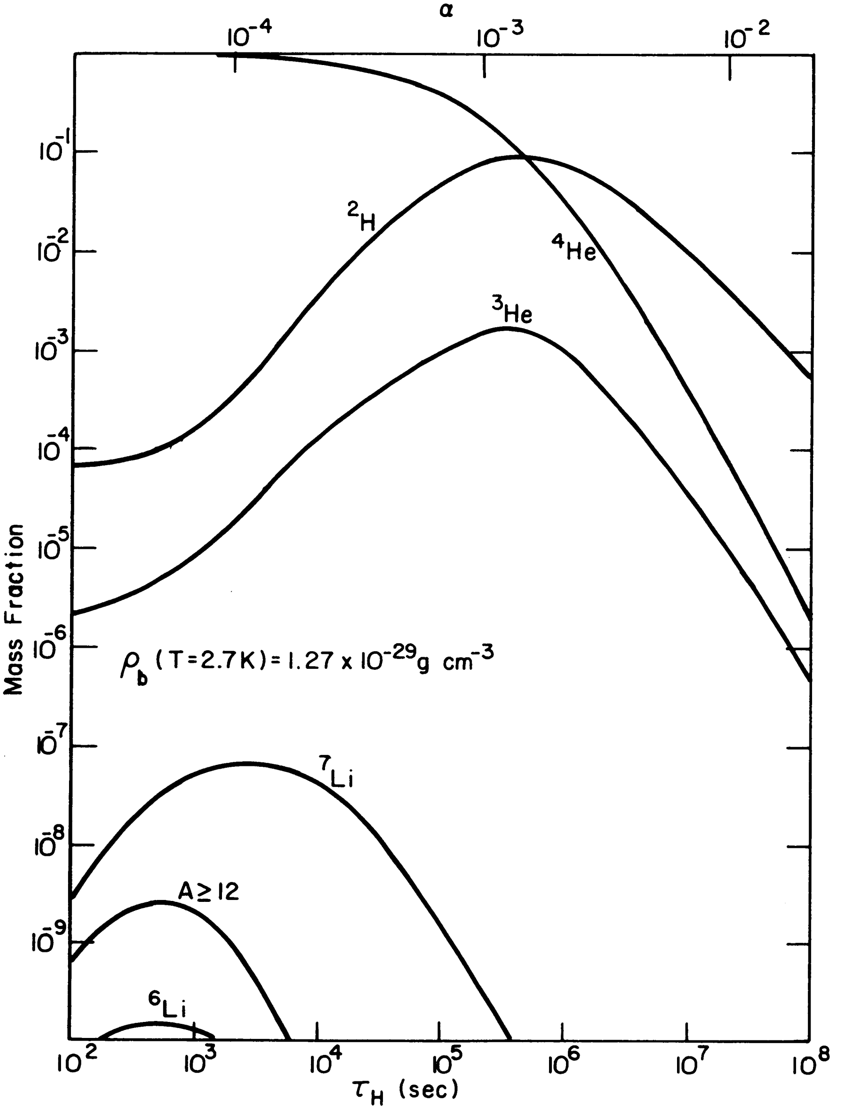

Fig. 8. Same as Figure 7 for a present baryon density of 1.27×10^{-29} g cm^{-3}.

https://doi.org/10.1017/S0074180900235535 Published online by Cambridge University Press

duction from the superbaryon decay and neutron decay. Deuterium production proceeds relatively easily even at these low temperatures, but heavier elements are suppressed by the Coulomb barriers.

The results of nucleosynthesis in these models are presented in Figures 7, 8, and 9. It should be noted that if $\tau_H \lesssim 10^{-2}$ s, the results will be identical to those in the standard model, while if $\tau_H \gtrsim 10^8$ s, the photons produced are not able to thermalize. We see that the lower density models with $\tau_H \sim 10^7$ s can produce the observed deuterium, but not enough helium. It is interesting that this value of the lifetime also optimizes the possibility of galaxy formation, according to the calculations of Carlitz *et al.* (1972). Smaller values of τ_H would in general result in too much helium or deuterium (unless, of course, deuterium destruction by stars in our Galaxy was exceedingly efficient).

6. Conclusions

We summarize our conclusions in Table III, which lists various statements one can make about the Universe, depending upon whether the observed deuterium and

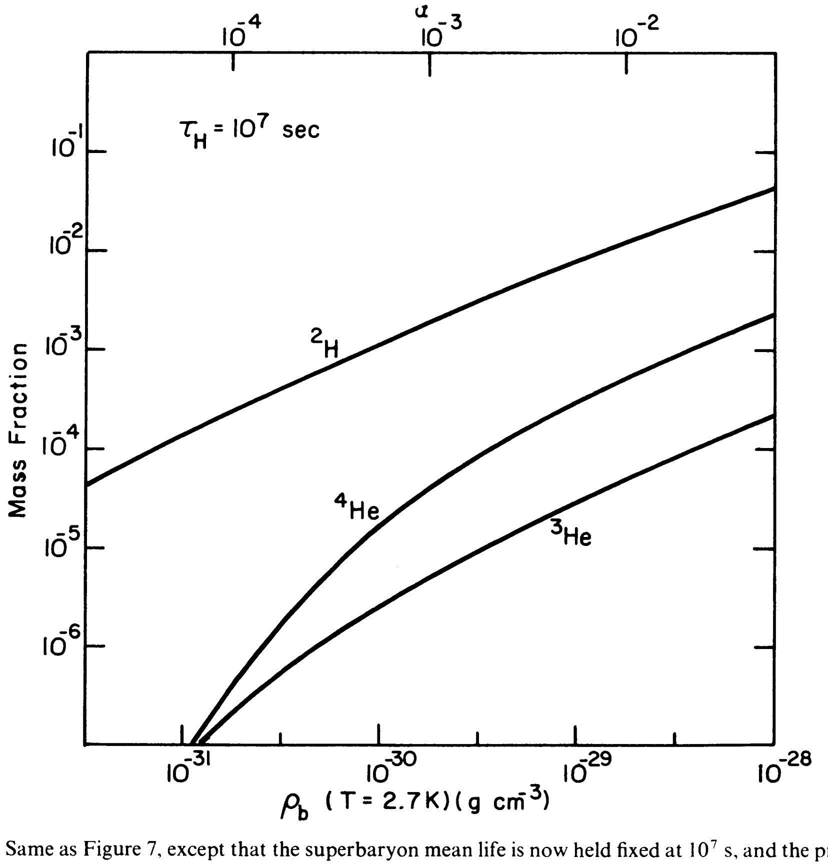

Fig. 9. Same as Figure 7, except that the superbaryon mean life is now held fixed at 10^7 s, and the present baryon density is varied.

https://doi.org/10.1017/S0074180900235535 Published online by Cambridge University Press

TABLE III

Conclusions

		^{2}H Galactic	^{2}H Cosmological
^{4}He	Galactic	$T \lesssim 10^{11}$ K, or $B = 0$ (?), or Degenerate neutrinos, or General relativity invalid, or ?	Statistical bootstrap model valid, or Slight neutrino degeneracy, or General relativity invalid, or ?
^{4}He	Cosmological	Standard model valid $\varrho_b(T = 2.7\,\mathrm{K}) \gtrsim 10^{-31}$ g cm^{-3}	Standard model valid $\varrho_b(T = 2.7\,\mathrm{K}) \leqslant 6 \times 10^{-31}$ g cm^{-3} Friedmann models open

helium were produced mainly during galactic evolution or in the primeval fireball. Recently, it has been claimed that significant deuterium production is possible in shock waves resulting from supernovae (Colgate, 1973) or explosions of more massive objects (Hoyle and Fowler, 1973). At present, the best way to investigate whether a given element is of galactic or cosmological origin is to search for inhomogeneities in its relative abundance.

If the observed ^{4}He is of cosmological origin, then it provides exceedingly powerful evidence for the validity of the standard model, since we have shown the sensitivity of the helium abundance to violations of its defining assumptions. If the observed ^{2}H is also of cosmological origin, then the present *baryon* density must be less than 6×10^{-31} g cm^{-3}. Friedmann models of the Universe ($\Lambda = p = 0$) with such *total* densities are open, since the density required for closure is $\varrho_c = 5 \times 10^{-30} (H_0/50)^2$.

On the other hand, if the observed helium and deuterium are of galactic origin, we must be prepared to accept at least one of the consequences also indicated in Table III. If only the deuterium is cosmological, we have seen how it can be produced without significant helium by the statistical bootstrap model. As alternatives, partially degenerate neutrinos or other theories of gravity (such as scalar-tensor theories, some of which involve extremely rapid expansion rates) can produce similar consequences.

Acknowledgement

The author would like to thank the Polish Academy of Sciences for support during the Extraordinary General Assembly of the International Astronomical Union.

References

Baschek, B., Sargent, W. L. W., and Searle, L.: 1972, *Astrophys. J.* **173**, 611.
Black, D. C.: 1972, *Geochim. Cosmochim. Acta* **36**, 347.
Carlitz, R., Frautschi, S., and Nahm, W.: 1972, Enrico Fermi Institute, preprint 72-56.

Cesarsky, D. A., Moffet, A. T., and Pasachoff, J. M.: 1973, *Astrophys. J. Letters* **180**, L1.
Colgate, S. A.: 1973, *Astrophys. J. Letters* **181**, L53.
Frautschi, S.: 1971, *Phys. Rev.* **D3**, 2821.
Geiss, J. and Reeves, H.: 1972, *Astron. Astrophys.* **18**, 126.
Hoyle, F. and Fowler, W. A.: 1973, *Nature* **241**, 384.
Jefferts, K. B., Penzias, A. A., and Wilson, R. W.: 1973, *Astrophys. J. Letters* **179**, L57.
Omnes, R.: 1971, *Astron. Astrophys.* **10**, 228.
Peebles, P. J. E.: 1966a, *Phys. Rev. Letters* **16**, 410.
Peebles, P. J. E.: 1966b, *Astrophys. J.* **146**, 542.
Peebles, P. J. E.: 1971, *Physical Cosmology*, Princeton University Press.
Peebles, P. J. E.: 1972, *Comments Astrophys. Space Phys.* **4**, 53.
Reeves, H., Audouze, J., Fowler, W. A., and Schramm, D. N.: 1973, *Astrophys. J.* **179**, 909.
Searle, L. and Sargent, W. L. W.: 1972, *Astrophys. J.* **173**, 25.
Shapiro, S.: 1971, *Astron. J.* **76**, 291.
Solomon, P. M. and Woolf, N. J.: 1973, *Astrophys. J. Letters* **180**, L89.
Spitzer, L., Drake, J. F., Jenkins, E. B., Morton, D. C., Rogerson, J. B., and York, D. G.: 1973, *Astrophys. J. Letters* **181**, L116.
Steigman, G.: 1974, this volume, p. 347.
Tammann, G. A.: 1974, this volume, p. 47.
Wagoner, R. V.: 1967, *Science* **155**, 1369.
Wagoner, R. V., Fowler, W. A., and Hoyle, F.: 1967, *Astrophys. J.* **148**, 3.
Wagoner, R. V.: 1973, *Astrophys. J.* **179**, 343.
Watson, W. D.: 1973, *Astrophys. J. Letters* **181**, L129.
Will, C. M.: 1973, *Proceedings of Course* **56**, International School of Physics 'Enrico Fermi', Varenna, Italy.
Wilson, R. W., Penzias, A. A., Jefferts, K. B., and Solomon, P. M.: 1973, *Astrophys. J. Letters* **179**, L107.