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# Accounting for systematic risk in benefit-cost analysis: a practical approach

**Abstract:** Circular A-94 specifies how analysts should discount costs and benefits of government projects, and thus how to account for risk. In this paper, we argue that the methods mandated by A-94 properly account for non-systematic and term risk, but not for systematic risk. A numerical example illustrates how improper accounting for systematic risk produces misleading results and social welfare loss. We conclude by proposing a simple modification of A-94's procedures that would allow analysts to at least partially account for systematic risk.

**Keywords:** A-94; benefit-cost analysis; discounting; risk.

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## 1 Introduction

Benefit-cost analyses (BCA) studies conducted by US government entities follow methodological guidelines outlined in the Office of Management and Budget's (OMB) Circular A-94: Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs.<sup>1</sup> According to A-94:

The goal of this Circular is to promote efficient resource allocation through well-informed decision-making by the Federal Government. It provides general guidance for conducting benefit-cost and cost-effectiveness analyses. It also provides specific guidance on the discount rates to be used in evaluating Federal programs whose benefits and costs are distributed over time. The general guidance will serve as a checklist of whether an agency has considered and properly dealt with all the elements for sound benefit-cost and cost-effectiveness analyses.<sup>2</sup>

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<sup>1</sup> This paper will focus exclusively on the proper discounting of risk in BCA. Procedures for conducting regulatory impact analyses (RIA) will not be addressed.

<sup>2</sup> See OMB (1992, p. 2).

Over the years, the discounting procedures mandated by Circular A-94 have engendered considerable debate and controversy. This is hardly surprising. BCA is inherently normative and it is by definition impossible to falsify any hypothesis regarding the “correct” discount rate. As Burgess and Zerbe (2011) note, “little consensus can be found on issues such as what should be discounted, or on the choice of a discount rate.”<sup>3</sup>

One of the many criticisms often leveled at Circular A-94 is that the guidelines do not successfully capture the actual social welfare implications of risk. Bazelon and Smetters cogently articulate this position in their 1999 evaluation of BCA practiced “within the Beltway,” in which they conclude that “the ongoing challenge is to discount future costs or benefits in a way that matches the project’s level of riskiness.”<sup>4</sup>

In this essay, we review the risk accounting methods mandated by Circular A-94.<sup>5</sup> We find that the procedures required by A-94 actually do an excellent job of accounting for what is known as “non-systematic risk,” and an admirably good job of accounting for what is often called “term risk.” Consistent with Bazelon and Smetters (1999), however, we find that A-94’s mandated methodology clearly fails to account for “systematic” risk – something that can lead benefit-cost analyses to draw deeply misleading conclusions regarding the actual social welfare implications of various projects. We argue, however, that currently there is no workable method of correctly accounting for systematic risk when evaluating the types of projects actually analyzed by government agencies and departments. In order to overcome this, we suggest a practical “ad hoc” treatment of systematic risk in government BCA that, while far from perfect, is superior to current practice. Specifically, we propose that analysts add 1% to the rate used to discount the value of a future flow of cost or benefit that is positively correlated with per capita consumption, and subtract 1% from the rate used to discount the value of a future flow of cost or benefit that is negatively correlated with per capita consumption.

This paper is divided into six sections. In Section 2, we review the basic logic underpinning the discounting of future benefits and costs and the role that attitudes towards risk play in informing that logic. In Section 3, we define three different types of risk and evaluate how Circular A-94 accounts for them in its discounting procedures. In Section 4, we provide a mathematical example that illustrates why failure to account for systematic risk in BCA is potentially dangerous.

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<sup>3</sup> See Burgess and Zerbe (2011, p. 1).

<sup>4</sup> See Bazelon and Smetters (1999, p. 214).

<sup>5</sup> In this paper, we will make no distinction between BCA and cost-effectiveness analysis. Circular A-94’s methodological guidelines apply to both types of analyses, and there is no meaningful difference in how flows of cash or flows of effectiveness should be discounted.

In Section 5, we discuss how the private sector accounts for systematic risk in the discounting of future costs and benefits, and argue that there is no workable way for government analysts to exploit similar techniques. Section 6 concludes the paper by proposing a methodology that would at least partially capture the welfare implications of systematic risk in the discounting of future costs and benefits, while applying methods that “real life” government analysts grappling with “real life” government BCA can actually apply in a consistent manner.

## 2 The risk free discount rate: theory

Modern thinking regarding the discount rate can be traced back to Ramsey’s (1928) seminal analysis of the optimal rate of savings – the rate of savings that will maximize the aggregate utility of all present and future citizens. Central to this approach is the observation that utility maximization requires that the marginal rate of substitution in the consumption of two goods must equal the marginal rate of transformation in the production of the two goods. Ramsey considers, under conditions of certainty, the optimum when the two goods in question are current aggregate consumption and future aggregate consumption. From this, the discount rate that maximizes aggregate utility follows:

$$r = \alpha + \eta\lambda, \quad (1)$$

where  $r$  is the discount rate,  $\alpha$  is the pure rate of time discount,  $\lambda$  is the rate at which per capita real consumption is expected to grow over the long term, and  $\eta$  is a marginal elasticity of utility with respect to real consumption.

There are two popular ways to interpret  $\alpha$ . One interpretation is that it captures the degree to which people are impatient and simply want things as early as possible because – like small children – they hate waiting for anything. The second is that it reflects the degree to which people fear that they will unexpectedly die, or in a social context, that their country or the world will be destroyed by war, disease, a meteor strike, or something like that.<sup>6</sup>

The value of  $\lambda$  captures the degree to which our progeny are likely to be wealthier than we are. For  $\lambda$  to equal zero, the multi-century boom in global living standards would have to come to a complete end. That seems rather unlikely. US living standards have risen by about 2% per year over the past 150 years, and

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<sup>6</sup> Arguments over the proper value for  $\alpha$  played an important role in the debate that followed the 2006 publication of the Stern Review on Economics of Climate Change (Stern 2006).

Gordon (2012) – the most recent effort to project future living standards – estimates that American living standards will rise over the long term by about 1% per year.

As for  $\eta$ , its value captures the degree to which people are risk averse, or in other words, the degree to which their marginal utility of consumption (or wealth) is declining. A value of zero for  $\eta$  implies that people are risk neutral and the marginal utility of consumption is constant. This is the equivalent of assuming that if the prosperous Fitzwilliam Darcy (from Jane Austen's *Pride and Prejudice*) found \$100 lying on the ground, it would offer him just as much utility as it would offer the impoverished Oliver Twist (from Charles Dickens' *Oliver Twist*) should he find the same sum. Clearly, this is wrong – people are risk averse and  $\eta$  does not equal zero.<sup>7</sup>

The presence of  $\eta$  as one of the determinants of the risk free rate illustrates that the discounting of future costs and benefits – even under conditions of certainty – is at least partially determined by risk aversion. Given this, it makes little sense to apply BCA under conditions of uncertainty without taking into account the impact of the resultant risk on future costs and benefits. To what extent does Circular A-94 actually do so?

### 3 Circular A-94's treatment of risk

Analytically, there are three very different types of risk that must be considered in evaluating new projects. The first is known as “non-systematic,” “diversifiable,” or “idiosyncratic” risk. Non-systematic risk involves risks and uncertainties that are unique to the specific project or policy being evaluated. For example, in assessing a project involving new technologies, there is a distinct risk that prototypes will simply fail to function in the manner envisioned by engineers. Such risks have nothing to do with, and are not correlated with, overall economic conditions.

OMB Circular A-94 states, “in general, variations in the discount rate are not the appropriate method of adjusting net present value for the *special risks* of particular projects.”<sup>8</sup> Instead, it mandates that such risks be accounted for by estimating expected values:

<sup>7</sup> Evans (2005), for example, estimates  $\eta$  for 20 OECD countries and derives estimates that range from 1.08 to 1.82, with values ranging from 1.15 to 1.45 for the US.

<sup>8</sup> See OMB (1992, p. 12).

The expected values of the distributions of benefits, costs and net benefits can be obtained by weighting each outcome by its probability of occurrence, and then summing across all potential outcomes. If estimated benefits, costs and net benefits are characterized by point estimates rather than as probability distributions, the expected value is the appropriate estimate for use.<sup>9</sup>

As an example, consider a new component for a naval propulsion system. The development of the component will cost \$10 MM this year. If it works, it will save \$20 MM (in net present value) worth of fuel and maintenance over the next decade under all circumstances, but there is an estimated 30% chance that the component will not actually work as planned. In such a case, A-94 would mandate that the estimated net benefits be calculated by multiplying \$20 MM by 0.7 and then subtracting \$10 MM, so that the project would generate a positive expected benefit of \$4 MM.

The estimate is correct because this \$10 MM research project is only one of hundreds that the government will fund in its R&D program – each being too small to influence the overall economy in any particular way. The “Law of Large Numbers” assures that for the research and development program as a whole, the aggregate net benefit will vary little from the weighted average of the estimates for each individual project: “As the number of trials of a random process increases, the percentage difference between the expected and actual values goes to zero” (Renze and Weisstein 2013). In other words, non-systematic risk should be ignored in BCA because at the aggregate level, it is not really a risk at all.

Now, let us consider the second type of risk. This is known as “term” or “liquidity” risk. It is the risk that while a project may have certain costs and benefits, the world in which the project is embedded remains uncertain and may change in some way that makes the project less valuable or more costly.

For example, say there are two alternative futures. In one, everyone will be as poor as Oliver Twist. In the other, everyone will be as wealthy as Fitzwilliam Darcy. We do not know which future awaits our progeny. We do know, however, that over time it will become increasingly clear whether the future is one of poverty or prosperity. Now let us consider a project that will cost us now but pay off with certainty long in the future. Based on analysis, we have concluded that this project will be more than worthwhile should the future be one of poverty, but will not be worthwhile should the future be one of prosperity.

Should we implement this project? To answer that question, we must compare the project with alternatives. For example, instead of implementing the project, we could stockpile resources that we intend to bequeath to our progeny should

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<sup>9</sup> *Ibid.*, p. 11.

they turn out to be poor. If over time, however, it becomes increasingly clear that our progeny are not going to actually be poor, we can liquidate the stockpile and have some belated fun.

Clearly, the relative attractiveness of these alternatives depends on net returns. But how can analysts determine whether the project's returns are sufficiently high? For a country with capital markets as efficient as those of the US, the government bond market offers analysts a good way to estimate the value that the public places on certain returns in an uncertain future relative to the value of "keeping options open." For all practical purposes, buying a zero coupon long-term government bond is the equivalent of irreversibly implementing a project. You are guaranteed a particular return, but you are locked into that return for a very long time. Short-term bonds, however, are the equivalent of stockpiling – you are keeping your options open. The difference in yields offered by these bonds of different maturities, something known as the "term structure," reflects the degree to which the public prizes flexibility over expected returns.<sup>10</sup>

OMB Circular A-94 accounts for term risk. Each year, OMB issues a revision to A-94's Appendix C that mandates the discount rates to be applied for costs and benefits taking place at different points in a project's future. These discount rates are set using prevailing yields for government bonds. The following, for example, are the real discount rates guidelines that OMB published for 2012 (OMB, n.d.):

| 3-Year | 5-Year | 7-Year | 10-Year | 20-Year | 30-Year |
|--------|--------|--------|---------|---------|---------|
| 0.0    | 0.4    | 0.7    | 1.1     | 1.7     | 2.0     |

Should the BCA be evaluating a project that – once implemented – can literally not be stopped or modified prior to its completion, then the Appendix C guidelines *perfectly capture* the term risk that should be accounted for in the analysis.

Many projects, of course, can be modified or cancelled prior to their completion, but that is not always easy and it is certainly not costless. Once projects get rolling, they often take on a life and logic (and lobby) of their own. Hence, while Circular A-94's approach to term risk is not perfect, it is actually quite reasonable given the lumpiness and inertia inherent in most government projects.

Now, let us turn to the last type of risk. This involves situations where a project's costs and benefits are correlated with future aggregate consumption, and is known as "systematic" risk. To illustrate the nature of systematic risk, let us once again call upon the services of Oliver Twist and Fitzwilliam Darcy. Consider two mutually exclusive projects that offer an expected return of \$100 in an uncertain future. There are two possible and equally likely futures – once again futures

<sup>10</sup> For an excellent discussion of term risk, see Abel (1999).

where everyone is as poor as Oliver Twist or as rich as Fitzwilliam Darcy. One project will return \$200 in the future where everyone is poor and nothing in the future where everyone is rich. The other project will return \$200 when everyone is rich and nothing when everyone is poor.

If we calculate expected values and discount using any given discount rate, these projects will appear to be identical. But is that really so? From the perspective of social welfare maximization, the answer is clearly no. The first project generates returns when the marginal utility of money is high, while the second generates returns when the marginal utility of money is low. As a result, the first project unambiguously generates more expected utility and is superior to the second. The authors of Circular A-94 recognize this reasoning: “The absolute variability of a risky outcome can be much less significant than its *correlation* with other significant determinants of social welfare, such as real national income.”<sup>11</sup>

The statement clearly establishes that risks correlated with “significant determinants of social welfare” are, to use A-94s own language, “significant.” This strongly implies that such risks are also *important* and should be accounted for in BCA. Remarkably, however, Circular A-94 then offers no methodology for accounting for this “significant” risk, and never mentions the subject again. In the next section, we offer a numerical example that illustrates why this is a serious problem.

## 4 The danger involved in failing to account for systematic risk

Consider a two-period model. A country is made up of a large number of identical citizens, who seek to maximize their private welfare subject to the constraint that they are adequately defended by their country’s navy.

In order to defend the country adequately, it has been determined that the navy will need an additional submarine in period two. Hence, the navy must build a submarine during the first period.

Now, the navy must choose between two alternative technologies for the propulsion of the submarine. The first alternative is to equip the submarine with a nuclear reactor. The second is to equip the submarine with diesel-electric propulsion. Each alternative has a known and identical initial cost. Let  $g_1$  be the fraction of that initial cost that each citizen pays in period 1. We also assume that the choice of propulsion has no impact on the submarine’s effectiveness.

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<sup>11</sup> See OMB (2012, p. 12).

In period 2, there are two possible states of nature. A state of war occurs with probability  $\pi$  and a state of peace with probability  $(1-\pi)$ . In times of war, submarines must steam for many hours, often at high speeds.<sup>12</sup> In times of peace, very little steaming is required. With nuclear propulsion, the cost of operating the submarine does not vary with the realized state of nature, but for diesel-electric propulsion, more steaming means more cost. Let the random variable  $g_2$  be the fraction of operating costs that each citizen must pay in period 2.

Now let us consider the expected utility of the representative citizen. In each period the agent receives an income,  $c$ , in the form of a consumption good that cannot be stored for later use. Assume that log-utility represents the agent's preferences. Then

$$EU = \ln(c - g_1) + \gamma E[\ln(c - g_2)], \quad (2)$$

where  $E$  is the expectations operator and  $\gamma = 1/(\alpha + 1)$ . To operationalize this simple model, we use the parameters in Table 1.

In Table 2, we assume values for operating costs of both alternative propulsion systems in both states of nature. We then substitute the values from Tables 1 and 2 into equation (2).

The result is that, in terms of expected second period cost, diesel-electric propulsion is the cheaper alternative. Given that first period cost and utility is identical for both alternatives, a BCA analysis of the propulsion choice would always recommend diesel-electric propulsion, regardless of the discount rate chosen. This is disconcerting, given that nuclear propulsion is the choice that maximizes the expected utility of the representative citizen.

But what is to be done? Bazelon and Smetters (1999) argue that, given the danger inherent in ignoring systematic risk, "there is little rationale for the government to discount future costs and benefits of any particular project or program differently than the private market," where valuations of future cash flows are

**Table 1** Numerical model input data.

| Symbol   | Definition                  | Value |
|----------|-----------------------------|-------|
| $\pi$    | Probability of war          | 0.30  |
| $\alpha$ | Pure rate of time discount  | 0.00  |
| $g_1$    | Agent share of initial cost | 1.20  |
| $c$      | Citizen income              | 2.00  |

<sup>12</sup> Naval pedants would be correct to point out that, formally, a diesel-electric submarine does not use steam propulsion, and hence cannot "steam."

**Table 2** Numerical model output data.

| Propulsion type | $g_2$ in Peace | $g_2$ in War | Expected cost | Expected utility |
|-----------------|----------------|--------------|---------------|------------------|
| Nuclear         | 0.3000         | 0.3000       | 0.3000        | 0.3075           |
| Diesel          | 0.1500         | 0.6450       | 0.2985        | 0.2986           |

adjusted for systematic risk as a matter of course.<sup>13</sup> In the following section, we consider that recommendation and conclude that it is unworkable.

## 5 Proper accounting for systematic risk: theory and practice

Ramsey's analysis of inter-temporal utility maximization resulted in an expression for the discount rate under conditions of certainty. Exploiting theoretical foundations that include Markowitz (1952) and Sharpe (1964), Breeden (1979) shows that under conditions of uncertainty, the inter-temporal maximization of *expected* utility is achieved by adjusting the discount rate applied to an uncertain future cost or benefit ( $X$ ) by an additional "risk premium" (RP):

$$RP_x = - \left[ \frac{1}{E(u_c)} \right] * \text{cov}(u_c, X), \quad (3)$$

where  $u_c$  is the marginal utility of future consumption.<sup>14</sup>

Estimation of the proper risk premiums for different cash flows plays an important role in private sector analyses of investment decisions, and is reflected in the common use of terms such as "risk adjusted return." But what method does the private sector employ to make such estimates? The standard approach is to estimate the RP using the following formula:

$$RP_x = \beta_x [E(R) - r], \quad (4)$$

where  $R$  is a variable that proxies for consumption or wealth, and  $\beta_x$  is the covariance of  $X$  and  $R$ , divided by the variance of  $R$ . In order to estimate the covariance of  $X$  and  $R$ , analysts look at the historical fluctuations in the valuation of assets with similar cash flows relative to the proxy.

<sup>13</sup> See Bazelon and Smetters (1999, p. 26).

<sup>14</sup> See Varian (1992, p. 379).

This can be something of an art. For example, there are many different future flows of cash involved in a hydroelectric project. Each requires its own estimated RP, but in practice, markets offer no information regarding historical fluctuations in the value of these separate cash flows since they are not listed securities that trade separately. Indeed hydroelectric projects do not trade as separate securities either. Instead, analysts will try to deduce the RPs from fluctuations in the valuation of firms that own a lot of hydroelectric assets.

What about private sector projects that are entirely novel? Markets offer no information regarding the past statistical relationships between similar assets and  $R$ , since there has never been a similar asset. In dealing with such cases, a popular graduate text helpfully tells young financial analysts in training that “you cannot hope to estimate the relative risk of assets with *any precision*,” but goes on to helpfully suggest that you “examine the project from a variety of angles and look for *clues* as to its riskiness.”<sup>15</sup>

This leads us to question whether private sector risk discounting methodologies are applicable to government BCA. First, let us observe that private sector techniques will work quite well if the government is dabbling in activities that are commonly done by the private sector. If, for example, the government has decided to set up a McDonald’s franchise, it can easily apply the same techniques for the evaluation of these investment proposals that are exploited by the private sector. Such investments are hardly novel – indeed “billions and billions have been served” – and market derived information can be obtained regarding the covariance of the value of such franchises and aggregate economic conditions.

Things, however, become far trickier when evaluating government projects that are focused on the provision of public goods or involve important positive or negative externalities. Markets provide no information on changes in the value of public goods, since such goods are not traded in private markets. As for externalities, there may indeed be market-traded assets associated with both positive and negative externalities but, *by definition*, the market ignores fluctuations in the value of the externalities since the buyers and sellers of the assets are not affected by them and hence could not care less. What this means is that, for the types of project that actually matter for government BCA – projects that involve the provision of public goods or important externalities – the preferred private sector method of exploiting the historical market relationship between the project’s costs and benefits and aggregate economic conditions is simply not applicable. For example, what should the risk premium be for the costs associated with a new torpedo, let alone the risk premium from the benefits that come from sinking ships with it?

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<sup>15</sup> See Brealey and Myers (1991, p. 200).

In the absence of market derived information that characterizes the costs and benefits of government projects, the private sector best practice is to “*look for clues*.” The problem with this is that there is no standardized and consistent way to mandate a search for clues, let alone signs and portents. Clearly, OMB A-94 cannot mandate the use of such vague suggestions as formal guidelines for project analysis.

## 6 A workable method of incorporating systematic risk into BCA analysis

As we have seen, failure to incorporate systematic risk into BCA analyses clearly can result in major misallocations of resources. But we have also found that there is actually a good reason why OMB Circular A-94 avoids addressing this issue and why risk premiums are not incorporated into US government estimates of costs and benefits. We would like to suggest, however, that there may be a simple and transparent approach to the problem that would at least partially capture the role of systematic risk in the estimation of the social discount rate while preserving the methodological standardization that government BCA requires.

Specifically, we propose that Circular A-94 be amended to require that analysts evaluate whether each flow of cost or benefit associated with a project is positively or negatively correlated with increases in per capita consumption.<sup>16</sup> In making his or her evaluation, the analyst would be required to write a paragraph explaining the reasoning behind the answer chosen.

If the answer is “positively correlated,” then the analyst would add 1% to the current discount rate set in Appendix C of Circular A-94. If, on the other hand, the answer is “negatively correlated,” then the analyst would subtract 1% from the discount rate. If the answer is “not correlated,” then no adjustment of the discount rate would be made.

Such an approach would be transparent and simple to execute. It is also extremely unlikely that an analyst would make a *qualitative* mistake regarding the relationships involved. To see why, let us consider two examples.

First, consider a proposal to install a new golf course at an air force base in Korea. Under what conditions would the golf course be a good investment for the

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<sup>16</sup> The focus on private consumption implicitly assumes that such consumption is additively separable from all other arguments in citizens’ utility functions. This assumption is regularly made in economic analyses, but cannot be tested or falsified.

government? Well, if the economy is thriving and officers and technical personnel are receiving attractive offers from civilian employers, then offering service members a golf course may be a valuable form of compensation that keeps them in the air force and preserves their valuable human capital. If, however, the economy is weak and service members do not have many civilian alternatives, then the golf course may have been – *ex post* – an unnecessary and wasteful expense. In this case, the golf course's payoff is positively correlated with per capita consumption.

What about a proposal to build a new bomb shelter at the same air base? Such a project would pay off the most in the event of a full scale war on the Korean peninsula, something likely to disrupt global trade and result in difficult economic conditions.<sup>17</sup> Hence, the bomb shelter's payoff is negatively correlated with per capita consumption.<sup>18</sup>

Under our proposal, the analyst would add 1% to the discount rate used to discount benefits flowing from the proposed golf course and subtract 1% from the rate used to discount benefits flowing from the proposed bomb shelter.

There is no way to be sure if those adjustments accurately capture the proper risk premiums that should be used. Most likely, however, these adjustments would underestimate the risk premium, perhaps by a wide margin. Quantitatively, a 1% risk premium suggests a very modest relationship. By comparison, the risk premium for valuing the US equity market as a whole is generally estimated to be between 4% and 7%.

Qualitatively, however, being able to notice – and justify – a positive or negative correlation between a project's future costs and benefits and private consumption would require a relationship likely to be pretty obvious. It seems to us most unlikely that many of the relationships evaluated in government BCA would prove simultaneously to be both very small and easy to notice.

To summarize, the proposed methodology would not result in the proper discounting of risk in government projects. That actually cannot be done. But it would do two things. First, it would force analysts to think about systematic risk and openly discuss projects' systematic risks as a formal part of the decision-making process. Second, it would provide estimates for the social discount rate that – while not accurate – are almost certainly closer to their true values than the methods currently mandated by Circular A-94. As such, adoption of this proposal would improve the quality of US government BCA.

Previously published online October 2, 2013

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<sup>17</sup> Needless to say, the consequences of such a conflict would be dire for the golf course.

<sup>18</sup> The correlation is clearly not 1, since there are conditions where consumption could be low while the Korean peninsula remains peaceful.

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