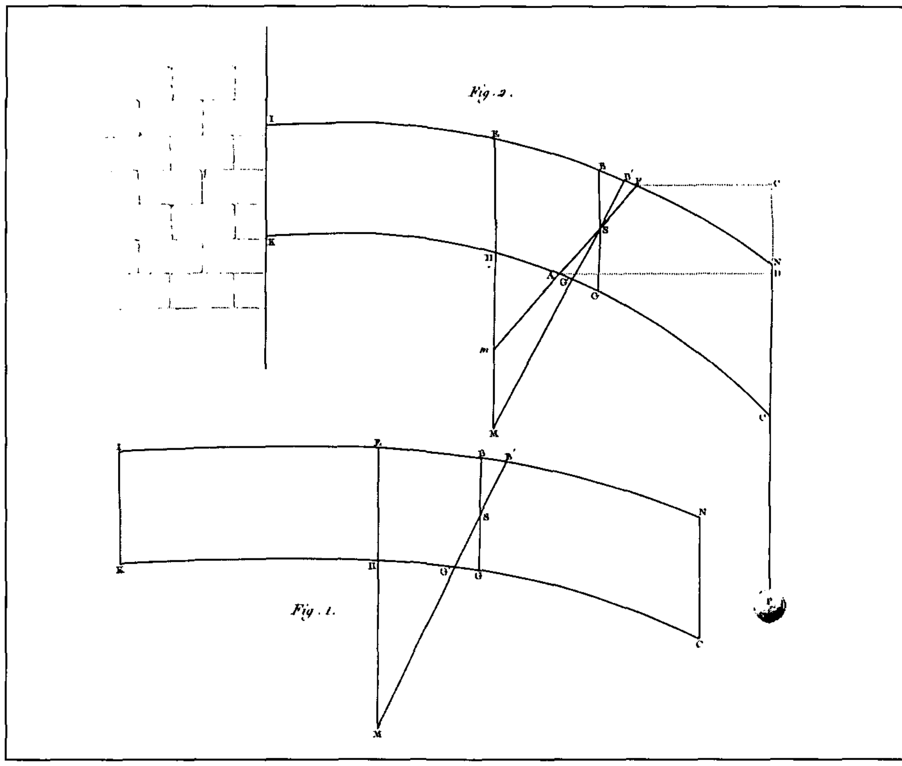
Sophie Germain's Early Contribution to the Elasticity Theory



Sophie Germain's sketch of an elastic bar and its radius of curvature when bent by an external force, taken from her book on the theory of elastic surfaces (1821).

Marie-Sophie Germain (1776–1831) was born in Paris to a family of the (increasingly) rich bourgeoisie. She blossomed, in her teens, into a self-taught mathematician. Her contributions to number theory, solutions to special cases of the Fermat theorem, are still remembered and referenced, along with a rich harvest of anecdotes. Her contribution to the development of the theory of elasticity is less known, though we owe her the first mathematical treatment of the resonance nodes of vibrating membranes.

Germain, by her own account, selected the study of mathematics to isolate herself from mounting social unrest and political revolution. Her father had been elected as a representative of the bourgeoisie—*le tiers état*—to the congress of Versailles. It was the prohibition of the meetings of this assembly that sparked the events of July 14, 1789—the French Revolution. Reportedly, when Germain read an account, in her father's library, of Archimedes who was slaughtered by a Roman soldier during the siege of Siracusa because he was too engrossed in a mathematical problem to notice the events, the young teenager singled out mathematics as a subject absorbing enough to distract one's mind from the unsettling events at hand.

Unfortunately, the study of mathematics was inappropriate for a merchant's daughter. Germain's parents tried to curb her worrisome activities by preventing her studies in every way. When she resorted to study at night, they hid her clothes, forbade a fire, and hid away candles. It is said that Germain would rise to study mathematics in the dead of night, wrapped in bedcovers, while it was so cold that the ink would freeze in the inkpot. Her family soon desisted. Although no institutions of higher education would admit women as students, those were times when the laws of society, and the role of women therein, were being rewritten. While Germain studied Gauss's *Disquisitiones Arithmeticae*, Paris grew with political chat-rooms, the salons, hosted by women. In 1789, Paris also witnessed the 12-mile march by 6,000 women on the royal palace of Versailles. This Amazon army, led by the Belgian Théroigne de Mericourt, once a courtesan, was instrumental in bringing Louis XVI to Paris under direct control of the French citizens.

Although women had no possibility of receiving a formal education, the Ecole Polytechnique, newly founded to consolidate the new role of France in positivist Europe, produced written notes for students, and requested that solutions be submitted to the problems. Germain obtained these notes under the name of Monsieur LeBlanc. She revealed herself as a woman when Lagrange desired to meet the student who submitted such brilliant solutions. Under the same name of LeBlanc she initiated a correspondence with Gauss, offering solutions to some of the problems of his *Disquisitiones Arithmeticae*, and asking for advice on how to proceed with her studies. Gauss was pleased with his correspondent from Paris, who was able to offer a partial proof of Fermat's theorem. In 1806 Napoleonic troops invaded Brunswick where Gauss resided. Germain recommended his safety to General Pernety, a friend of the family, and it is he who revealed to Gauss the true identity of his Paris correspondent.

The appreciation for her work allowed Germain to be invited to attend the demonstration which set the basis for interest in vibrating surfaces. The acoustician Chladni demonstrated in 1808 at the French Academy of Sciences the vibrating modes of plates with grains of sand, which distributed themselves in fixed patterns on membranes made to resonate with the arch of a violin. This experiment duly impressed Napoleon, who approved a public contest for the theoretical description of these patterns, with a prize of 3,000 francs. The beginnings of the modern theory of elasticity did not thus spring from interest in practical engineering problems, but from a desire of finding, with the new mathematical tools at hand (the variational methods of Lagrange), a solution to physics problems. The call, open for two years, received only one entry, by Sophie Germain.

This entry contained a basic hypothesis, namely, that the elasticity was proportional to the sum of the inverses of the principal radii of curvature of a surface. Unfortunately, no proof was offered for the hypothesis and the derivation of the subsequently inferred equations was not correct, so the prize was not awarded, but the contest was extended. Lagrange, who was a member of the commission that judged the entry to the contest, offered the correct partial differential equation. However, he died two years later and Germain remained unable to derive Lagrange's equation by

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variational calculus. Her ability resided mostly in algebraic manipulations, and she tried to generalize the work of Euler on the elasticity of bars, extending it to two dimensions. Germain related elasticity to the sum of the principal curvatures of a surface.

The objective of the contest was to offer a mathematical explanation of Chladni's experiment, and Germain successfully experimented with laminae. In her second entry (again, the only one) to the extended contest, she modified her equation and proved that it was correct in a number of special cases. The experimental part earned her an honorable mention from the jury, and led her to publish her work—at author's cost—in 1821. Another stimulus to publication was the fact that Poisson was devoting his attention to the problem of elasticity, offering another approach and yet another equation, and consistently avoided to duly acknowledge Germain's preceding and ongoing work. Partly to patch up this breach of confidentiality, the contest was called again, with a gold medal as a prize. The gold medal was awarded to Germain, but, to the annoyance of those attending the public ceremony, she decided not to appear in public.

In 1828, a new approach was given to the solution of the plate equation: an equation by Fourier, in the form of series of sines and cosines. Navier, a former student of Fourier, offered a new expression for the elastic moment: It was proportional to an elastic constant, the cube of the plate's thickness (instead of the fourth power of Germain's), and also contained the product of the principal curvatures. This research in turn stimulated Cauchy to define how the stress of an elastic plate depended on the applied strain in terms of tensors while introducing a second elastic constant. Cauchy, and subsequently Kirchhoff, set the foundations of the present theory of elasticity.

These were Germain's final years. She fought a losing battle against breast cancer, while continuing to write her thoughts over the sciences and the arts, "Real superiority is nothing more than the means of considering difficult problems from a point of view whence they become easy, where the spirit can embrace them and follow them without effort." Germain did not live long enough to ripen the fruits of Gauss's request to the University of Göttingen to award her an honorary doctorate.

CRISTINA P. TANZI

FOR FURTHER READING: A.D. Dalmédico, "Sophie Germain," *Scientific American* Dec. (1991) p. 117; A.D. Dalmédico, "Mécanique et théorie des surfaces: les travaux de Sophie Germain," *Historia Math.* **14** (4) (1987) p. 347; L.L. Bucciarelli and N. Dworsky, *Sophie Germain: An Essay in the History of the Theory of Elasticity* (D. Reidel, Dordrecht, 1980); S. Germain, *Recherches sur la théorie des surfaces élastiques, par Sophie Germain* (Mme. V. Courcier, Paris, 1821); H. Stupuy, "Notice sur la vie et les ouvres de Sophie Germain," *Oeuvres philosophique de Sophie Germain* (Paul Ritti, Paris, 1879), pp. 1–92.

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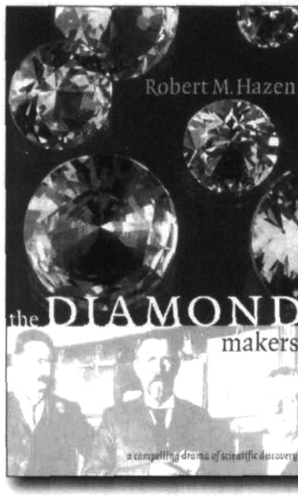

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