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CHARACTERIZATION OF A FAMILY OF SIMPLE GROUPS BY THEIR CHARACTER TABLE

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Abstract

The paper establishes a method for bounding the 2-rank of a simple group with one conjugacy class of involutions, by means of its character table. For many groups of 2-rank $\leqq 4$, this bound is shown to be exact. The main result is that the simple groups $G_2(q)$, $(q,6)=1$, are characterized by their character table.

1. Introduction

The main purpose of this paper is to characterize the simple groups $G_2(q)$, $q=p^n$, $p\neq 2,3$ by their character tables. The first important step is to establish a method for bounding the 2-rank of a simple group with one conjugacy class of involution by means of its character table. Next it is shown that a group $^*G$ having the same character table as $G_2(q)$, $(q,6)=1$, has precisely one conjugacy class of involutions. The method referred to above yields 2-rank $^*G\leqq 3$. Such groups have recently been classified by Stroth (1976). Using this classification it is shown that $G_2(q)$ is the only choice for $^*G$.

Section 2 deals with the determination of upper bounds for the 2-rank of a simple group from a limited knowledge of the character table, see Lemmas 2.1 and 2.7. The upper bounds give the correct 2-rank for all simple groups of 2-rank 2, many simple groups of 2-rank 3 or 4 and for some families of unbounded rank, see Corollaries 2.3, 2.4, 2.8 and Remarks 2.6, 2.9.

Section 3 is devoted to the characterization of $G_2(q)$, $(q,6)=1$, by its character table.

For a summary of the known characterizations of simple groups by their character table, see Lambert (1972) and Pahlings (1974).

The source of the character tables used in this paper are: $PSL\,(2,q)$, Jordan (1907, pp. 402–403); $PSL\,(3,q)$, $PSU\,(3,q)$, Simpson and Frame (1973,

p. 492); A_7, A_8, A_9, A_{10}, Littlewood (1935, pp. 178–184); A_{11}, Zia-ud-Din (1935); M_{11}, M_{12}, M_{22}, M_{23}, M_{24}, James (1973, pp. 100–105); $G_2(q)$, $(q,6)=1$, Chang and Ree (1974); Ree type, $Re\,(q)$, Ward (1966, pp. 87–88); Suzuki groups, $Sz\,(2^{2m+1})$, Suzuki (1962, p. 143); Janko's group, J_1, Janko (1966, pp. 148–149); Hall–Janko group, *HaJ*, Hall and Wales (1968, p. 435); Higman–Janko–McKay group, *HJM*, Janko (1969, p. 61); Higman–Sims group, *HiS*, Frame (1972, p. 346); Lyons–Sims group, *LyS*, Lyons (1972, pp. 557–559); Conway's Group .3, Fendel (1973, pp. 188–191); O'Nan's sporadic simple group, O'Nan (1976, pp. 461–463).

The notation is standard. All groups are finite and all characters are over the complex field.

2. Preliminary lemmas and some applications

In this section we prove some elementary results concerning groups with one or two conjugacy classes of elements of order p. Some easy applications are also derived. The main application is Theorem 3.2 which is proved in Section 3.

LEMMA 2.1. *Let G be a group of p-rank n and let H be an elementary abelian subgroup of G of order p^n. Let X be a character of G of degree x, taking value x_1 on each element of G of order p. Then*

(a) *x_1 is an integer,*
(b) $p^n \mid (x - x_1)$,
(c) *if* $\ker X \cap H = \{1\}$ *then* $x \geqq p^n - 1$, *and*
(d) *if* $x_1 < 0$ *then* $x/|x_1| \geqq p^n - 1$.

PROOF. Let $q = (X_{|H}, 1_H)_H$. Then

$$x + (p^n - 1)x_1 = (x - x_1) + p^n x_1 = qp^n$$

and (a), (b), (d) follow from the fact that x_1 is an algebraic integer and q is a non-negative integer. Now (c) follows from

$$p^n x - (p^n - 1)(x - x_1) = qp^n$$

as $x - x_1$ is then a positive multiple of p^n.

NOTE. If G has one conjugacy class of elements of order p, then any X clearly satisfies the condition of Lemma 2.1.

As an application we mention:

COROLLARY 2.2. *Let G be a simple group with one conjugacy class of involutions. Suppose that G has a non-principal irreducible character of degree $x \leqq 14$. Then 2-rank $G \leqq 3$ and G is known.*

PROOF. By Lemma 2.1(c) 2-rank $G \leqq 3$. Hence G is one of the groups classified by Stroth (1976).

NOTE. *PSL* (2, 16) is of 2-rank 4 and has an irreducible character of degree 15.

All simple groups of 2-rank 2 have one conjugacy class of involutions [see classification in Alperin, Brauer and Gorenstein (1973)]. The next two corollaries show that Lemma 2.1(b) can be used in order to establish the 2-rank of a simple group of 2-rank 2, with the exception of *PSU* (3, 4), for which part (d) of Lemma 2.1 has been used (see Remark 2.6).

COROLLARY 2.3. *Let G be a simple group with one class of involutions. Suppose that G has an irreducible character of degree x taking value x_1 on an involution. Then the following statements hold*:

(i) *if* $x_1 \leqq -5$, $x_1 \equiv 3 \pmod 4$ *and* $x = x_1^2 - x_1 + 1$ *then* $x_1 = -q$, $q \equiv 1 \pmod 4$, $q \geqq 5$ *and* $G \simeq PSL(3, q)$.

(ii) *if* $x_1 \leqq -6$, $x_1 \equiv 2 \pmod 4$ *and* $x = (1 - x_1)^3 - 1$ *then* $x_1 = -(q-1)$, $q \equiv 3 \pmod 4$, $q \geqq 7$ *and* $G \simeq PSL(3, q)$,

(iii) *if* $x_1 \leqq -2$, $x_1 \equiv 2 \pmod 4$ *and* $x = -(1 + x_1)^3 + 1$ *then* $x_1 = -(q+1)$, $q \equiv 1 \pmod 4$, $q \geqq 5$ *and* $G \simeq PSU(3, q)$,

(iv) *if* $x_1 \geqq 3$, $x_1 \equiv 3 \pmod 4$ *and* $x = x_1^2 - x_1 + 1$ *then* $x_1 = q$, $q \equiv 3 \pmod 4$, $q \geqq 3$ *and* $G \simeq PSU(3, q)$,

(v) *if* $x_1 = 0$, $x = 4$ *then* $G \simeq PSL(2, 5)$,

where q is a power of an odd prime.

PROOF. In each of the five cases it is easy to verify that $x - x_1 \equiv 4 \pmod 8$. It follows from Lemma 2.1(b) that G has 2-rank 2. By Alperin, Brauer and Gorenstein (1973) the only simple groups of 2-rank 2 are the groups *PSL* $(2, q)$, *PSL* $(3, q)$, *PSU* $(3, q)$, q odd, *PSU* (3, 4), A_7 and M_{11}. Note that in the tables of Simpson and Frame (1973) the class of involutions in *PSL* $(3, q)$ and *PSU* $(3, q)$ is $C_4^{(k)}$, $k = \frac{1}{2}r'$, when q is odd and is C_2 when $q = 4$. Also, $x - x_1 \equiv 0 \pmod{16}$ for each character of *PSU* (3, 4), hence $G \not\simeq PSU(3, 4)$. Only part (iii) will be proved in detail, the remaining parts being similar. The method is as follows. A search is conducted through the character tables of *PSL* $(2, q)$, *PSL* $(3, q)$, *PSU* $(3, q)$, q odd, A_7 and M_{11} to discover a character X whose value x_1 on an involution is even and less than zero. Next check the condition $X(1) - x_1 \equiv 4 \pmod 8$. For those characters that have not already been eliminated, the equation $X(1) = x = -(1 + x_1)^3 + 1$ is shown to only have the solutions, $x_1 = -(q+1)$, $X = \chi_{rt}^{(u)}$, u odd, $G \simeq PSU(3, q)$, q a power of an odd prime. Now for the details of part (iii). If $x_1 = -2$ then $x = 2$ which is impossible since G is simple. Hence $x_1 \leqq -6$. Case (a) $G \simeq$

$PSL\,(2,q)$, A_7 or M_{11}. No choice of x_1 exists within this range of values. Case (b) $G \simeq PSL\,(3,q)$. There are two non-trivial cases. Firstly, $x_1 = -(q+1)$, $X = \chi_{st}^{(u,v,w)}$, u, v, w not all even. Then $q \equiv 1 \pmod 4$ and $\chi_{st}^{(u,v,w)}(1) - x_1 = (q+1)(q^2+q+2) \equiv 0 \pmod 8$. Secondly, $x_1 = -q+1$, $X = \chi_{rt}^{(u)}$, u odd. The equation $\chi_{rt}^{(u)}(1) = x$ gives $q^3 - 1 = -(2-q)^3 + 1$ i.e. $6(q-1)^2 = 0$ so that $q = 1$. Case (c) $G \simeq PSU\,(3,q)$. There are three possibilities that need to be dealt with, namely, $x_1 = -q+1$, $X = \chi_{qs}$; $x_1 = -q+1$, $X = \chi_{st}^{(u,v,w)}$, u, v, w not all even; and $x_1 = -q-1$, $X = \chi_{rt}^{(u)}$, u odd. Now, $\chi_{qs}(1) - (-q+1) = q^2 - 1 \equiv 0 \pmod 8$ and $\chi_{st}^{(u,v,w)}(1) - (-q+1) = (q-1)(q^2-q+2) \equiv 0 \pmod 8$, as $q \equiv 3 \pmod 4$. The last case shows that $G \simeq PSU\,(3,q)$, $x_1 = -q-1$ does occur. This completes the proof.

Corollary 2.4. *Let G be a simple group with one class of involutions. Suppose that G possesses two characters of degree x, y respectively, taking values x_1, y_1, respectively on an involution. Then the following statements hold*:

(i) *if* $x - y = 2$, $x_1 = -2$, $y_1 = 0$ *then* $x = q+1$, $y = q-1$, $q \equiv 1 \pmod 4$, $q \geqq 9$ *and* $G \simeq PSL\,(2,q)$,

(ii) *if* $x - y = -2$, $x_1 = 2$, $y_1 = 0$ *then* $x = q-1$, $y = q+1$, $q \equiv 3 \pmod 4$, $q \geqq 7$ *and* $G \simeq PSL\,(2,q)$,

(iii) *if* $x - y = 20$, $x_1 = y_1 = -1$ *then* $x = 35$, $y = 15$ *and* $G \simeq A_7$,

(iv) *if* $x - y = 10$, $x_1 = -2$, $y_1 = 0$ *then* $x = 26$, $y = 16$ *and* $G \simeq PSL\,(3,3)$,

(v) *if* $x - y = -6$, $x_1 = -2$, $y_1 = 0$ *then* $x = 10$, $y = 16$ *and* $G \simeq M_{11}$,

where q is a power of an odd prime.

Proof. In each case $(x - x_1) - (y - y_1) \equiv 4 \pmod 8$. By Lemma 2.1(b) it follows that G has 2-rank 2. The rest of the argument is similar to that of Corollary 2.3.

Corollaries 2.3 and 2.4 yield:

Corollary 2.5. *Let G be a simple group possessing one conjugacy class of involutions and $G \not\simeq PSU\,(3,4)$. Then 2-rank $G = 2$ if and only if there exists a character X satisfying $8 \nmid (X(1) - x)$, where x is the value of X on an involution.*

Note. For $G \simeq PSU\,(3,4)$, $16 \mid (X(1) - x)$ for every irreducible character of G, (in the notation of Corollary 2.5).

Remark 2.6. The upper bounds for the 2-rank established in Lemma 2.1 yield the correct 2-rank for the following groups, in addition to those mentioned in Corollaries 2.3 and 2.4:

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Group	2-rank	deg X	X(inv)	Part of Lemma 2.1 Used
M_{22}	4	21	5	(b)
M_{23}	4	22	6	(b)
HJM	4	85	5	(b)
HiS	4	154	10	(b)
LyS	4	45,694	110	(b)
$Re(3^{2k+1})$	3	$(3^{2k+1})^3$	3^{2k+1}	(b)
$Sz(2^{2m+1})$	$2m+1$	$2^m(2^{2m+1}-1)$	-2^m	(d)
$G_2(q)$, $q \equiv 1$ or 3 (mod 8)*	3	$q(q+1)(q^4+q^2+1)$	$-(q+1)$	(b)
$G_2(q)$, $q \equiv 5$ or 7 (mod 8)*	3	$q(q-1)(q^4+q^2+1)$	$q-1$	(b)
$PSL(2, 2^n)$	n	2^n	0	(b)
$PSL(3, 2^n)$	$2n$	$2^n(2^n+1)$	2^n	(b)
$PSU(3, 2^n)$	n	$2^n(2^n-1)$	-2^n	(d)

* $3 \nmid q$.

Next we obtain some results concerning the p-rank of a group with two conjugacy classes of elements of order p.

LEMMA 2.7. *Let G be a group of p-rank n and let H be an elementary abelian subgroup of G of order p^n. Let X, Y be characters of G of degrees x and y, respectively. Suppose that G has two conjugacy classes of elements of order p, C_1 and C_2, on which X takes the values x_1, x_2 and Y takes the values y_1, y_2, respectively. Then*

(a) *If $x_1 - x_2 = y_1 - y_2$ then $(x - x_2 - y + y_2)/p^n$ is an algebraic integer, and*

(b) *If $x_1 - x_2 = y_1 - y_2$ and $p = 2$, then $x - x_2 - y + y_2$ is an integer divisible by 2^n.*

PROOF. Denote $|H \cap C_i| = A_i$, $i = 1, 2$. Then $A_2 = p^n - 1 - A_1$ and denoting $(X_{|H}, 1_H)_H = q_x$ and $(Y_{|H}, 1_H)_H = q_y$ we get by subtraction

$$x - x_2 - y + y_2 = p^n(q_x - x_2 - q_y + y_2)$$

yielding (a) and (b).

As an application we prove

COROLLARY 2.8. *Let G be a simple group with two conjugacy classes of involutions, C_1 and C_2. Let X and Y be irreducible characters of G of degrees x*

and y, respectively, taking the values x_1, y_1 and x_2, y_2 on C_1 and C_2, respectively. If $x_1 - x_2 = y_1 - y_2$ and $16 \nmid (x - x_2 - y + y_2)$ then $G \simeq M_{12}$.

PROOF. By Lemma 2.7 2-rank $G \leqq 3$ and by Stroth (1976), M_{12} is the only such group with two classes of involutions. On the other hand, M_{12} does satisfy the assumptions of Corollary 2.7 with respect to the following characters:

	degree	C_1	C_2
X	11	3	-1
Y	176	0	-4

REMARK 2.9. The upper bound for the 2-rank established in Lemma 2.7 yields the correct 2-rank for the following groups in addition to M_{12} mentioned in Corollary 2.8. In this table X and Y denote two irreducible characters and v_1, v_2 are representatives of the two conjugacy classes of involutions.

Group	2-rank	Deg X	$X(v_1)$	$X(v_2)$	Deg Y	$Y(v_1)$	$Y(v_2)$
HaJ	4	36	4	0	90	10	6
.3	4	9625	105	-55	23,000	280	120
A_8	4	7	3	-1	21	1	-3
A_9	4	8	4	0	27	7	3
A_{10}	4	9	5	1	84	0	-4
A_{11}	4	45	13	-3	120	8	-8
M_{24}	6	45	5	-3	990	-10	-18

NOTE. O'Nan's sporadic simple group has 2-rank 3 and one conjugacy class of involutions. However, the methods of Lemma 2.1.b yield only 2-rank $\leqq 4$ ($x = 64790$, $x_1 = 70$).

3. A characterization of $G_2(q)$

The purpose of this section is to establish that $G_2(q)$ is characterised by its character table, when $q = p^m$, p prime, $p \neq 2, 3$. The notation for elements and characters of $G_2(q)$ is the same as in Chang and Ree (1974).

Suppose that $^*G$ is a group with the same character table as $G_2(q)$, $(q, 6) = 1$. Put an asterisk in front of each conjugacy class representative, character, etc., to distinguish it from the same in $G_2(q)$. Since a character table determines the order of the group and the lattice of normal subgroups, see Feit (1967), $^*G$ is simple with $|^*G| = q^6(q^2 - 1)(q^6 - 1)$. The first step is to establish that $^*G$ has a unique conjugacy class of involutions.

LEMMA 3.1. *The only conjugacy class of involutions in $^*G$ is that represented by $^*k_2$.*

PROOF. By Chang and Ree (1974, p. 409), $^*k_2$ is the only class representative with the full 2-power dividing the order of its centraliser. Hence, $^*k_2$ is a central involution. By Chang (1968, p. 199) u_1 and u_2 are conjugate to elements in the unipotent subgroup. Also, u_3, u_4, u_5 are given to be unipotent elements and u_6 is unipotent by Chang (1968, p. 195) Theorem 3.1. In $G_2(q)$ each u_i, $1 \leqq i \leqq 6$ is a p-element, so by Lambert (1972, Property 2.5) each $^*u_i$ is a p-element. Similarly, as k_3 has order 3, $^*k_3$ is a 3-element. Let $^*c$ be a conjugacy class representative in $^*G$, $^*c \neq 1$, $^*k_2$, $^*k_3$, $^*u_i$, $1 \leqq i \leqq 6$. Then $|C_{^*G}(^*c)| \leqq \max(q^3(q-\varepsilon),\ q(q-1)(q+1)^2) \leqq q^2(q+1)^2$, see Chang and Ree (1974, p. 409). Thus the minimum order of the conjugacy class containing $^*c$ is

$$\begin{aligned}&\geqq |G|/q^2(q+1)^2 = q^4(q-1)^2(q^4+q^2+1)\\ &= q^4(q^6-2q^5+2q^4-2q^3+2q^2-2q+1)\\ &> q^4(q^6-2q^5) = q^{10}-2q^9.\end{aligned}$$

Suppose that $^*c$ is an involution. If t is the number of involutions in $^*G$ then

$$t+1 \leqq \sum_i {}^*X_i(1) = \sum_i X_i(1)$$

see Feit (1967, p. 23), where $^*X_i$ (X_i) runs through a complete set of irreducible characters of $^*G$ $(G_2(q))$. Chang and Ree (1974, p. 412) list the irreducible characters of $G_2(q)$. To obtain an upper bound for $\Sigma_i X_i(1)$ the following method is employed: for each parenthesis in the degree column first forget the negative terms and then replace each remaining term by the highest power of q occurring in the parenthesis; the number of characters of each type can be approximated by the term involving the highest power of q with the one exception of characters of type χ_3 when $(q^2+q-1-\varepsilon)/6 \leqq q^2/3$. This yields

$$\sum_i X_i(1) < 2\tfrac{11}{12}q^8 + 9q^7 + 15q^6 + 20\tfrac{1}{2}q^5 + 3q^4 + 2q^3 + 1 < 6q^8 + 1,$$

as $2q^3 < q^4$, $4q^4 < q^5$, $21\frac{1}{2}q^5 < 5q^6$, $20q^6 \leqq 4q^7$, $13q^7 < 3q^8$. The conjugacy class $^*k_2$ has order $q^4(q^4+q^2+1) > q^8$. Substituting in the formula for the number of involutions gives

$$1 + q^8 + q^{10} - 2q^9 < 6q^8 + 1$$

i.e.

$$(q-1)^2 < 6,$$

a contradiction to $q \geqq 5$. Thus $^*c$ is not an involution, proving the lemma.

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THEOREM 3.2. *The character table of* $G_2(q)$, $q = p^m$, $p \neq 2, 3$ *determines* $G_2(q)$.

PROOF. With the above notation Lemma 3.1 allows us to use Lemma 2.1(b) to bound the 2-rank of $^*G$. Using the notation of Chang and Ree (1974, p. 399), let π be a linear character of the multiplicative group F_q^x of F_q such that the isotropic groups of π_1 in W_1 and π_a in W_a are both $\{1, w_a\}$. If x is a primitive $(q-1)$th root of unity then π may be defined by $\pi(x) = x^t$. Then

$$X_{1a}(k_2) = \begin{cases} (q+1)(2q+1), & t \text{ even}, \\ -(q+1), & t \text{ odd}, \end{cases}$$

where t lies in the range $1 \leqq t < q-1$, $t \neq \frac{1}{2}(q-1)$ and when $q \equiv 1 \pmod 3$, $t \neq \frac{1}{3}(q-1)$, $t \neq \frac{2}{3}(q-1)$. The choice $t = 1$ is always permissible. Thus $G_2(q)$ has a character in the family X_{1a} for which

$$X_{1a}(k_2) = -(q+1).$$

Now $\deg X_{1a} = q(q+1)(q^4+q^2+1)$. Thus, when $q \equiv 1, 3 \pmod 8$

$$X_{1a}(1) - X_{1a}(k_2) = (q+1)(q^5+q^3+q+1) \equiv 8 \pmod{16}.$$

By Lemma 2.1(b) 2-rank $^*G \leqq 3$, provided $q \equiv 1, 3 \pmod 8$. Now consider π' to be a linear character of the subgroup $\langle s^{q-1} \rangle$ of order $q+1$ of the multiplicative group $F_{q^2}^x = \langle s \rangle$ of F_{q^2} such that the isotropic groups of π_2 in W_2 and of π_a in W_a are both $\{1, w_a w_2\}$, see Chang and Ree (1974, p. 400). π' may be defined by $\pi'(s^{q-1}) = y^t$, y a primitive $(q+1)$th root of unity. Then

$$X_{2a}(k_2) = \begin{cases} (q-1)(2q-1), & t \text{ even}, \\ q-1, & t \text{ odd}, \end{cases}$$

where t lies in the range $1 \leqq t \leqq q$, $t \neq \frac{1}{2}(q+1)$. Also, $\deg X_{2a} = q(q-1)(q^4+q^2+1)$. The choice $t = 1$ leads to a character X_{2a} which for $q \equiv 5, 7 \pmod 8$ satisfies

$$X_{2a}(1) - X_{2a}(k_2) = (q-1)(q^5+q^3+q-1) \equiv 8 \pmod{16}.$$

By Lemma 2.1(b), and the above, it follows that 2-rank $^*G \leqq 3$ in all cases. Since $G_2(q)$ has 2-rank 3, by Lemma 2.1(b), $X(1) - X(k_2) \equiv 0 \pmod 8$ for all characters X of $G_2(q)$. By Corollary 2.5 and the Note immediately after it, 2-rank $^*G = 3$. Stroth (1976) tells us that $^*G$ is isomorphic to one of: $G_2(q_1)$, $D_4^2(q_1)$, $Re(q_1)$, (q_1 odd), $PSL(2,8)$, $Sz(8)$, $PSU(3,8)$, J_1, M_{12} or O'Nan's sporadic simple group. It is easy to check that $|G_2(q)| \neq |D_4^2(q_1)|$ for any choices of q, q_1. $Re(q_1)$ have a Sylow subgroup of order 8 which is too

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small. The order of O'Nan's sporadic simple group is not divisible by an odd prime to the sixth power. The remaining groups, except $G_2(q_1)$, have order $\leqq 2.10^9 < |G_2(5)|$. Hence $^*G \simeq G_2(q_1)$ and trivially $q = q_1$, proving the theorem.

References

J. Alperin, R. Brauer and D. Gorenstein (1973), 'Finite simple groups of 2-rank two', *Scripta Math.* **29**, 191–214.

Bomshik Chang (1968), 'The conjugate classes of Chevalley groups of type (G_2)' *J. Algebra* **9**, 190–211.

Bomshik Chang and Rimhak Ree (1974), 'The characters of $G_2(q)$', *Symposia Mathematicā*, Volume XIII, 395–413 (Academic Press, London and New York, 1974).

Walter Feit (1967), 'Characters of finite groups', *Mathematics Lecture Notes*, (W.A. Benjamin, Inc., New York, 1967).

Daniel Fendel (1973), 'A characterization of Conway's group .3', *J. Algebra* **24**, 159–196.

J.S. Frame (1972), 'Computation of characters of the Higman–Sims group and its automorphism group', *J. Algebra* **20**, 320–349.

Marshall Hall, Jr. and David Wales (1968), 'The simple group of order 604,800', *J. Algebra* **9**, 417–450.

G. D. James (1973), 'The modular characters of the Mathieu groups', *J. Algebra* **27**, 57–111.

Zvonimir Janko (1966), 'A new finite simple group with abelian Sylow 2-subgroups and its characterization', *J. Algebra* **3**, 147–186.

Zvonimir Janko (1969), 'Some new simple groups of finite order, I', *Symposia Mathematica*, Volume I, 25–64 (Academic Press, London and New York, 1969).

Herbert E. Jordan (1907), 'Group-characters of various types of linear groups', *Amer. J. Math.* **29**, 387–405.

P. J. Lambert (1972), 'Characterizing groups by their character tables, I', *Quart. J. Math. Oxford* (2), **23**, 427–433.

D. E. Littlewood (1935), 'Group characters and the structure of groups', *Proc. London Math. Soc.* (2), **39**, 150–199.

Richard Lyons (1972), 'Evidence for a new finite simple group', *J. Algebra* **20**, 540–569.

Michael E. O'Nan (1976), 'Some evidence for the existence of a new simple group' *Proc. London Math. Soc.* (3) **32**, 421–479.

H. Pahlings (1974), 'On the character tables of finite groups generated by 3-transpositions', *Comm. Algebra* **2** (2), 117–131.

William A. Simpson and J. Sutherland Frame (1973), 'The character tables for $SL(3,q)$, $SU(3,q^2)$, $PSL(3,q)$, $PSU(3,q^2)$', *Canad. J. Math.* **25**, 486–494.

G. Stroth (1976), 'Über Gruppen mit 2-Sylow-Durchschnitten von Rang $\leqq 3$', I and II, *J. Algebra* **43**, 398–456 and 457–505.

Michio Suzuki (1962), 'On a class of doubly transitive groups', *Ann. of Math.* **75**, 105–145.

Harold N. Ward (1966), 'On Ree's series of simple groups', *Trans. Amer. Math. Soc.* **121**, 62–89.

M. Zia-ud-Din (1935), 'The characters of the symmetric group of order 11 !', *Proc. London Math. Soc.* (2) **39**, 200–204.

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