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Synthetic Options and Implied Volatility for the Corporate Bond Market

Steven Shu-Hsiu Chen
A. R. Sanchez, Jr. School of Business, Texas A&M International University
shu-hsiu.chen@tamiu.edu

Hitesh Doshi
Bauer College of Business, University of Houston
hdoshi@bauer.uh.edu

Sang Byung Seo
Wisconsin School of Business, University of Wisconsin–Madison
sang.seo@wisc.edu (corresponding author)

Abstract

We synthetically create option contracts on a corporate bond index using CDX swaptions, overcoming the limitations that stem from the lack of traded corporate bond options. Our approach allows us to estimate forward-looking moments concerning the corporate bond market in a model-free manner. By constructing an aggregate volatility measure and the associated variance risk premium, we examine the role of volatility risk in the corporate bond market. We highlight that the ex ante conditional second and higher moments we estimate from synthetic corporate bond options carry important implications for credit risk models, providing an extra basis for testing their validity.

I. Introduction

The availability of stock index options with a wide range of moneyness has brought about a significant breakthrough in understanding the dynamics of the aggregate equity market. As exemplified by the VIX, the price data on these options make it possible to estimate forward-looking moments concerning the equity market in a model-free manner. Furthermore, stock options help us study investors' risk preferences, which have direct implications for risk premia associated with the equity market.

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Considering the utility of option contracts in the equity market, we expect similar contracts in the bond market, which occupies an even larger portion of the entire capital market. This is indeed the case for the Treasury market: Options on Treasury futures are actively traded. However, this is not the case for the corporate bond market: Neither options written on aggregate corporate bond indices nor ones on corporate bond ETFs are actively traded with meaningful cross sections. Corporate bonds are issued with various maturities, coupons, and embedded provisions, which makes it difficult to trade standardized option contracts on an aggregate price index. The lack of traded options poses a serious challenge to examining the innate risk in the corporate bond market. Not surprisingly, the behavior of the aggregate corporate bond market has received far less attention in the literature, despite its large size and economic significance.

In this article, we develop a new tool to investigate the corporate bond market. Using novel data on credit derivatives, we synthetically create options on a price index of 5-year corporate bonds with floating coupons (namely, floating rate notes (FRNs)). First, we exploit the fact that a defaultable FRN issued by a certain firm can be replicated by the portfolio of a default-free FRN and a credit default swap (CDS) contract that is exposed to the firm's credit risk. This implies that under no arbitrage, the price of the firm's FRN can be inferred from the CDS pricing data. The replicability of FRNs is an important advantage. Since CDS contracts are highly standardized, we are able to create a cross section of synthetic FRNs whose coupons and maturities are exactly identical. The same cannot be achieved using the bond price data because the terms of the bonds that are issued and traded vary across different firms.

We construct our synthetic corporate bond index as the average price of synthetic FRNs issued by a large number of investment-grade firms. To capture the risk of the aggregate corporate bond market, we choose the pool of firms in the CDX North American Investment Grade index (in short, CDX). Since the CDX represents 125 equal-weighted investment-grade single-name CDS contracts across 5 industrial sectors, it can serve as a good proxy for the average behavior of the corporate bond market. Consequently, we show that the synthetic corporate bond index can be expressed in terms of the quoted upfront fee for the CDX and the cumulative loss from defaults in the CDX.

Then, we use the data on CDX swaptions. An important advantage of using the CDX for our analysis is that its "calls" and "puts" are actively traded. These options are called credit swaptions because they grant holders the right, not the obligation, to enter into a 5-year CDX contract. A receiver swaption allows the holder to enter into a contract as the protection seller, whereas a payer swaption allows the holder to do so as the protection buyer. We show that using CDX swaptions, it is possible to replicate call and put options on the 5-year synthetic corporate bond index, which enable us to calculate various forward-looking moments in a model-free fashion.

One such example is the corporate bond VIX (CBVIX), which represents the risk-neutral expectation of future 1-month volatility on our synthetic corporate bond index. Existing option-based volatility measures (such as the equity VIX and the Treasury VIX) have attracted ample attention from academics and practitioners alike, as volatility risk is a central and universal subject in modern

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finance irrespective of the market.[1] Despite the much earlier introduction of these volatility indices, a volatility measure for the corporate bond market has been missing due to the unavailability of corporate bond options. We fill this gap by introducing the CBVIX.

The resulting CBVIX from Mar. 2012 to Sept. 2018 shows significant time-series variations with a sample mean of 1.72%. The CBVIX is high when the synthetic corporate bond index is low, implying asymmetric volatility in the corporate bond market. Although its level is much lower compared to the equity VIX, they fluctuate in a consistent manner with a correlation of 0.71. Although both are based on bonds, the CBVIX and the Treasury VIX have a weaker correlation of 0.44, showing similar yet distinct patterns.

Based on the CBVIX, we examine the role of variance risk in the corporate bond market by constructing the monthly time series of the variance risk premium. We find that most of the time, the corporate bond variance risk premium is positive and shows substantial time variations. Furthermore, although the corporate bond variance risk premium has negative contemporaneous correlations with bond and equity returns, it positively predicts future bond and equity returns. The predictability remains significant even when we control for the equity variance risk premium. These robust empirical results suggest that the variance risk premium in the corporate bond market captures an important source of systematic risk shared by both bond and equity markets.

Our model-free option-based estimation is not limited to the second moment: We further estimate higher-order moments of the corporate bond index and find large magnitudes of skewness and kurtosis, especially during the eurozone debt crisis. In addition, we nonparametrically estimate the distributions of the price relative under the risk-neutral and physical measures and discover that they imply a nonmonotonic U-shaped pricing kernel.

Importantly, the conditional moments we estimate from synthetic corporate bond options provide extra grounds for testing the validity of credit risk models. Particularly relevant are structural credit risk models: In such models, the firm's equity and bond dynamics both originate from the firm's asset dynamics. Motivated by this insight, we examine the relation between the equity VIX and the CBVIX through the lens of a simple structural credit risk model. Specifically, we develop and test a model implication that the ratio between the equity VIX and the CBVIX should equal the price elasticity between the equity and the bond. Our estimation reveals that the price elasticity well captures the relative magnitude of the 2 volatility indices, upholding the simple economic intuition behind the model. Through this exercise, we demonstrate that there are potentially many aspects of credit risk models that can be studied using our synthetic options and the resulting forward-looking conditional moments.

Overall, this article highlights the usefulness of CDX swaptions in studying the corporate bond market. Unlike the equity and Treasury markets, the corporate bond market lacks a cross section of traded options on an aggregate price index,

[1]The CBOE Volatility Index (so-called the VIX) represents the risk-neutral expectation of future 1-month volatility of the S&P 500. To avoid potential confusion, we refer to this index as the equity VIX. The Treasury VIX represents the risk-neutral expectation of future 1-month volatility of 10-year maturity Treasury note futures.

which makes it difficult to analyze its behavior. We circumvent this problem by replicating option contracts on our synthetic corporate bond index using CDX swaptions. Equipped with these synthetic options, one can take advantage of a vast array of option-based tools that the literature has developed for the equity and Treasury markets and gain more insight into the corporate bond market.

Our article builds on prior studies that propose a model-free approach to estimate the risk-neutral moments of future returns based on options. According to Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003), we can span the risk-neutral expectation of a twice-differentiable payoff function by using prices of out-of-the-money (OTM) European calls and puts. In general, these articles apply this model-free approach to equity options in order to study the properties of the equity market. Although we adopt similar techniques, we instead apply them to synthetic bond index options to examine the properties of the corporate bond market.

With the aid of the model-free option-based approach, we make a contribution to the literature on the variance risk premium. Bollerslev, Tauchen, and Zhou (2009) find that their constructed equity variance risk premium is significantly positive and predicts future equity market returns.[2] Consistent with the findings in the equity market, we obtain a positive variance risk premium in the corporate bond market, which robustly predicts future bond and equity returns.

In the credit risk literature, credit derivatives have become increasingly important instruments for understanding the underlying firm dynamics behind the high credit spread in the data. For example, based on an estimation with CDS spreads, Du, Elkamhi, and Ericsson (2019) emphasize the role of time-varying asset volatility in resolving the credit spread puzzle.[3] Relatedly, Kelly, Manzo, and Palhares (2018) create a credit-implied volatility surface from CDS spread data via the Merton (1974) model and find a 3-factor structure in the surface. Whereas these articles largely focus on asset volatility, our main interest is in extracting bond market volatility.

Our article also relates to the extensive literature on corporate bond returns. In this literature, empirical work is typically based on the time series and the cross section of corporate bond returns. Fama and French (1993) stress the importance of the default and term premia in explaining corporate bond returns.[4] As additional examples, Bai, Bali, and Wen (2016) study the distributional characteristics of historical corporate bond returns, and Bai, Bali, and Wen (2019) investigate the cross-sectional determinants of corporate bond returns. Our article differs from these articles in that we extract ex ante conditional moments using CDX swaptions.

CDX swaptions are similar to CDX tranches in that they are both derivative contracts on the CDX. Prior to the subprime mortgage crisis, CDX tranches were

[2]Related articles in the literature include Coval and Shumway (2001), Bakshi and Kapadia (2003), Carr and Wu (2009), Todorov (2010), Eraker and Wu (2017), and Aït-Sahalia, Karaman, and Mancini (2018).

[3]The credit spread puzzle reflects the inability of structural credit risk models in explaining observed credit spreads. See, for example, Eom, Helwege, and Huang (2004), Huang and Huang (2012), Feldhutter and Schaefer (2018), and Huang, Shi and Zhou (2019).

[4]See, also, Gebhardt, Hvidkjaer, and Swaminathan (2005), Lin, Wang, and Wu (2011), Acharya, Amihud, and Bharath (2013), Jostova, Nikolova, Philipov, and Stahel (2013), Bongaerts, de Jong, and Driessen (2017), and Choi and Kim (2018).

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much more actively traded than CDX swaptions and, therefore, received greater attention in the empirical literature.[5] However, the market for CDX tranches abruptly went cold after the crisis: Investors became reluctant to trade collateralized debt obligation products like CDX tranches because they were singled out for their role in the crisis. The irony is that this opened up a new opportunity for CDX swaptions. To hedge credit risk, investors started to rely more on CDX swaptions, and as a result, their liquidity has drastically been improving since 2012. To the best of our knowledge, this is the first article to make use of the information contained in CDX swaptions whose importance in the credit market is on the rise.

The article proceeds as follows: Section II presents the methodology for constructing our synthetic corporate bond index and its options. Section III describes the pricing data on CDX and CDX swaptions. Section IV estimates option-implied moments concerning the corporate bond market and conducts empirical analysis. Section V discusses the implications of our results for credit risk models. Section VI concludes.

II. Constructing Options on a Corporate Bond Index

A. Synthetic Corporate Bond Index

We first synthetically create a price index for investment-grade corporate bonds. Let $P_t^{(T)}$ denote an equal-weighted price index, which consists of T-maturity corporate bonds issued by a cross section of investment-grade firms $i \in \{1,\ldots,N\}$:

$$P_t^{(T)} = \frac{1}{N}\sum_{i=1}^{N} P_{i,t}^{(T)}.$$

Here, $P_{i,t}^{(T)}$ represents the time-t price of firm i's bond, which is expressed as a fraction of the bond's face value (i.e., price per a dollar face value). Although corporate bonds can be issued with various coupon structures and embedded provisions, what truly sets them apart from government bonds is their credit risk exposures. To focus on the credit component of the corporate bond market, we construct our index with FRNs, which are immune to fluctuations in default-free interest rates. Typically, FRNs pay quarterly coupons that are calculated as the sum of i) a default-free benchmark interest rate, which resets every 3 months, and ii) a quoted margin, which is an additional spread that remains fixed as compensation for credit risk.

An important benefit of working with FRNs is that their payoffs can be replicated using CDS contracts (e.g., Duffie and Singleton (2003)). Selling protection on a firm's standard CDS, which pays 1% coupons as the insurance premium, together with investing in a default-free FRN exactly replicates the firm's defaultable FRN with a 1% quoted margin. First, each coupon paid by the defaultable FRN is reproduced by the sum of the coupon payment from the default-free FRN (the benchmark interest rate) and the premium payment from the CDS contract

[5]For example, Coval, Jurek, and Stafford (2009) and Collin-Dufresne, Goldstein, and Yang (2012) debate about the mispricing of CDX tranches during the period preceded by the 2008 financial crisis. Other empirical articles that concern CDX tranches are Longstaff and Rajan (2008), Seo and Wachter (2018), and Choi, Doshi, Jacobs, and Turnbull (2019).

(1%). Second, if the firm survives until maturity, the defaultable FRN expires and pays out its face value. The same payoff is delivered by the replicating portfolio because the default-free FRN contained in it matures at the same time with the same face value. Lastly, in the event of the firm's default, the defaultable FRN experiences a loss. The replicating portfolio suffers an identical amount of loss due to the protection sell position on the firm's CDS contract.

Therefore, under no arbitrage, the price of the defaultable FRN $P_{i,t}^{(T)}$ should equal the cost of implementing this replication strategy.[6] Purchasing a default-free FRN costs a dollar, as its price remains close to its face value.[7] Entering into a standard CDS contract also incurs a fee exchanged upfront. Combining the 2 costs, it follows that

$$P_{i,t}^{(T)} = 1 - U_i(t, t+T), \tag{1}$$

where $U_{i,t} = U_i(t, t+T)$ is the quoted upfront fee paid by the protection buyer to the protection seller for firm i's T-maturity CDS contract. Note that $U_{i,t}$ can be negative. In a standard investment-grade CDS, the coupon spread is fixed at 1%, not at the fair market spread, so the upfront fee exchanged at the beginning of the contract settles this difference. If the fair market spread is lower than 1%, the coupons that the protection seller receives are too high compared to the fair level. Thus, the protection seller should make an upfront payment to the protection buyer, resulting in a negative $U_{i,t}$. Appendix A provides further details on how CDS contracts are quoted and traded.

The replicability of FRNs is a major advantage because CDS contracts are highly standardized in terms of their coupons and maturities. Constructing a corporate bond index directly using the bond price data is tricky; firms do not issue bonds with identical coupons and maturities. Using the CDS data, however, we are able to create a cross section of synthetic FRNs whose coupons and maturities are exactly identical. Equation (1) implies that the synthetic corporate bond index is computed as:

$$P_t^{(T)} = \frac{1}{N}\sum_{i=1}^{N} P_{i,t}^{(T)} = 1 - \frac{1}{N}\sum_{i=1}^{N} U_i(t, t+T). \tag{2}$$

From equation (2), we can see that $P_t^{(T)}$ can be replicated by purchasing a default-free FRN with \$1 face value and by taking protection sell positions on N single-name CDS contracts, each of which has a notional value of $\$(\frac{1}{N})$.

How does our index evolve as time progresses? After a τ period, the time to maturity of the FRNs reduces to $T - \tau$, and the index becomes $P_{t+\tau}^{(T-\tau)} =$

[6]Although, in theory, this replication argument should always hold for every firm, the CDS-bond basis significantly deviated from zero during the Great Recession period, which poses a puzzle (see, e.g., Bai and Collin-Dufresne (2019)). During our sample period, this basis is relatively small and stable. As long as the basis is not too volatile, it should not bias our ex ante risk measures.

[7]This is because a default-free FRN pays coupons that exactly mirror the discount rates. The price of a default-free FRN is exactly par at coupon reset dates. Even between two adjacent coupon reset dates, the price is essentially par because the effective duration is less than 3 months regardless of bond maturity.

$\frac{1}{N}\sum_{i=1}^{N} P_{i,t+\tau}^{(T-\tau)}$, the average FRN price at time $t+\tau$. Each firm either survives or defaults up until time $t+\tau$. If firm i survives, it follows from the same no-arbitrage argument used in equation (1) that the firm's FRN price becomes $P_{i,t+\tau}^{(T-\tau)} = 1 - U_i(t+\tau, t+T)$. If the firm goes into default, the value of the firm's FRN simply becomes $P_{i,t+\tau}^{(T-\tau)} = 1 - L_i$, where L_i denotes the firm's loss rate given default. Hence, the index value at time $t+\tau$ is given by

$$\text{(3)} \qquad \begin{aligned} P_{t+\tau}^{(T-\tau)} &= \frac{1}{N}\sum_{i \notin \mathbb{D}_{t+\tau}} [1 - U_i(t+\tau,\, t+T)] + \frac{1}{N}\sum_{i \in \mathbb{D}_{t+\tau}} [1 - L_i] \\ &= 1 - \frac{1}{N}\sum_{i \notin \mathbb{D}_{t+\tau}} U_i(t+\tau,\, t+T) - \frac{1}{N}\sum_{i \in \mathbb{D}_{t+\tau}} L_i, \end{aligned}$$

where $\mathbb{D}_{t+\tau}$ is the set of indices for the firms defaulted by time $t+\tau$.

Equation (3) reveals that our synthetic corporate bond index varies due to two reasons. First, each firm in the index might go into default, which causes the value of the FRN to fall significantly from its face value. Second, even if the firm survives, fluctuations in the firm's default risk, which are reflected in variations in the firm's CDS upfront fee, can change the FRN price.

B. Selection of Index Constituents Using the CDX

To capture the aggregate behavior of the investment-grade corporate bond market, it is important to choose a pool of firms that is representative of the entire market. To this end, we exploit the CDX, a credit index that consists of 125 investment-grade debt obligations, evenly distributed across 5 different industrial sectors (Consumer; Energy; Financial; Industrial; Technology, Media, and Telecommunications). The CDX rolls on a semiannual basis every March and September to ensure that it tracks the most liquid investment-grade entities and to keep the maturity of the index roughly constant. When the new series, so-called "on-the-run" series, is introduced, the previous series then becomes "off-the-run." Most importantly, among various credit derivatives, the CDX is the most popular with the highest trading volume. For these reasons, we believe that the CDX can serve as a good proxy for the average behavior of the credit market.

Although the CDX is an index that represents the average CDS spread of 125 firms, it is traded as an independent product whose quoted upfront fee, denoted as U_{CDX}, is readily observable in the market. Since entering into a CDX contract as the protection seller is essentially equivalent to taking equal-weighted protection sell positions on all of the 125 single-name CDS contracts that comprise the index, the synthetic corporate bond index $P_t^{(T)}$ in equation (2) can simply be calculated as follows:

$$\text{(4)} \qquad P_t^{(T)} = 1 - \frac{1}{N}\sum_{i=1}^{N} U_i(t, t+T) = 1 - U_{\mathrm{CDX}}(t, t+T).$$

The future index value $P_{t+\tau}^{(T-\tau)}$ in equation (3) can also be interpreted in the context of the CDX. After a total of $N_{t+\tau}^{\mathrm{d}}$ defaults, the CDX represents the average of the remaining $N^{\mathrm{s}}_{t+\tau} = N - N^{\mathrm{d}}_{t+\tau}$ firms in the pool. Accordingly, the upfront fee for the CDX is quoted to capture the average upfront fee for the surviving firms, $\frac{1}{N^{\mathrm{s}}_{t+\tau}} \sum_{i \notin \mathbb{D}_{t+\tau}} U_i(t+\tau, t+T)$. This implies that equation (3) can be reformulated as follows:

$$(5) \qquad P^{(T-\tau)}_{t+\tau} = 1 - \left(\frac{N^{\mathrm{s}}_{t+\tau}}{N}\right) U_{\mathrm{CDX}}(t+\tau, t+T) - L_{\mathrm{CDX},\, t+\tau},$$

where $L_{\mathrm{CDX},t+\tau} = \frac{1}{N} \sum_{i \in \mathbb{D}_{t+\tau}} L_i$ is the cumulative loss of the CDX from defaults of $N^{\mathrm{d}}_{t+\tau}$ firms. In a nutshell, we are able to characterize the time-series evolution of our corporate bond index from the upfront fee (U_{CDX}), the fraction of firms defaulted ($N^{\mathrm{s}}_{t+\tau}/N$), and the accumulated loss (L_{CDX}) from the CDX.

In the data, it is extremely rare to observe a case in which an investment-grade entity goes into default within a short period of time. In fact, since the CDX was introduced in 2003, there have been zero defaults in the CDX on-the-run series.[8] In other words, once an investment-grade entity entered into a new on-the-run CDX series, it survived for at least 6 months. This means that the realized time series of our synthetic corporate bond index in the data has been entirely driven by fluctuations in default risk, not the occurrence of defaults itself.

However, this does not mean that we can simply disregard future possible realizations of defaults in the index. For instance, the expected loss of the index from potential defaults can have a nonnegligible effect on ex ante or forward-looking volatility of the index. The impact of default occurrences can play a more significant role when it comes to risk-neutral volatility. Instances in which investment-grade firms default within a short period of time are likely to coincide with a very bad economic state with high marginal utility, and, thus, it is possible that the risk-neutral measure puts much more weight on such scenarios.

For this reason, the historical time series of the synthetic corporate bond index alone cannot paint the whole picture of the investment-grade corporate bond market. Instruments that can help are option contracts written on the future index level, which we construct in Section II.C using CDX swaptions.

C. Synthetic Options

How do we obtain prices of call and put options written on the future level of the synthetic corporate bond index, $P^{(T-\tau)}_{t+\tau}$? In this section, we show that they can be obtained by making use of the time-t prices of CDX swaptions that expire at time $t+\tau$.

CDX swaptions are credit swaptions that allow investors to enter into a CDX contract in the future at a given upfront fee. A payer CDX swaption provides the holder the right to enter into a $(T-\tau)$-maturity CDX contract after a τ period from today as the protection buyer (who "pays" insurance premiums) at a strike upfront fee K_{U}. On the other hand, a receiver swaption provides the holder the right to enter into the same contract as the protection seller (who "receives" insurance premiums).

[8]Two credit events did occur: Fannie Mae and Freddie Mac. However, these events were not really defaults. These 2 companies were acquired by the government and became default-free entities (i.e., conservatorship).

These instruments are typically European-style options that can only be exercised at maturity. Under our notation, τ represents the time to maturity of CDX swaptions.

For simplicity, first consider a case in which no firm went into default by time $t+\tau$, the maturity date of CDX swaptions. A payer CDX swaption is only exercised when the quoted upfront fee at maturity, $U_{\text{CDX}}(t+\tau,t+T)$, is larger than the strike K_{U}. This is because in this case, the holder can buy protection for paying a lower-than-the-fair upfront fee. Specifically, the holder can lock in a positive payoff of $[U_{\text{CDX}}(t+\tau,t+T)-K_{\text{U}}]$ at maturity: The holder pays K_{U} when exercising the option to acquire a protection buy position, and receives $U_{\text{CDX}}(t+\tau,t+T)$ when immediately closing out this position by taking an opposite position (i.e., protection sell position) at the market rate. In other words, the payoff of a payer swaption is written as follows:

$$(6)\quad V^{\text{PAY}}_{\text{CDX},\,t+\tau} = \max\left[U_{\text{CDX}}(t+\tau,t+T)-K_{\text{U}},0\right],\ \ \text{provided no defaults by time } t+\tau.$$

In contrast, a receiver CDX swaption is only exercised when the quoted upfront fee at maturity is smaller than the strike because the holder can sell protection for receiving a higher-than-the-fair upfront fee. The payoff of a receiver swaption is expressed as follows:

$$(7)\quad V^{\text{RCV}}_{\text{CDX},\,t+\tau} = \max\left[K_{\text{U}}-U_{\text{CDX}}(t+\tau,t+T),0\right],\ \ \text{provided no defaults by time } t+\tau.$$

It is important to note that the payoffs of CDX swaptions in equations (6) and (7) are derived under the assumption that there were zero defaults by the expiration date of CDX swaptions. What would happen at maturity if some firms in the CDX went into default over the life of CDX swaptions? In such occasions, CDX swaptions provide so-called "front-end" protection. When a payer swaption is exercised, the holder not only obtains the protection buy position on a CDX contract, but also collects the protection payment regarding past defaults from the option writer. When a receiver swaption is exercised, the holder should provide the option writer the protection payment with regard to the previous defaults when receiving the protection sell position.[9] Therefore, even when $U_{\text{CDX}}(t+\tau,t+T)$ is smaller than K_{U}, it is possible that a payer option is exercised and a receiver swaption is not, due to front-end protection.

To be concrete, consider a scenario in which 5 out of the 125 firms (i.e., 4% of the index) went into default before the exercise date. To simplify the example, assume that each firm's bond price after its default was 50% of the par value. This means that the CDX as a whole experienced a loss of $4\% \times (1-0.5) = 2\%$. Without any defaults, the holder of a payer (receiver) CDX swaption would pay (receive) K_{U} dollars and enter into a dollar CDX contract as the protection buyer (seller). However, since 4% of the index was already defaulted in this example, the holder enters into a CDX contract only with a notional amount of 96 cents. In addition, due to the loss caused by the defaulted firms, the holder receives (pays) an immediate compensation, or front-end protection, of 2 cents. We can mathematically formulate these payoffs as follows:

[9]This front-end protection is contingent on options being exercised: If options are not exercised, the protection payment is not made in either type of options.

$$V^{\mathrm{PAY}}_{\mathrm{CDX},\,t+\tau} = \max\left[\left(\frac{N^{\mathrm{s}}_{t+\tau}}{N}\right)U_{\mathrm{CDX}}(t+\tau,t+T)+L_{\mathrm{CDX},t+\tau}-K_{\mathrm{U}},0\right],$$
$$V^{\mathrm{RCV}}_{\mathrm{CDX},\,t+\tau} = \max\left[K_{\mathrm{U}}-\left(\frac{N^{\mathrm{s}}_{t+\tau}}{N}\right)U_{\mathrm{CDX}}(t+\tau,t+T)-L_{\mathrm{CDX},t+\tau},0\right],$$

where $\left(\frac{N^{\mathrm{s}}_{t+\tau}}{N}\right)$ is the fraction of firms that survived by the options' maturity, and $L_{\mathrm{CDX},t+\tau}$ is the cumulative loss of the CDX, consistent with the notation in Section II.B. The 2 equations reduce to equations (6) and (7) when there are no defaults, as $N^{\mathrm{s}}_{t+\tau}=N$ and $L_{\mathrm{CDX},t+\tau}=0$.

These payoff structures clearly indicate that using CDX swaptions, it is possible to replicate payoffs of hypothetical option contracts written on our synthetic corporate bond index. Since equation (5) implies that

$$\left(\frac{N^{\mathrm{s}}_{t+\tau}}{N}\right)U_{\mathrm{CDX}}(t+\tau,t+T)+L_{\mathrm{CDX},t+\tau}=1-P^{(T-\tau)}_{t+\tau},$$

the payoffs of CDX swaptions can be rewritten in terms of the synthetic corporate bond index at time $t+\tau$:

$$V^{\mathrm{PAY}}_{\mathrm{CDX},\,t+\tau} = \max\left[(1-K_{\mathrm{U}})-P^{(T-\tau)}_{t+\tau},0\right], \tag{8}$$

$$V^{\mathrm{RCV}}_{\mathrm{CDX},\,t+\tau} = \max\left[P^{(T-\tau)}_{t+\tau}-(1-K_{\mathrm{U}}),0\right]. \tag{9}$$

Essentially, equation (8) establishes that each payer CDX swaption with strike K_{U} has the same payoff as the European put option on the synthetic corporate bond index with the corresponding strike $K_{\mathrm{P}}=1-K_{\mathrm{U}}$. Since the 2 options generate identical payoffs at maturity, their prices at time t should also be identical under no arbitrage. Similarly, equation (9) demonstrates that the price of each receiver CDX swaption with strike K_{U} should be the same as that of the European call option on the synthetic corporate bond index with the corresponding strike K_{P}. In sum, the prices of CDX swaptions enable us to directly acquire the prices of puts and calls written on an aggregate price index for the corporate bond market, as long as we adjust the strike dimension.

III. Data

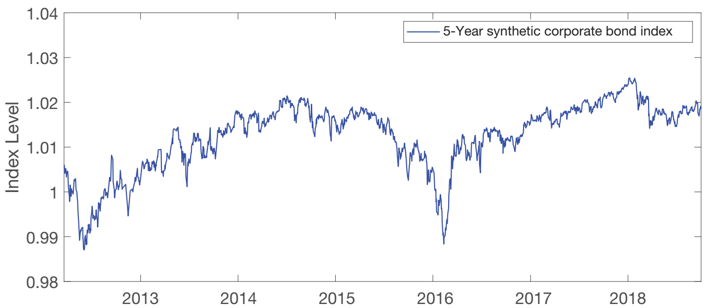
We obtain the daily on-the-run CDX and CDX swaptions data from a major investment bank. Our sample period is from Mar. 2012 to Sept. 2018, as CDX swaptions started actively trading from 2012.[10] The CDX swaptions in our sample are with a 1-month maturity ($\tau=1/12$). Once exercised, option contracts deliver an on-the-run CDX contract that expires approximately 5 years from

[10]For further information about the liquidity and trading volume of CDX swaptions, see Collin-Dufresne, Junge, and Trolle (2021).

FIGURE 1

Synthetic Corporate Bond Index

Figure 1 presents the daily time series of the 5-year constant maturity synthetic corporate bond index from Mar. 2012 to Sept. 2018. The index represents the average price of 125 synthetic floating rate notes, each of which is with a dollar face value.



today ($T=5$).[11] Accordingly, we set the maturity of our synthetic corporate bond to 5 years. Figure 1 plots the time series of the 5-year constant maturity synthetic corporate bond index during our sample period.

In the market, CDX swaptions are quoted in terms of Black-implied volatilities: For each swaption, the volatility term in the Black formula (provided in the Supplementary Material) is backsolved to match its market price, and the resulting volatility is called Black-implied volatility. This is simply the market practice for quoting credit swaptions, which has nothing to do with the validity of the Black model. In the Supplementary Material, we describe the derivation of the Black formula in detail. It is worth noting that the Black model concerns nonstandard credit swaptions. As discussed in Section II.C, when standard credit swaptions are exercised, the holder enters into a standard CDS contract with a 1% fixed coupon spread and instead receives/pays the strike upfront fee K_{U}. In contrast, when nonstandard credit swaptions are exercised, the holder enters into a traditional/nonstandard CDS contract at a given strike coupon spread, say K_{S}, with zero upfront payment. Therefore, under the Black model, the moneyness of credit swaptions is expressed in terms of K_{S}, not K_{U}.

The Black-implied volatilities in our sample are for at-the-money (ATM) and OTM CDX swaptions across a wide range of strike coupon spreads K_{S}. Specifically, for each day, strike spreads are from 65% to 135% of the 1-month forward CDX spread with 5% intervals. Graph A of Figure 2 calculates the average Black-implied volatilities from the data across different moneyness values (in terms of K_{S}). The graph shows a positive volatility skew: The higher the moneyness, the higher the implied volatility. This is consistent with economic intuition. A right-skewed spread distribution corresponds to a left-skewed bond price distribution, indicating a negative volatility skew for our bond index. Graph B of Figure 2 displays the daily time series of the implied volatilities at the 65%, 100% (i.e., ATM forward), and 135% moneyness values.

[11]In fact, the maturity of each on-the-run CDX series experiences small variations from 5.25 years (when it is first introduced) to 4.75 years (when it becomes off-the-run). For computational simplicity, we assume that the maturity of the CDX is always 5 years.

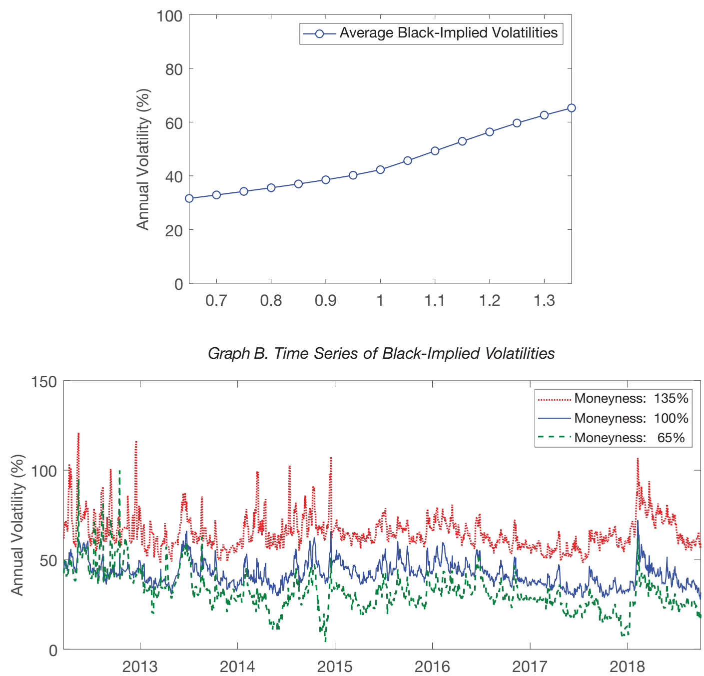
FIGURE 2

Data on CDX Swaptions

Figure 2 displays the pricing data on CDX swaptions. Graph A calculates the average Black-implied volatilities from the data across various moneyness values. Graph B plots the daily time series of the Black-implied volatilities at the 65%, 100%, and 135% moneyness values. The sample period is from Mar. 2012 to Sept. 2018.

Graph A. Average Black-Implied Volatility Skew

Graph B. Time Series of Black-Implied Volatilities



We convert the Black-implied volatilities from the data into CDX swaption prices using the Black formula. We also convert each strike coupon spread K_S into the corresponding strike upfront fee K_U; it is calculated as the CDX upfront fee when the CDX spread is K_S.[12] Through this entire process, for each day, we are able to collect the cross section of CDX swaption prices across various strike upfront fees. To signify that CDX swaption prices depend on the strike upfront fee K_U, we denote the time-t CDX swaption prices as $V_{\mathrm{CDX},t}^{\mathrm{PAY}}(\tau;K_U)$ and $V_{\mathrm{CDX},t}^{\mathrm{RCV}}(\tau;K_U)$. Using the prices of CDX swaptions, we finally obtain the prices of puts and calls written on the synthetic corporate bond index, which are denoted as $V_t^{\mathrm{PUT}}(\tau;K_P)$ and $V_t^{\mathrm{CALL}}(\tau;K_P)$. As we discuss in Section II.C, they can be found from CDX swaption prices based on the following mapping:

$$\begin{aligned} V_t^{\mathrm{PUT}}(\tau;K_P) &= V_{\mathrm{CDX},t}^{\mathrm{PAY}}(\tau;K_U), \\ V_t^{\mathrm{CALL}}(\tau;K_P) &= V_{\mathrm{CDX},t}^{\mathrm{RCV}}(\tau;K_U), \text{ where the strike price } K_P = 1 - K_U. \end{aligned}$$

[12]To be concrete, $K_U = [K_S - 0.01] \times \Pi(K_S)$, where we define $\Pi(K_S)$, by slight abuse of notation, as the risky PV01 when the CDX term structure is flat at K_S.

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IV. Option-Implied Risk in the Corporate Bond Market

Equipped with the prices of the synthetic options constructed in Section II, we estimate the forward-looking conditional moments of the synthetic corporate bond index. This allows us to study aggregate risk in the corporate bond market in a model-free fashion. We first examine the role of variance risk in the corporate bond market by creating an aggregate corporate bond volatility index and estimating the associated variance risk premium (Section IV.A). We also consider higher-order moments such as risk-neutral conditional skewness and kurtosis (Section IV.B). Taking a further step, we construct the physical and risk-neutral densities of our corporate bond index and examine their implications for the pricing kernel (Section IV.C).

A. Variance Risk

1. Corporate Bond VIX

We create a model-free volatility index for the corporate bond market, which we call the CBVIX. Similar to the "equity" VIX, the CBVIX measures the risk-neutral expectation of future 1-month volatility on the 5-year synthetic corporate bond index. For the remainder of this article, we assume that $\tau = 1/12$ and $T = 5$. Since there is no ambiguity, we drop the superscripts from $P_t^{(T)}$ and $P_{t+\tau}^{(T-\tau)}$ and denote them as P_t and $P_{t+\tau}$ to simplify the notation. For the sake of brevity, the term "bond index" refers to our 5-year synthetic corporate bond index.

As the first step, we calculate the risk-neutral expectation of the realized log bond index variance over the next month:

$$E_t^{\mathbb{Q}}[RV_{t\to t+\tau}] = E_t^{\mathbb{Q}}\left[\int_t^{t+\tau} d[\log P]_u\right],$$

where $\mathbb{Q}$ represents the risk-neutral measure and $[\log P]_u$ refers to the quadratic variation of the log bond index up to time u. We calculate this risk-neutral expectation using calls and puts under the assumption that the bond index follows an Ito process, as in the case of the equity VIX.[13] In Appendix B, using the general spanning formula in Bakshi and Madan (2000) and Carr and Madan (2001), we show that the risk-neutral expectation of the bond index variance is expressed as follows:

$$\text{(10)} \qquad E_t^{\mathbb{Q}}[RV_{t\to t+\tau}] = 2e^{r_f\tau}\left(\int_0^{F_t} \frac{V_t^{\text{PUT}}(\tau;K)}{K^2}dK + \int_{F_t}^{\infty} \frac{V_t^{\text{CALL}}(\tau;K)}{K^2}dK\right),$$

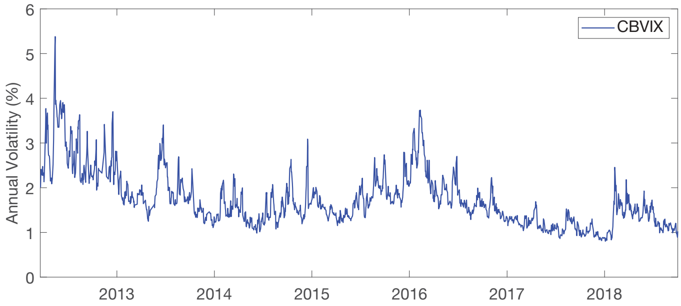
where r_f is the risk-free rate and F_t is the τ-maturity forward price of the bond index. Since this quantity is calculated using options, it is often referred to as implied variance. We also use this term throughout the article. Following other volatility indices, the CBVIX is expressed in annualized percentage volatility:

[13] An extensive literature discusses the calculation of the VIX. Examples include, but are not limited to, Dupire (1994), Neuberger (1994), Carr and Madan (1998), Britten-Jones and Neuberger (2000), Jiang and Tian (2005), and Carr and Wu (2006).

FIGURE 3

Corporate Bond VIX

Figure 3 presents the daily time series of the CBVIX from Mar. 2012 to Sept. 2018. At each point in time, the CBVIX represents the risk-neutral expectation of future 1-month volatility on the 5-year synthetic corporate bond index.



$$\text{CBVIX}_t = 100 \times \sqrt{\frac{1}{\tau} E_t^{\mathbb{Q}}[RV_{t \to t+\tau}]}.$$

Figure 3 plots the resulting time series of the CBVIX during our sample period. The average CBVIX is at 1.72%. Considering that the average ATM Black-implied volatility for CDX swaptions is 42% (see Figure 2), this may sound too small at first glance. However, a simple back-of-the-envelope calculation suggests that the level of the CBVIX is sensible. With the average CDX spread of 97 basis points, the 42% Black-implied volatility (i.e., 42% spread volatility) corresponds to $97 \times 0.42 = 40$ basis point fluctuations in the CDX spread on average. This, in turn, corresponds to less than $5 \times 40/10{,}000 = 2\%$ fluctuations in the bond index because 5-year coupon bonds have a duration shorter than 5 years. In fact, the small magnitude of the CBVIX is not surprising for two reasons. First, the CBVIX is based on corporate bonds with floating rate coupons, which are immune to interest rate fluctuations unlike fixed-coupon bonds. Second, the CBVIX measures the bond index volatility, which is much smaller than individual bond volatilities due to diversification.

Nevertheless, we can see from Figure 3 that the CBVIX has significant time variations. It fluctuates from 0.80% to 5.38%, depending on the market condition. The beginning of the sample corresponds to the post Great Recession period. During this period, the CBVIX maintained a high level, reflecting fears of the eurozone debt crisis. The CBVIX started to decline as uncertainty reduced after the European Central Bank took an aggressive measure to support eurozone countries. The reduced level of the CBVIX started to rise again from mid-2014. Besides the resurfacing of the Greek government debt issue, China's economic slowdown resulted in the turmoil of global financial markets, and, as a result, the CBVIX increased to almost 4% in Feb. 2016. Yet, the steady recovery of the U.S. economy stabilized the financial market and pushed the CBVIX to a lower level during 2016 and 2017. Despite minor spikes due to events such as Brexit in June 2016 and the presidential election in Nov. 2016, the CBVIX reached 1% by the end of 2017. Although the CBVIX temporarily went up as high as 2.5% in Feb. 2018 when high

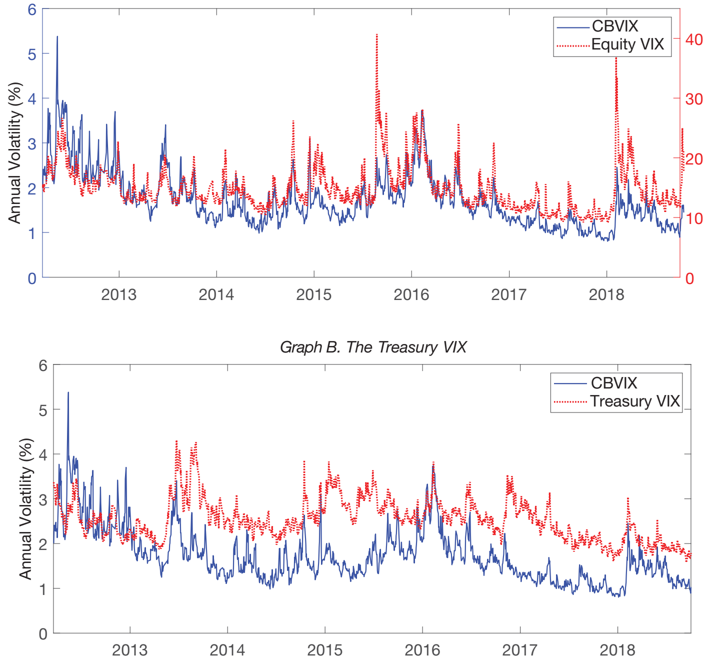
FIGURE 4

Comparison with Other Volatility Indices

Figure 4 contains side-by-side comparisons of the time series of the CBVIX with that of the equity VIX and that of the Treasury VIX, from Mar. 2012 to Sept. 2018. The solid blue lines represent the CBVIX, and the dotted red lines represent the equity VIX (Graph A) and the Treasury VIX (Graph B). When plotted, the original Treasury VIX is scaled by 2 so that the Treasury VIX and the CBVIX are based on bonds that have roughly the same duration. All time series are in terms of annual percentage volatility.

Graph A. The Equity (S&P 500) VIX

Graph B. The Treasury VIX



inflation concerns caused the stock market to plummet, it quickly returned to a lower level between 1% and 2%.

By comparing Figure 3 with Figure 1, we can clearly see that the level of the index negatively comoves with the CBVIX. In the data, the 2 time series exhibit a negative correlation of −0.89. This implies that a positive shock to the CBVIX tends to be associated with a negative shock to the aggregate bond price, creating asymmetric volatility.[14] This finding can be explained as the volatility feedback effect, which is well documented in the stock market.[15] If volatility risk is priced, a higher level of volatility induces investors to demand a higher premium for holding corporate bonds, which leads to lower corporate bond prices.

Graph A of Figure 4 compares the time series of the CBVIX with that of the equity VIX. We can see that the CBVIX has a much smaller magnitude compared to

[14]Specifically, regressing excess corporate bond returns on contemporaneous changes in the CBVIX results in a negative slope coefficient with high significance. This result is robust, controlling for changes in the equity VIX and the Treasury VIX as well as for excess equity market returns and excess long-term Treasury returns.

[15]See French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), Bekaert and Wu (2000), and Bollerslev, Litvinova, and Tauchen (2006) for more details.

the equity VIX. During our sample period, the average equity VIX is 14.86%, which is roughly 9 times larger than the average CBVIX. This is intuitive because the bond is a senior claim on a firm's income or asset, whereas the equity is a residual claim. Thus, when shocks to the firm arrive, the bond should respond to them with less sensitivity. Despite a substantial difference in their levels, the CBVIX and the equity VIX show fairly similar patterns: They fluctuate in a consistent manner, peaking or dipping around the same times. In line with this, these 2 volatility indices are correlated at 0.71. Such a strong association between the 2 volatility indices is expected through the lens of a structural credit risk model where both bond and equity dynamics originate from the same source: the firm's asset dynamics. In Section V, we further investigate the consistency between the CBVIX and the equity VIX with the aid of such a model.

In Graph B of Figure 4, we also plot the time series of the Treasury VIX together with the CBVIX. The Treasury VIX reflects the expected volatility of 10-year Treasury note future prices under the risk-neutral measure. To facilitate comparison with the CBVIX, which is based on a 5-year bond, we first divide the Treasury VIX by 2. This allows us to roughly approximate the Treasury VIX that corresponds to a 5-year maturity because the duration of 5-year Treasury notes is roughly half of the duration of 10-year Treasury notes.[16] In the beginning of the sample, the scaled Treasury VIX exhibits a lower level and a lower variability compared to the CBVIX. Although the eurozone debt crisis heavily influenced the corporate bond market, it had a limited impact on the Treasury market because U.S. government bonds were regarded as the safest asset. However, the Treasury VIX doubled in June 2013, when Ben Bernanke alluded that the Federal Reserve may reduce the size of quantitative easing policies. From this point in time, the Treasury VIX consistently showed a higher magnitude than the CBVIX, reflecting higher uncertainty regarding "tapering" and postcrisis monetary policies.

The average scaled Treasury VIX is 2.54% in our sample. That is, compared with the equity VIX, the Treasury VIX has a level much closer to the CBVIX. This is intuitive because both the CBVIX and the Treasury VIX are based on bonds whose volatilities are much smaller than stocks. However, in terms of patterns, the CBVIX has a weaker correlation with the Treasury VIX (0.44) compared to that with the equity VIX. Considering that the CBVIX measures the future volatility of FRNs, which are insensitive to interest rate risk, such a small correlation is reasonable because Treasury notes are sensitive to interest rate risk alone. Nonetheless, the 2 volatility indices are still correlated because the level of interest rates is directly and indirectly associated with the level of credit risk in the economy.

2. Realized Variance Measure

Whereas the implied variance is estimated using options, the realized variance is estimated using the realized time series of the bond index. Specifically, we construct the monthly realized variance measure $\widehat{RV}_{t\rightarrow t+\tau}$ between times t and

[16]The (modified) duration is a price sensitivity measure with respect to parallel shifts in the yield curve. Thus, if the duration is half, the price volatility should also be approximately half. Of course, this approximation ignores second- and higher-order effects of parallel shifts as well as the effects of nonparallel shifts, such as changes in the slope and curvature of the yield curve.

$t+\tau$ by tracking the daily levels of the bond index whose maturity starts out as 5 years at time t. To simplify our computation, we assume that each monthly horizon corresponds to 22 trading days, and that each interval between 2 adjacent trading days is $\delta = \tau/22$. Then, following French et al. (1987), our realized variance measure is estimated as the sum of squared daily log price relatives of the bond index, adjusted for the first-order autocorrelation:[17]

$$\widehat{RV}_{t\to t+\tau} = \sum_{n=1}^{22} p_{t+\delta n}^2 + 2\sum_{n=1}^{21} p_{t+\delta n}\cdot p_{t+\delta(n+1)},$$

where $p_{t+\delta n} = \log\left(\frac{P_{t+\delta n}}{P_{t+\delta(n-1)}}\right)$.

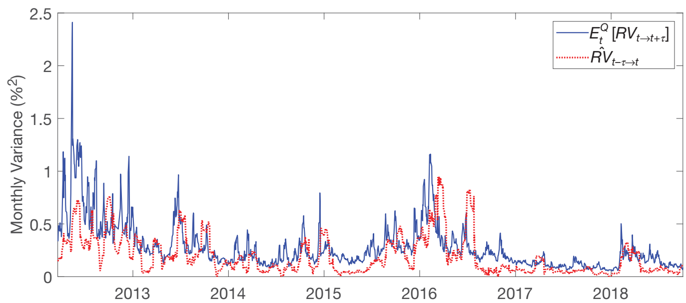
Figure 5 shows the daily time series of our realized variance measure together with the model-free implied variance. Note that the value of $RV_{t\to t+\tau}$ is displayed at time $t+\tau$ (not time t) because this realized variance is observed at time $t+\tau$. That is, the value plotted at time t is $RV_{t-\tau\to t}$, the realized variance from time $t-\tau$ up to time t. We multiply the 2 time series by 10^4 in order to express them in monthly percentage squared terms, consistent with the literature.

Figure 5 reveals that the time series of the realized variance has a fairly similar pattern compared to the implied variance. More importantly, we can see that the level of the implied variance is considerably higher than the realized variance for the majority of our sample period. The daily average of the implied variance in our sample is 0.28, whereas that of the realized variance is 0.19. To gauge how large this difference is, we express the 2 variables in annual percentage volatility terms: The average implied volatility (i.e., CBVIX) is 1.72%, whereas the average realized volatility is 1.35%. In other words, the implied volatility of the bond index is 30%

FIGURE 5

Implied and Realized Variance Measures

Figure 5 plots the time series of the model-free implied variance measure (the solid blue line) and that of the realized variance measure (the dotted red line), from Mar. 2012 to Sept. 2018. The implied variance measure is estimated using call and put options on the synthetic corporate bond index. The realized variance measure is estimated from daily log price relatives of the synthetic corporate bond index, following French et al. (1987). Both time series are expressed in monthly percentage squared terms.



[17]In the case of the stock market, it is well known that the realized variance can be accurately estimated using high-frequency (typically, 5-minute) time series (e.g., Andersen, Bollerslev, Diebold, and Labys (2003), Andersen, Fusari, and Todorov (2015)). Unfortunately, high-frequency data on the CDX are not available.

higher than its realized counterpart, on average. The large gap between the implied variance and the realized variance suggests that variance risk is significantly priced in the corporate bond market.

3. Variance Risk Premium

To quantitatively assess the importance of variance risk in the corporate bond market, we construct the monthly time series of the corporate bond variance risk premium, which is defined as the difference between the risk-neutral and physical expectations of future bond index variance:

$$\text{VRP}_t = E_t^{\mathbb{Q}}[RV_{t\to t+\tau}] - E_t^{\mathbb{P}}[RV_{t\to t+\tau}]. \quad (11)$$

The monthly time series of the risk-neutral expectation in equation (11) is simply obtained by extracting the implied variance series on the last day of each month. In contrast, constructing the monthly time series of the physical expectation is subject to various approaches. For example, Bollerslev et al. (2009) assume a unit-root process for the realized variance so that the previous month's realized variance serves as a proxy for the physical expectation of the upcoming month's realized variance. Drechsler and Yaron (2011) run a linear forecast model where the realized variance is projected onto the lagged realized variance and the lagged implied variance. Finally, Zhou (2018) additionally suggests methods based on a moving average and on an autoregressive model. In our article, we estimate the physical expectation in equation (11) as the exponentially weighted average of the last 12 monthly realized variances. As Zhou (2018) discusses, this smoothing method is simple as it does not require parameter estimation.

Table 1 contains the summary statistics for the variables used in our empirical analysis. The first 3 variables are the variance risk premium, implied variance, and realized variance in the corporate bond market. The next 3 variables are the corresponding variables in the equity market.[18] The last 2 variables are the monthly percentage excess log returns on the bond index and on the CRSP value-weighted index.[19] The resulting sample consists of 79 months from Mar. 2012 to Sept. 2018. In the Supplementary Material, we provide a further description of these variables.

Table 1 shows that the corporate bond variance risk premium is positive on average and is substantially time-varying. Furthermore, it has negative contemporaneous correlations with bond and equity market returns. These characteristics are similar to those of the equity variance risk premium. Given that the literature finds that the equity variance risk premium predicts future equity returns, a natural question that follows is: Can the corporate bond variance risk premium predict future bond returns? Moreover, is it also capable of predicting future equity returns? What is the joint predictability of the two variance risk premium measures?

Although our sample is relatively short with 79 monthly time series, it is still possible to run predictability regressions to address the previously mentioned questions. First, we investigate the predictability of 1-month ahead bond market

[18]We obtain these 3 variables from Hao Zhou's web page.

[19]Returns on the synthetic corporate bond index include not only capital gains/losses but also coupon payments.

TABLE 1

Descriptive Statistics for Predictability Regressions

Table 1 reports descriptive statistics for the variables used in our empirical analysis. The first 3 variables pertain to the corporate bond market: namely, the corporate bond variance risk premium (CB VRP), the corporate bond implied variance (CB IV), and the corporate bond realized variance (CB RV). The next 3 variables are the corresponding variables in the equity market: the equity variance risk premium (Equity VRP), the equity implied variance (Equity IV), and the equity realized variance (Equity RV). The last 2 variables are the monthly percentage excess log returns on the synthetic corporate bond index ($\log R_B^e$) and the monthly percentage excess log returns on the CRSP value-weighted index ($\log R_E^e$). Panel A lists the summary statistics, and Panel B lists the correlations.

Panel A. Summary Statistics

	N	Mean	Median	Std. Dev.	Skew.	Kurt.	AR(1)
CB VRP	79	0.03	0.00	0.12	1.80	7.86	0.24
CB IV	79	0.26	0.21	0.17	2.02	9.51	0.59
CB RV	79	0.19	0.13	0.19	1.63	5.03	0.26
Equity VRP	79	9.59	9.01	6.36	−0.14	4.73	−0.09
Equity IV	79	20.06	16.78	9.99	2.03	8.98	0.39
Equity RV	79	10.47	7.08	10.50	3.38	17.73	0.38
$\log R_B^e$ (%)	79	0.12	0.13	0.36	0.10	6.15	−0.13
$\log R_E^e$ (%)	79	1.12	1.17	2.83	−0.42	3.29	−0.16

Panel B. Correlations

	CB VRP	CB IV	CB RV	Equity VRP	Equity IV	Equity RV	$\log R_B^e$
CB VRP	–						
CB IV	0.68	–					
CB RV	0.24	0.61	–				
Equity VRP	0.27	0.39	0.02	–			
Equity IV	0.71	0.70	0.42	0.24	–		
Equity RV	0.52	0.43	0.39	−0.38	0.81	–	
$\log R_B^e$ (%)	−0.66	−0.36	−0.04	−0.26	−0.53	−0.34	–
$\log R_E^e$ (%)	−0.60	−0.41	−0.14	−0.16	−0.67	−0.54	0.77

returns based on the corporate bond variance risk premium as well as other variance-related variables, as shown in Table 2.[20] From column 1, we can see that higher levels of the corporate bond variance risk premium significantly predict higher future returns over the next month. The slope coefficient is 0.79 with a *t*-statistic of 2.86. In other words, a 1-standard-deviation increase in the corporate bond variance risk premium (i.e., 0.12) leads to a roughly $0.12 \times 0.79 = 0.1\%$ point higher bond return the next month. This regression generates a sizable adjusted R^2 over 6%.

Moreover, higher levels of the corporate bond implied variance and realized variance also predict larger future bond returns, as can be seen in columns 2 and 3 of Table 2. The coefficients are roughly similar in size and are highly significant. In the case of the corporate bond implied variance, it generates an even higher adjusted R^2 of 9.42%. The fact that both the implied and realized variances predict future returns is unique to the bond market. In the equity market, as Bollerslev et al. (2009) document, the variance risk premium predicts future equity returns, but the implied and realized variances do not, which we also confirm in our sample. The fact that all 3 variance-related variables in the corporate bond market predict future bond returns suggests that variance risk is an important source of risk that drives bond returns.

Column 4 of Table 2 indicates that the equity variance risk premium predicts future bond returns, despite the significance level only being marginal (*t*-stat

[20]Note that all standard errors in Tables 2 and 3 are corrected according to Newey and West (1987) with 4 lags.

TABLE 2
Bond Return Predictability

Table 2 presents the results of predictability regressions of 1-month ahead bond market returns. Predictor variables include variance-related variables in the corporate bond market (the corporate bond variance risk premium, the corporate bond implied variance, and the corporate bond realized variance) and the corresponding variables in the equity market (the equity variance risk premium, the equity implied variance, and the equity realized variance). The dependent variable is the monthly percentage excess log returns on the synthetic corporate bond index. All standard errors are Newey–West corrected with 4 lags.

	Excess Return on the Synthetic Corporate Bond Index							
	1	2	3	4	5	6	7	8
Intercept	0.09 (2.26)	−0.06 (−1.03)	0.02 (0.45)	0.02 (0.26)	0.03 (0.43)	0.12 (3.03)	0.03 (0.36)	−0.08 (−1.01)
CB VRP	0.79 (2.86)						0.71 (2.03)	
CB IV		0.68 (3.39)						0.63 (2.56)
CB RV			0.51 (2.98)					
Equity VRP				0.01 (1.68)			0.01 (0.86)	0.00 (0.46)
Equity IV					0.00 (1.02)			
Equity RV						0.00 (0.03)		
Adj. R^2 (%)	6.33	9.42	6.19	1.94	0.11	−1.31	6.33	8.55

of 1.68). The slope coefficient is 0.01, implying that a 1-standard-deviation increase in the equity variance risk premium (i.e., 6.36) leads to a $0.01 \times 6.36 = 0.06\%$ higher bond return the next month, approximately half compared to the corporate bond variance risk premium. This predictive regression has an adjusted R^2 of around 2%, which is also smaller than that of the corporate bond variance risk premium. The equity implied and realized variances do not predict future bond returns: Their coefficients are insignificant, as shown in columns 5 and 6.

We also run multiple regressions to assess the joint predictability of our variables of interest. In column 7 of Table 2, we enter the corporate bond variance risk premium and the equity variance risk premium into the same regression.[21] The result shows that the 2 slope coefficients are both positive. However, the corporate bond variance risk premium remains significant with a t-statistic of 2.03, whereas the equity variance risk premium becomes insignificant with a t-statistic of 0.86. Comparing this regression with the simple regression solely based on the corporate bond variance risk premium (column 1), the addition of the equity variance risk premium does not alter the adjusted R^2. This shows that although the equity variance risk premium itself predicts bond returns, when the corporate bond variance risk premium is present, it does not add any extra predictive power. We can observe a similar pattern in column 8 when the corporate bond implied variance and the equity variance risk premium are entered into the same regression.

Now, we turn to the predictability of equity returns. Consistent with prior studies, we find that higher levels of the equity variance risk premium predict higher

[21]We find that the predictability power of the corporate bond variance risk premium is robust to controlling for other bond return predictors such as the default spread.

TABLE 3

Equity Return Predictability

Table 3 presents the results of predictability regressions of 1-month ahead equity returns. Predictor variables include variance-related variables in the corporate bond market (the corporate bond variance risk premium, the corporate bond implied variance, and the corporate bond realized variance) and the corresponding variables in the equity market (the equity variance risk premium, the equity implied variance, and the equity realized variance). The dependent variable is the monthly percentage excess log returns on the CRSP value-weighted index. All standard errors are Newey–West corrected with 4 lags.

	Excess Return on the CRSP Value-Weighted Index							
	1	2	3	4	5	6	7	8
Intercept	0.92	0.21	0.98	0.00	0.46	1.23	0.07	−0.33
	(2.96)	(0.45)	(2.71)	(0.00)	(0.84)	(4.25)	(0.11)	(−0.53)
CB VRP	5.35						4.10	
	(3.52)						(2.31)	
CB IV		3.37						2.05
		(2.39)						(1.17)
CB RV			0.56					
			(0.40)					
Equity VRP				0.11			0.09	0.09
				(2.35)			(1.71)	(1.74)
Equity IV					0.03			
					(1.19)			
Equity RV						−0.01		
						(−0.62)		
Adj. R^2 (%)	4.33	2.99	−1.17	5.33	−0.06	−1.06	7.19	5.44

equity returns the next month, whereas higher levels of the implied and realized variances do not. This is summarized in columns 4–6 of Table 3. What we newly discover and add to the literature is that the corporate bond variance risk premium also predicts future equity returns. For instance, column 1 shows that the corporate bond variance risk premium has a positive slope coefficient of 5.35. This implies that a 1-standard-deviation increase in the corporate bond variance risk premium leads to a $5.35 \times 0.12 = 0.64\%$ point higher equity return the next month. The adjusted R^2 is fairly high at 4.33%. While the corporate bond implied variance also predicts future equity returns (column 2), the corporate bond realized variance does not (column 3), with a t-statistic of only 0.40.

We examine the equity return predictability based on the corporate bond variance risk premium or the corporate bond implied variance, controlling for the equity variance risk premium. In column 8 of Table 3, when the corporate bond implied variance is entered into the same regression with the equity variance risk premium, it becomes insignificant. This is the exact opposite of what we observed with the bond return predictability: The equity variance risk premium is driven out by the corporate bond implied variance when predicting future bond returns. In contrast, as can be seen in column 7, when we put the two variance risk premium measures from both markets into the same regression, they both remain statistically significant (with the corporate bond variance risk premium at 5% and the equity variance risk premium at 10%). The 2 variables jointly generate high predictive power with an adjusted R^2 of 7.19%.

In sum, Tables 2 and 3 consistently find that the corporate bond variance risk premium positively predicts future returns in both markets. This result signifies that variance risk in the corporate bond market captures an important source of systematic risk, shared not only by the bond market but also by the equity market.

B. Higher-Order Risk

Having studied variance risk in the corporate bond market, the natural next step is to examine the significance of higher-order moments, such as skewness and kurtosis. Again, we exploit the general spanning formula to estimate risk-neutral conditional higher-order moments using option prices on the bond index. In Appendix B, we show that the nth order noncentral moment of the future price relative is determined as follows:

$$\begin{aligned} m_{n,t} &= E_t^{\mathbb{Q}}\left[\left(\frac{P_{t+\tau}}{P_t}\right)^n\right] \\ &= \left(\frac{F_t}{P_t}\right)^n + \frac{n(n-1)e^{r_f\tau}}{P_t^n}\left(\int_0^{F_t} V_t^{\text{PUT}}(\tau;K)K^{n-2}dK + \int_{F_t}^{\infty} V_t^{\text{CALL}}(\tau;K)K^{n-2}dK\right). \end{aligned}$$

Note that this formula is distinct from the one in Bakshi, Kapadia, and Madan (2003): Whereas Bakshi, Kapadia, and Madan (2003) consider the moments of the log price relative, we consider the simple price relative. For the detailed derivation, refer to Appendix B. Once we estimate the first 4 noncentral moments of the price relative based on the equation previously mentioned, the risk-neutral conditional skewness and kurtosis are calculated as follows:

$$\begin{aligned} \text{SKEW}_t\left(\frac{P_{t+\tau}}{P_t}\right) &= \left[m_{3,t} - 3m_{1,t}m_{2,t} + 2m_{1,t}^3\right] / \left[m_{2,t} - m_{1,t}^2\right]^{\frac{3}{2}}, \\ \text{KURT}_t\left(\frac{P_{t+\tau}}{P_t}\right) &= \left[m_{4,t} - 4m_{1,t}m_{3,t} + 6m_{1,t}^2 m_{2,t} - 3m_{4,t}\right] / \left[m_{2,t} - m_{1,t}^2\right]^2. \end{aligned}$$

In Figure 6, the solid blue lines represent the time series of the risk-neutral conditional return skewness (Graph A) and excess kurtosis (Graph B). For comparison, the dotted red lines plot the corresponding time series, calculated under the lognormal assumption. If the conditional distribution indeed follows a lognormal distribution, the solid blue lines and the dotted red lines should be close.

As can be seen in Figure 6, the lognormal assumption generates nearly zero (but slightly positive) skewness and excess kurtosis. This is clearly not the case in the data. First of all, the actual skewness values are substantially negative over the entire sample. The conditional skewness is, on average, −1.76, fluctuating between −3.94 and −0.42. Furthermore, the actual excess kurtosis values far exceed 0 with a mean of 5.41. Especially in the early part of the sample, the risk-neutral excess kurtosis goes even beyond 15, implying an extremely fat-tailed distribution. In sum, we can conclude that risk-neutral conditional distributions of the price relative are highly skewed to the left and are heavily fat-tailed, which set them far apart from a lognormal distribution.

C. Implied Pricing Kernel

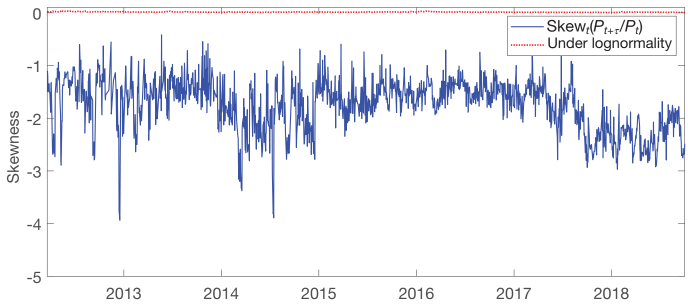
Going even further, we now turn to distributions of the price relative, which are affected not just by the first 4 moments, but by all moments. We follow Aït-Sahalia and Duarte (2003) to nonparametrically estimate densities of the price relative.

FIGURE 6

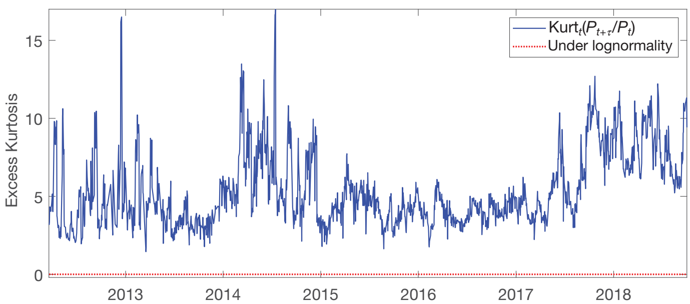
Higher Moments of the Synthetic Corporate Bond Index

Figure 6 depicts the higher moments of the price relative of the synthetic corporate bond index, from Mar. 2012 to Sept. 2018. The solid blue lines represent the time series of the 1-month risk-neutral conditional skewness (Graph A) and excess kurtosis (Graph B). The dotted red lines represent the corresponding time series of skewness and kurtosis under lognormality.

Graph A. Conditional Skewness



Graph B. Conditional Kurtosis



Specifically, we first solve the optimization problem under conic constraints, as proposed in Dykstra (1983), to ensure that option prices are arbitrage-free across different strike values.[22] As Breeden and Litzenberger (1978) and other subsequent studies show, the risk-neutral density can be calculated as the second-order partial derivative of the call price with respect to the strike. Since we are interested in the price relative $P_{t+\tau}/P_t$ rather than the future index level $P_{t+\tau}$, we normalize the price of each call option and its strike by the current index level P_t. Then, the density of the price relative is calculated as follows:

$$f^{\mathbb{Q}}(m) = e^{r_f \tau} \frac{\partial^2 V_n^{\text{CALL}}(\tau;m)}{\partial m^2}, \tag{12}$$

where $V_n^{\text{CALL}}(\tau;m)$ is the normalized price of a τ-maturity call option whose moneyness is m. The second-order partial derivative in equation (12) is estimated by running a locally linear regression.[23]

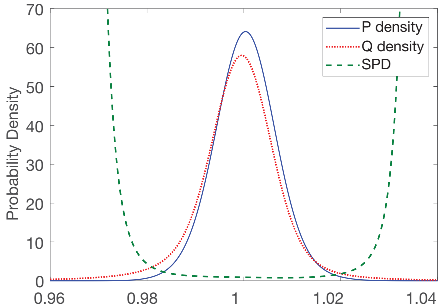
[22]For the details of the algorithm, see Dykstra (1983) and Aït-Sahalia and Duarte (2003).

[23]The locally linear estimator of the second-order partial derivative is provided in Appendix B of Aït-Sahalia and Duarte (2003).

FIGURE 7

Risk-Neutral, Physical, and State-Price Densities

Figure 7 shows the results from our nonparametric density estimations. The solid blue line and the dotted red line indicate the physical and risk-neutral density functions of the 1-month price relative of the synthetic corporate bond index, respectively. Both density functions are estimated with the normal kernel and a bandwidth of 0.50%. The dashed green line represents the state-price density, which is the ratio between these 2 functions.



Whereas the approach of Aït-Sahalia and Duarte (2003) can be used to generate conditional densities by only choosing options from certain dates, our main objective is to obtain the unconditional, or average, density over our sample period. We therefore apply their estimation methodology to average normalized option prices.[24] The dotted red line in Figure 7 shows the resulting risk-neutral density function with the normal kernel and a bandwidth of 0.50%.

We also estimate the density of the price relative under the physical measure. Using the daily time series of the price relative, which is calculated according to equations (4) and (5), we run kernel density estimation. The solid blue line in Figure 7 shows the resulting physical density function based on the normal kernel with a bandwidth of 0.50%.

Finally, we estimate the implied pricing kernel (or state-price density) projected on the 1-month future price relative as the ratio between the risk-neutral and physical densities. We overlay the resulting pricing kernel with a dashed green line in Figure 7.

Comparing the solid blue line and the dotted red line, it is apparent that the risk-neutral distribution has a larger variance. Furthermore, it is more skewed to the left and exhibits fatter tails on both sides, compared to the physical distribution. More importantly, as the future price moves away from its mode, the physical density converges to 0 much faster than the risk-neutral density does, which makes the ratio between the risk-neutral and physical densities sharply rise in both directions. As a result, we can observe that the state price density is U-shaped.

Prior studies such as Rosenberg and Engle (2002) consistently document a nonmonotonic or U-shaped pricing kernel, when projected on the future stock

[24]Specifically, for each moneyness, we take the time-series average of normalized option prices. Since moneyness values of available options do not coincide every day, we use cubic spline interpolation. Note that aside from using average option prices, an alternative way to obtain the unconditional distribution is to run the estimation based on all available options during our sample period. This is computationally very challenging due to the conic-constrained optimization problem.

market return.[25] This finding is not limited to the stock market. Li and Zhao (2009), Song and Xiu (2016), and Christoffersen, Jacobs, and Pan (2022) discover similar patterns of the pricing kernel using options on interest rates, the VIX, and crude oil prices, respectively. Using options on the synthetic corporate bond index, we confirm that this pattern also exists in the corporate bond market.

V. Implications for Credit Risk Models

The ex ante risk measures we estimate from synthetic options not only help understand the behavior of the corporate bond market, but also provide important implications for credit risk models. So far, the validity of these models mainly relied on their abilities to match the level of credit spreads in the data. The conditional second and higher moments constructed in our article can serve as extra grounds for assessing the dynamics of credit risk models.

In line with this, of particular interest is a structural credit risk model (e.g., Merton (1974)), in which a default event occurs when the firm's asset value falls below a certain threshold. The firm's floating-coupon bond is a senior claim whose value is entirely driven by the firm's credit risk, and the equity is a residual claim whose payoff resembles a call option on the firm's asset. Therefore, in such a model, both bond and equity dynamics endogenously arise from the firm's asset dynamics.

Based on this insight, we examine the relation between the CBVIX and the equity VIX using a simple structural credit risk model. Section IV.A established that the CBVIX and the equity VIX are highly correlated. Can a model then tell us something about their relative magnitude? For each firm $i \in \{1, \ldots, N\}$, let $A_{i,t}$ denote the firm's asset dynamics, which follow a Geometric Brownian motion:

$$\frac{dA_{i,t}}{A_{i,t}} = \mu_i^A dt + \sigma_i \left[\rho_i dB_{m,t} + \sqrt{1-\rho_i^2} dB_{i,t} \right],$$

where σ_i represents the asset volatility. The firm's diffusive risk decomposes into the systematic Brownian motion $B_{m,t}$ and the idiosyncratic Brownian motion $B_{i,t}$. The coefficient ρ_i captures the loading on the systematic component. All Brownian motions are independent of one another.

Under this structural setup, the equity price becomes a function of the asset: $E_{i,t} = E_i(A_{i,t})$. Similarly, the floating-coupon bond price $P_{i,t}$ is expressed as $P_i(A_{i,t})$. Then, it follows from Ito's lemma that

$$\frac{dE_{i,t}}{E_{i,t}} = \mu_i^E dt + E_i'(A_{i,t}) \frac{A_{i,t}}{E_{i,t}} \sigma_i \left[\rho_i dB_{m,t} + \sqrt{1-\rho_i^2} dB_{i,t} \right],$$
$$\frac{dP_{i,t}}{P_{i,t}} = \mu_i^P dt + P_i'(A_{i,t}) \frac{A_{i,t}}{P_{i,t}} \sigma_i \left[\rho_i dB_{m,t} + \sqrt{1-\rho_i^2} dB_{i,t} \right].$$

[25]There is an extensive literature that examines the properties of the pricing kernel estimated based on equity index options. Examples include Aït-Sahalia and Lo (1998), Jackwerth (2000), Chabi-Yo, Garcia, and Renault (2007), Bakshi, Madan, and Panayotov (2010), Chabi-Yo (2012), and Christoffersen, Heston, and Jacobs (2013).

These equations imply that the firm's equity and bond volatilities are $\sigma^E_{i,t} = E'_i(A_{i,t})\frac{A_{i,t}}{E_{i,t}}\sigma_i$ and $\sigma^P_{i,t} = P'_i(A_{i,t})\frac{A_{i,t}}{P_{i,t}}\sigma_i$, respectively. Note that the two firm-level volatilities are unaffected by ρ_i. Individual firm securities are affected by the total risk, regardless of whether it is from a systematic or idiosyncratic source.

However, when it comes to the volatilities of the equity and bond indices, ρ_i becomes relevant due to diversification. For parsimony, we consider a homogeneous pool of ex ante identical firms. If the number of firms in the index is large (i.e., $N \to \infty$), we can show that the equity and bond index volatilities are given by

$$\sigma^E_{\text{INDEX},t} = E'_i(A_{i,t})\frac{A_{i,t}}{E_{i,t}}\sigma_i\rho_i \quad \text{and} \quad \sigma^P_{\text{INDEX},t} = P'_i(A_{i,t})\frac{A_{i,t}}{P_{i,t}}\sigma_i\rho_i.$$

Although the equity and bond index volatilities depend on ρ_i, the previously mentioned expressions indicate that their ratio does not. In fact, taking this ratio cancels out the direct effects from the asset dynamics and results in:

$$\text{(13)} \qquad \frac{\sigma^E_{\text{INDEX},t}}{\sigma^P_{\text{INDEX},t}} = \frac{E'_i(A_{i,t})}{P'_i(A_{i,t})}\frac{P_{i,t}}{E_{i,t}} = \frac{dE_{i,t}}{dP_{i,t}}\frac{P_{i,t}}{E_{i,t}}.$$

That is, under our model, the ratio between the two index volatilities should equal the elasticity of the equity price with respect to the bond price.[26]

Equation (13) provides an implication that is directly testable. This is because both sides of the equation are measurable. The data counterparts of $\sigma^E_{\text{INDEX},t}$ and $\sigma^P_{\text{INDEX},t}$ are the equity VIX and the CBVIX, respectively.[27] Thus, for each day, the left-hand side of equation (13) can be estimated as the ratio between the equity VIX and the CBVIX. The price elasticity term on the right-hand side of equation (13) can be estimated from the past time series of the equity index and the bond index. For each day, we estimate the elasticity as the slope coefficient from regressing the log equity index on the log bond index over the past 63 days (i.e., approximately, 3 months).

Figure 8 compares the resulting time series of the equity VIX divided by the CBVIX (solid blue line) and that of the estimated price elasticity (dotted red line). Notably, the 2 time series show very similar levels. The time series average of the volatility ratio is 9.20, whereas that of the price elasticity is 7.95. Put differently, the estimated price elasticity suggests that the equity VIX should be roughly 8 times larger than the CBVIX, when it is 9 times larger in the actual data. This result implies that the levels of the 2 volatility indices are fairly compatible even through the lens of a simple structural credit risk model.

Although the relative magnitude is well captured, the correlation between the 2 time series is moderate at 35%. Nevertheless, we observe that the price elasticity (which is estimated based on the 3-month rolling window) tracks the index volatility ratio reasonably well. In particular, the estimated elasticity is able to capture the sharp

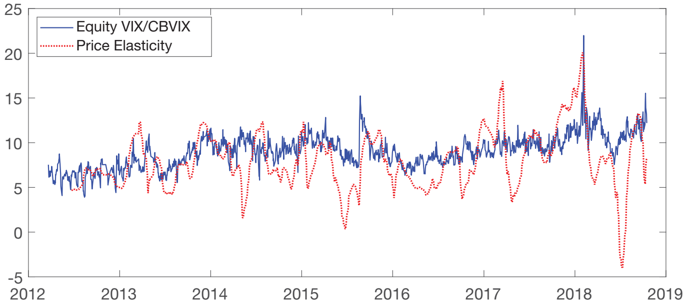
[26]This relation still holds in an extended model where the firm's asset dynamics are subject to idiosyncratic jumps.

[27]We implicitly assume that the CDX and the S&P 500 are based on the same pool of firms, although their index constituents are not exactly identical. Collin-Dufresne et al. (2021) find that this assumption is relatively harmless.

FIGURE 8

Ratio Between the Equity VIX and the Corporate Bond VIX

Figure 8 examines the relative magnitude of the equity VIX and the corporate bond VIX (CBVIX). The solid blue line plots the time series of the ratio between the equity VIX and the CBVIX. The dotted red line plots the time series of the elasticity of the equity price with respect to the bond price, which should equal the volatility ratio under our simple structural credit risk model. The sample period is from Mar. 2012 to Sept. 2018.



increase in the volatility ratio in mid-2015 as well as around the end of 2017. This is quite impressive, considering that our exercise is completely "out-of-sample."

As a precaution, the fact that our simple model can match the ratio between the equity VIX and the CBVIX does not preclude the possibility of a model failure in other dimensions. Our exercise simply shows that the CBVIX provides additional grounds for testing various aspects of a credit risk model. In addition to the CBVIX, which captures the second corporate bond moment, higher-order moments can also be useful for testing much more sophisticated structural credit risk models. For instance, conditional skewness and kurtosis may help us identify relative contributions of diffusive risk vs. jump risk to the credit spread. All in all, our exercise, together with other potential analyses that are beyond the scope of this article, demonstrates the usefulness of the ex ante risk measures estimated from synthetic corporate bond options.

VI. Conclusion

The credit derivatives market experienced a setback during the subprime mortgage crisis. For example, the trading volume of tranche products decreased significantly, as they were stigmatized for instigating the crisis. Ironically, this provided an opportunity for CDX swaptions: Market participants began to actively use CDX swaptions to hedge credit risk in place of tranche products.

Our article highlights the utility of CDX swaptions whose trading volume has steadily been rising. CDX swaptions enable us to obtain prices of option contracts written on an aggregate corporate bond index. Based on these option prices, we create the CBVIX, a model-free volatility measure for the corporate bond market. We further estimate the corporate bond variance risk premium, which reveals how much compensation investors demand to take variance risk in the corporate bond market. Our model-free option-based analysis extends beyond the second moment: Synthetic bond index options make it possible to investigate higher-order conditional moments as well as whole probability distributions. We demonstrate that our estimation results have major implications for credit risk models and allow us to assess their validity.

In this article, our findings are based on 1-month CDX swaptions. However, swaptions with longer maturities (such as 3 months and 6 months) also trade in the market, and they can be used to study the term structure of risks in the corporate bond market. We can further extend our analysis to the market for speculative grade bonds by exploiting swaptions on the CDX North American High Yield index. We plan to explore these dimensions in future research.

Appendix A. Single-Name CDS Contracts

A single-name CDS contract transfers the credit risk of a certain reference entity from one party to another. In the event of the reference entity's default (more generally, a credit event), the protection seller should either undertake a defaulted bond from the protection buyer at par (physical settlement), or directly make up for the loss of the protection buyer by paying the difference between the par value and the market value of the defaulted bond (cash settlement). In return, the protection buyer periodically pays an insurance premium to the protection seller until a credit event occurs or until the CDS contract matures, whichever comes first.

In Apr. 2009, the International Swap and Derivatives Association introduced the Standard North American Contract (SNAC) in an attempt to standardize CDS transactions and facilitate central clearing. The most distinctive feature of the SNAC is that the coupon spread for an investment-grade CDS is always set to be 1%.[28] Since the coupon spread is fixed, market fluctuations are instead captured by the upfront fee exchanged between the 2 involved parties at the beginning of the contract.

To illustrate this, let $S_{i,t} = S_i(t, t+T)$ denote the T-maturity fair market CDS spread for firm i at time t. We define $\Pi_{i,t} = \Pi_i(t, t+T)$ as the time-t present value of a risky annuity that pays a series of unit coupons until maturity or firm i's default, whichever occurs first. Under a traditional/nonstandard contract, the coupon spread is determined as the CDS spread, and, therefore, the present value of future premium payments equals $S_{i,t} \times \Pi_{i,t}$. In contrast, under a standard contract, the present value of future premium payments is $0.01 \times \Pi_{i,t}$ because the coupon spread is fixed at 1%. This discrepancy is resolved by an upfront fee $U_{i,t} = U_i(t, t+T)$ paid by the protection buyer to the protection seller at time t:

$$U_{i,t} + 0.01 \times \Pi_{i,t} = S_{i,t} \times \Pi_{i,t}.$$

In sum, under the SNAC, the protection seller enters into a standard CDS contract by receiving the quoted upfront fee from the protection buyer.

Appendix B. Spanning Formulas Based on Option Prices

Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003) show that the risk-neutral expectation of a twice-differentiable payoff function $H(\cdot)$ can be spanned by prices of European calls and puts. Expanding $H(P_{t+\tau})$ around the forward price $F_t = E_t^{\mathbb{Q}}[P_{t+\tau}]$ results in

[28]The coupon spread is fixed at 5% for speculative grade entities.

$$\text{(B-1)}\quad E^{\mathbb{Q}}[H(P_{t+\tau})] = H(F_t) + \underbrace{E^{\mathbb{Q}}[H'(F_t)(P_{t+\tau} - F_t)]}_{=0} + e^{r_f \tau}\left(\int_0^{F_t} H''(K) V_t^{\text{PUT}}(\tau;K)dK + \int_{F_t}^{\infty} H''(K) V_t^{\text{CALL}}(\tau;K)dK\right).$$

The forward price is calculated as $F_t = P_t e^{r_f \tau} - C_{t+\tau}$, where $C_{t+\tau}$ is the accrued coupon from the index between times t and $t+\tau$. Although the coupon is paid out at time $t+\tau$, this quantity is known at time t; it is calculated based on the coupon rate that is reset at time t.

B.1. CBVIX

By setting $H(P_{t+\tau}) = \log(P_{t+\tau}/F_t)$, equation (B-1) implies that

$$\text{(B-2)}\quad E_t^{\mathbb{Q}}\left[\log\left(\frac{P_{t+\tau}}{F_t}\right)\right] = -e^{r_f \tau}\left(\int_0^{F_t} \frac{V_t^{\text{PUT}}(\tau;K)}{K^2}dK + \int_{F_t}^{\infty} \frac{V_t^{\text{CALL}}(\tau;K)}{K^2}dK\right),$$

because $H''(K) = -1/K^2$. By assuming that the price process follows an Ito process, the left-hand side of equation (B-2) equals

$$E_t^{\mathbb{Q}}[\log P_{t+\tau}] - \log F_t = -\frac{1}{2}E_t^{\mathbb{Q}}\left[\int_t^{t+\tau} d[\log P]_u\right].$$

Therefore, it follows that

$$\tau \cdot \text{CBVIX}_t^2 = E_t^{\mathbb{Q}}\left[\int_t^{t+\tau} d[\log P]_u\right] = 2e^{r_f \tau}\left(\int_0^{F_t} \frac{V_t^{\text{PUT}}(\tau;K)}{K^2}dK + \int_{F_t}^{\infty} \frac{V_t^{\text{CALL}}(\tau;K)}{K^2}dK\right),$$

which provides us the expression in equation (10).

B.2. Risk-Neutral Noncentral Moments

We consider the case in which $H(P_{t+\tau}) = (P_{t+\tau}/F_t)^n$. In this case, it follows that $H''(K) = n(n-1)K^{n-2}/F_t^n$. By plugging the expressions for $H(P_{t+\tau})$ and $H''(K)$ into equation (B-1), we obtain

$$E_t^{\mathbb{Q}}\left[\left(\frac{P_{t+\tau}}{F_t}\right)^n\right] = 1 + \frac{n(n-1)e^{r_f \tau}}{F_t^n}\left(\int_0^{F_t} K^{n-2} V_t^{\text{PUT}}(\tau;K)dK + \int_{F_t}^{\infty} K^{n-2} V_t^{\text{CALL}}(\tau;K)dK\right).$$

Multiplying both sides of this equation by $(F_t/P_t)^n$ generates the expression for the nth order noncentral moment.

Supplementary Material

To view supplementary material for this article, please visit http://doi.org/10.1017/S0022109022000096.

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