Scale independence of till rheology

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ABSTRACT. Representation of till rheology in glaciological models of ice motion over deformable sediments has, until now, focused largely on two end-member cases: (1) linear, or mildly non-linear, viscous rheology and (2) (nearly) plastic rheology. Most laboratory and in situ experiments support the latter model. Hindmarsh (1997) and Fowler (2002, 2003) proposed that experimental results represent the behavior of small till samples (characteristic length scales of ~0.1 to ~1 m) but that till behaves viscously over length scales that are relevant to determination of ice-flow rates in glaciers and ice sheets (~1 km or more). Observations of short speed-up events on the ice plain of Whillans Ice Stream, West Antarctica, provide an opportunity to compare the in situ rheology of this till, integrated over ~10–100 km, with the rheology of till from beneath the same ice stream determined on small laboratory samples and in local borehole experiments. This comparison indicates that the rheology of the subglacial till beneath Whillans Ice Stream is independent of scale.

INTRODUCTION

The fact that many ice masses move over a layer of potentially deformable till introduces the need to represent till rheology in glaciological models. Laboratory and borehole data indicate that till has a (nearly) plastic rheology (Kamb, 1991; Fisher and Clarke, 1994; Hooke and others, 1997; Iverson and others, 1998). However, use of viscous till rheology is still common, partly due to its computational simplicity and partly due to continuing uncertainty as to whether results of laboratory and borehole measurements, representing spatial scales of ~0.1 to ~1.0 m, truly reflect till behavior at the spatial scales >1 km over which basal processes help control ice velocity.

Hindmarsh (1997), and more recently Fowler (2002, 2003), provided extensive discussions of this scaling problem. Hindmarsh conjectured that summation of many local plastic failures in subglacial till could lead to viscous-type till behavior over long spatial scales. If this conceptual picture were correct, it would have significant implications not only for glaciological models but also for understanding of soft sediment deformation in other environments, such as during formation of geologic structures.

Obviously, the proposition that till rheology may be different over long and short spatial scales is inherently difficult to test because building laboratory equipment for testing samples ≫1 m in size is prohibitively expensive. However, it is possible to consider the scale dependence of till rheology for Whillans Ice Stream (WIS), West Antarctica, thanks to the fortuitous availability of laboratory and field observations, which make it possible to compare behavior of till beneath WIS on scales ranging from ~0.1 m to dozens of kilometers.

DISCUSSION

Local rheological behavior of WIS till has been studied through a combination of shear-box, triaxial and ring-shear laboratory tests as well as borehole torvane measurements (Kamb, 1991, 2001; Tulaczyk and others, 2000). All these methods have consistently shown that at spatial scales ~0.1 to ~1 m, WIS till behaves as a highly non-linear material. This high rheological non-linearity means that small changes in shear stress applied to the WIS till may result in very large changes in shear strain rate. For instance, ten-fold acceleration in the deformation rate can be achieved by increasing shear stress by just 3–10%. This behavior contrasts strongly with linearly viscous till rheology, which implies that a, say, 10% increase in shear stress should produce only a 10% increase in strain rate.

Recently, Bindschadler and others (2003) have reported GPS (global positioning system) measurements of ice velocity from a location on the so-called 'ice plain' of WIS, ~200 km downstream of the UpB camp, where the studies of local till rheology were performed. They measured large and abrupt fluctuations in ice-stream velocity. On average, the ice plain is moving at ~400 m a^{-1} (Joughin and others, 2002). However, Bindschadler and others' high-resolution GPS measurements show that the plain either stagnates or moves at a rate of <0.1 m h^{-1} for most of the time and rushes forward in short bursts, ~10–30 min long, during which ice velocity is ~1–2 m h^{-1} (equivalent to ~10 000 m a^{-1}). This rapid acceleration of WIS is modulated tidally and is associated with a drop in the back-stress calculated to be just ~0.3 kPa. The model used to estimate the stress drop assumes that the speed-ups correspond to release of elastic stress accumulated during stagnation but does not invoke a decrease in bed strength as a causal mechanism.

The GPS data of Bindschadler and others indicate no measurable changes in vertical station elevation, suggesting that during cycles of quiescence and rapid motion the ice stream remains coupled to its bed. However, their measurements cannot reliably resolve displacements that are smaller than ~0.02 m, so it is worth further considering the possibility of decoupling at the ice base due to changes in basal water storage (e.g. Gray and others, 2005) during the observed ice-plain speed-ups. Since the ice plain is (nearly) stagnant between speed-ups, it apparently stays coupled to the bed strongly enough for the basal shear stress to balance the driving stress, which is just 2–5 kPa (Bindschadler and others, 1987; Joughin and others, 2004). If the till beneath this part of the ice stream is as weak as till found in boreholes at UpB (a few kilopascal; Kamb, 2001), this would require coupling over practically the whole area of the WIS ice plain.

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The volume of basal water needed to lift the ice plain of WIS, with an area of ~10 000 km^2, by ~0.001–0.01 m is ~0.01–0.1 km^3. To achieve such surface-elevation changes in a period of just 10–30 min, the basal water system beneath the ice plain would have to transfer, in and out, a water volume equivalent to about 3–30% of the estimated total annual basal meltwater outflow from the whole drainage basin of WIS (Joughin and others, 2003). This process would then be repeated on tidal timescales driving the speed-up events (Bindschadler and others, 2003). Since the tidal range for the Ross Ice Shelf is just ~1 m (Padman and others, 2002) and the seaward ice surface gradient is ~0.001 (Bindschadler and others, 1987), sea water cannot penetrate much further than ~1 km upstream of the grounding line. Hence, only meltwater flowing from beneath the ice stream can be responsible for any potential uplift and decoupling of the ice plain during the speed-ups.

An argument against a transient ice–base decoupling due to an increase in basal water storage can be developed by considering the basal water flux rates needed to inject water and initiate or increase decoupling beneath an area as large as the ice plain of WIS, within a short time period (10–30 min). To illustrate this argument quantitatively, I use the model of basal ice-stream drainage from Kamb (1991, his equations (15) and (16)) and require that the total change in basal water storage beneath the ice plain is equal to the total water inflow through the basal water system integrated over the time interval of a single speed-up event. I make the additional assumption that during the event the characteristic thickness of the basal water system grows linearly from D_0 in the beginning to D_f at the midpoint of the event, Δt (here ~1000 s). After performing the necessary time integration of water flux in the basal water system, one can express the requirement of water balance by:

$$\frac{(D_f - D_0)L}{\Delta t} = \frac{\phi}{48\eta}\frac{\partial P_w}{\partial x}(D_f + D_0)\left(D_f^2 + D_0^2\right) \tag{1a}$$

or

$$D_f - D_0 = C_0(D_f + D_0)\left(D_f^2 + D_0^2\right), \tag{1b}$$

where

$$C_0 = \frac{\Delta t}{L}\frac{\phi}{48\eta}\frac{\partial P_w}{\partial x} \tag{1c}$$

and L is the characteristic length-scale of the ice plain (~100 km), ϕ is the fractional aerial coverage of the bed by the water system (here assumed to be 1), η is the water viscosity (~0.0018 Pa s) and $\partial P_w/\partial x$ is the basal water-pressure gradient (~10 Pa m^{-1} for the ice plain; Alley and others, 1987). Given these assumptions, the coefficient C_0 is equal to ~1.16 m^{-2}. When solved for D_f, Equation (1b) has complicated roots with one non-negative, non-trivial root, which is characterized by a surprisingly low level of sensitivity to D_0 over a wide range of values (tested from 1 μm to 0.1 m). This is because both C_0 and D_f are large compared to D_0 so that the solution to Equation (1b) can be approximated by:

$$D_f = \sqrt{\frac{1}{C_0}}. \tag{2}$$

Given that the annual water flux beneath the ice plain of WIS should be ~0.3 km^3 a^{-1} (Joughin and others, 2003), reasonable values for D_0 are in the range of millimeters (see also Alley and others, 1986). For such small D_0, the change in water storage, $D_f - D_0$ for $\phi = 1$, within the time-frame of ~1000 s would have to reach ~0.9 m to be consistent with physical constraints encapsulated in Equations (1b) and (2). This is because Equations (1) require that water be pumped into the area incredibly fast under a relatively low hydraulic gradient, and to do so requires a rapid increase in thickness of the water layer. Obviously, there is some flexibility in choosing the assumed values of parameters comprising C_0. However, the mismatch between observations and calculated values, <0.02 m (Bindschadler and others, 2003) vs ~0.9 m, spans two orders of magnitude, and the square root in Equation (2) mutes the impact of uncertainties in C_0 on the final result. This analysis shows that decoupling of the ice plain of WIS triggered by changes in water storage is probably impossible over the short timescales of the speed-up events observed on WIS.

The theoretical considerations pointing to continued ice–bed coupling during speed-up events are consistent with geological and geophysical evidence, which indicates the presence of a meters-thick till layer grading into a 'till delta' (grounding-zone wedge) in the downstream direction (Alley and others, 1989). The existence of the delta demonstrates that till is being dragged by the ice and, thus, that ice motion is accompanied by till deformation. Hence, available theoretical and observational constraints suggest that the bursts of forward motion observed on the ice plain of WIS are associated with variations in shear strain rate in the subglacial till by an order of magnitude or more. Joughin and others (2002, 2004) have previously shown that the local force balance of the WIS ice plain is determined largely by basal resistance. Effectively, in this region, nature is running a giant subglacial shear-box experiment, with dimensions of dozens of kilometers, up to six orders of magnitude larger than the typical size of laboratory till samples.

Qualitatively, it is easy to see that the non-linear till rheology demonstrated by laboratory experiments is consistent with the high flow variability exhibited by the ice plain of WIS. If the till underlying WIS had linearly viscous rheology and remained coupled to the ice base then an approximately ten-fold acceleration of flow would have to be associated with an approximately ten-fold increase in driving shear stress. In contrast, velocity variations of at least an order of magnitude are driven on the ice plain by variations in driving stress of only ~10% (stress drop of ~0.3 kPa with overall level of driving stress of ~2–5 kPa (Bindschadler and others, 1987, 2003; Joughin and others, 2004). In laboratory triaxial tests on three samples of the UpB till (Tulaczyk and others, 2000), a ~10% increase in shear stress was also associated with a ten-fold increase in shear strain rate.

I use a plot relating non-dimensional shear strain rates to shear stresses to illustrate the agreement between the results of three laboratory triaxial tests on samples of the UpB till and regional observations from the ice plain of WIS (Fig. 1). The triaxial data have been previously presented and discussed by Tulaczyk and others (2000; R1, R2, R3 in their fig. 5). The primary difference between the three tests was the level of effective stress applied to the samples (~30, ~175, ~250 kPa). Here, I plot a total of 3708 readings made during the tests after the state of failure had been achieved. The data are plotted after non-dimensionalization of all strain-rate readings by the minimum strain rate and all shear stress readings by the minimum shear stress achieved during each of the three tests. A power law of the following form was fitted to the three groups of data:

$$\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} = k\left(\frac{\tau}{\tau_0}\right)^m, \tag{3}$$

where $\dot{\varepsilon}$ is the shear strain rate, $\dot{\varepsilon}_0$ is the reference strain rate (here the minimum strain rate in each test), k is the coefficient of proportionality (non-dimensional), τ is the shear stress, τ_0 the reference shear stress (here the minimum τ in each test) and m is the stress exponent. The fits are shown in Figure 1 as solid straight lines. The values of k are close to 1 and the values of the stress exponent vary between 9.4 and 13.3 with the measure of the goodness of fit, R^2, ranging between 0.55 and 0.77. For comparison, Figure 1 also shows a dashed line illustrating the predictions of a linearly viscous model. Clearly, in a logarithmic plot this line must pass through points (1,1), (10,10) and so forth. It is worth noting that in Figure 1 any power-law rheology should plot as a straight line crossing the point where shear stress and shear strain-rate ratios are exactly equal to 1 (hence $k = 1$ in Equation (3)). The value of k deviates from 1 for the lines fitted to the laboratory data because of measurement errors, which caused the minimum shear strain rates to not correspond to minimum shear stresses. Under ideal conditions, i.e. no measurement errors, the fitted lines would all pass through point (1,1).

The velocity and stress data of Bindschadler and others (2003, their fig. 1) can be plotted in Figure 1. Following Alley and others (1987), I make the simplifying assumption that the surface ice velocity is achieved by uniform, pervasive simple shear within a till layer of thickness H. Since Figure 1 involves a ratio of two strain rates, it is unimportant how thick this deforming layer is, provided its thickness remains relatively constant as ice velocity changes, thus:

$$\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} = \frac{\frac{u}{H}}{\frac{u_0}{H}} = \frac{u}{u_0}. \tag{4}$$

As the reference ice velocity, u_0, I use ~0.1 m h^{-1}, which is the velocity with which the ice plain of WIS is 'drifting' around in different directions between the short bursts of fast velocity, ~1–2 m h^{-1}. The strain-rate ratio is thus at least ~10. In Figure 1, this value is plotted as a lower bound, because the average forward ice-flow rate between the short velocity bursts is much less than ~0.1 m h^{-1} and may be zero given the precision of GPS measurements (fig. 1 in Bindschadler and others, 2003) (grey box in Fig. 1). To estimate the relevant stress ratios, I take the reference stress, τ_0, to be the regional driving stress, τ_d and take τ to be $\tau + \Delta\tau$ where $\Delta\tau$ is the change in stress that results in the acceleration. The conservative limits, $2 < \tau_d < 5$ and $0.1 < \Delta\tau < 1$, give a range in τ/τ_0 of 1.02–1.5.

I emphasize that the similarity between the narrow high shape of the shaded area in Figure 1 and the lines through the data points is irrelevant. The important point is that plausible values of the strain-rate ratio and the stress ratio place deformation of in situ WIS till in the region covered by the laboratory experiments. In Figure 1, perfectly plastic behavior would be indicated by overlap of data with the vertical axis along which variations in strain rate occur without changes in stress. Clearly, the unsteady behavior of WIS ice plain is consistent with the highly non-linear till rheology determined in the laboratory by testing small till samples. This good agreement between the two sets of observations occurs despite the fact that they were made in two localities on WIS separated by ~200 km and on spatial scales that vary from ~0.1 m to ~10 000 m. Apparently, till rheology beneath WIS is not changing significantly along the course of this ice stream and is independent of scale.

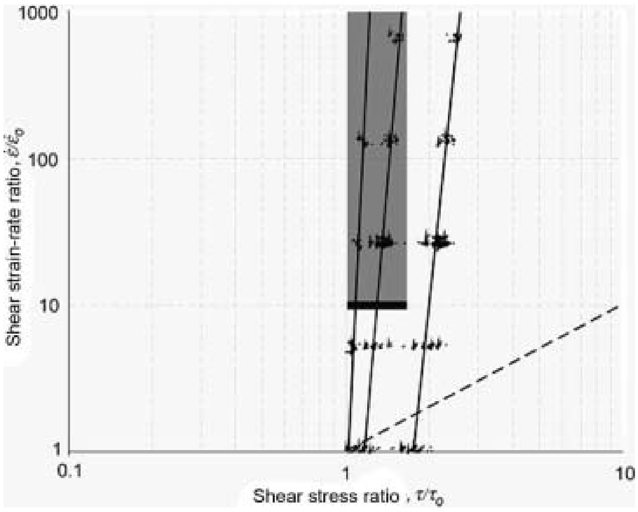

Fig. 1. Log–log plot of non-dimensional shear strain rate as a function of non-dimensional shear stress. Clouds of individual points represent measurements performed during three separate triaxial compression tests on three samples of UpB till (Tulaczyk and others, 2000). Solid lines show least-squares fits of Equation (3) to the data. The dashed line illustrates behavior of a linearly viscous rheology. The thick black bar shows the lower bound on the estimate of the shear strain-rate ratio for the ice plain of WIS, with the grey box reflecting the uncertainty caused by the fact that u_0 may be <0.1 m h^{-1}.

The lack of along-flow variability in till behavior is consistent with the conjecture that the till beneath WIS is formed through recycling of widespread Tertiary glacio-marine sediments without much change in along-flow till characteristics due to comminution (Tulaczyk and others, 1998; Studinger and others, 2001). If these source sediments are relatively homogeneous, as available evidence suggests, there is no reason for the till to have spatially variable composition or mechanical behavior. The lack of scale dependence of till rheology indicates that the mechanism of till deformation remains the same irrespective of the horizontal extent of shearing, at least over length scales between ~0.1 m and ~10 km.

CONCLUSION

The agreement between laboratory tests and field observations discussed herein weakens the interesting proposition that till could have very different behaviors on different spatial scales (Hindmarsh, 1997; Fowler, 2002, 2003). Rather, it shows that till, like many other deforming sediments (e.g. landslide debris), has the same rheology in small laboratory samples as during deformation in nature. It also demonstrates the power of laboratory experiments, which have the capability to elucidate glacial processes through controlled manipulation of small ice or till samples. Unlike other branches of geosciences, laboratory experiments appear to be under-appreciated in our field, despite their clear scientific potential. The example shown here demonstrates that laboratory tests were showing us the true rheology of till beneath a West Antarctic ice stream well before field observations started to point to the same conclusion (Kamb, 1991; Bindschadler and others, 2003).

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