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AUTOMORPHISMS OF A CERTAIN SKEW POLYNOMIAL RING OF DERIVATION TYPE

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1. Introduction. Throughout this paper, all rings have the identity 1 and ring homomorphisms are assumed to preserve 1. We use p to denote a prime integer and F to denote a field of characteristic p. For an element α in F, we set

$$A = F[x]/(x^p - \alpha)F[x].$$

Moreover, by D and R, we denote the derivation of A induced by the ordinary derivation of $F[x]$ and the skew polynomial ring $A[X;D]$ where $aX = Xa + D(a)$ $(a \in A)$, respectively (cf. [**2**]).

In [**3**], R. W. Gilmer determined all the B-automorphisms of $B[X]$ for any commutative ring B. Since then, some extensions or generalizations of his results have been obtained ([**1**], [**2**] and [**5**]). As to the characterization of automorphisms of skew polynomial rings, M. Rimmer [**5**] established a thorough result in case of automorphism type, while M. Ferrero and K. Kishimoto [**2**], among others, have made some progress in case of derivation type.

But, [**2**] is a study on B-automorphisms of $B[X;\delta]$ in case that B is a ring with a derivation δ satisfying the condition $\delta(N) \subset N$ where N is the union of all nilpotent ideals of B. Moreover, in that study, it is shown that this condition is fulfilled in the following cases: B is torsion free; B is semiprime. However, apart from these cases, we can find no information about this condition. Hence the results on [**2**] can not necessarily be applied to rings of characteristic p. In particular, we can never apply it to the ring $F[x]/(x^p)F[x]$ with the derivation D which is useful in studies of algebra.

On the other hand, for the algebra A, Jacobson [**4**, p. 190] mentions a certain kind of A-automorphisms of R in case that $x^p - \alpha$ is irreducible in $F[x]$. In this case there exists no another kind of A-automorphisms of R, which can be easily seen from our theorem or [**2**]. However, if $x^p - \alpha$ is not irreducible then A is isomorphic to $F[x]/(x^p)F[x]$, hence the problem to determine all the A-automorphisms of R has never been solved except in the case that A is a field.

The aim of this paper is to solve this problem and, as a result, to show an automorphism whose type is quite different from ones in [**1**], [**2**], [**3**] and [**5**].

To study this object, we consider the following conditions for the A-linear map ϕ of R to itself defined by

$$(\#) \quad X^k \longrightarrow \left(\sum_{i=0}^{n} X^i a_i\right)^k, \quad k = 0, 1, 2, \ldots (n \geqq 2, a_n \neq 0).$$

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(In case $n = 1$, see Remark 2.)

(a-i) $a_1 = 1$.

(a-ii) $a_i = 0$ for all $i \in \{j : 2 \leqq j \leqq n \text{ and } p \nmid j\}$.

Assume that (a-ii) is fulfilled. Then, it follows from $n \geqq 2$ and $a_n \neq 0$ that $p \mid n$. Hence there exist integers s and t which satisfy $ps = n$ and $pt \leqq s < p(t+1)$. Thus the following conditions can be considered.

(b-i) $D^{p-1}(a_p) + 1 \neq 0$.

(b-ii) $D^{p-1}(a_{p^2 i}) + a_{pi}^p = 0$ for all $i \in \{j : 1 \leqq j \leqq t\}$.

(b-iii) $a_{pi}^p = 0$ for all $i \in \{j : t+1 \leqq j \leqq s\}$.

(b-iv) $D^{p-1}(a_{pi}) = 0$ for all $i \in \{j : 2 \leqq j \leqq s \text{ and } p \nmid j\}$.

2. The main theorem. Our study starts by stating our main theorem.

THEOREM 1. *The map ϕ is an A-automorphism of R if and only if* (a-i), (a-ii), *and* (b-i)–(b-iv) *hold. Furthermore in this case, the inverse map ϕ^{-1} of ϕ is induced by*

$$X^k \longrightarrow \left(X + \sum_{j=0}^{s} X^{pj} b_{pj} \right)^k, \quad k = 0, 1, 2, \ldots$$

where

$$b_{pj} = \sum_{i=j}^{s} (-1)^{i-j+1} \binom{i}{j} (D^{p-1}(a_0) + a_0^p)^{i-j} (D^{p-1}(a_p) + 1)^{-i} a_{pi}$$

for each j. In particular, every A-automorphism ψ is necessarily of the above form.

To prove this theorem, we need several lemmas.

LEMMA 2. *Let i, j and k be non-negative integers.*
(1) *If $0 < i < pk$ and $p \nmid i$ then*

$$\binom{pk}{i} \equiv 0 \pmod{p}.$$

(2) *If $0 \leqq j \leqq k$ then*

$$\binom{pk}{pj} \equiv \binom{k}{j}^p \equiv \binom{k}{j} \pmod{p}.$$

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Proof. Since F is of characteristic p, we have the following equalities in $F[X]$ by the binomial theorem:

$$(1+X)^{pk} = \sum_{h=0}^{pk} \binom{pk}{h} X^h,$$

$$(1+X)^{pk} = \{(1+X)^k\}^p = \left(\sum_{h=0}^{k} \binom{k}{h} X^h\right)^p = \sum_{h=0}^{k} \binom{k}{h}^p X^{ph}$$

and

$$(1+X)^{pk} = \{(1+X)^p\}^k = (1+X^p)^k = \sum_{h=0}^{k} \binom{k}{h} X^{ph}.$$

Hence we have (1) and (2) by comparing the coefficients of X^i and X^{pj}, respectively.

LEMMA 3. *Let B be a commutative algebra over the prime field $GF(p)$ and δ a derivation of B such that $\delta^p = 0$. Assume that $\delta(z) = 1$ for some z in B. Then (1) and (2) hold.*

(1) *The map $X \longrightarrow \sum_{i=0}^{n} X^i b_i$ $(b_i \in B, n \geqq 1, b_n \neq 0)$ induces a B-endomorphism of $B[X;\delta]$ if and only if*

(i) $b_1 = 1$ *and*

(ii) $b_i = 0$ *for all $i \in \{j : 2 \leqq j \leqq n$ and $p \nmid j\}$.*

When this is the case, the image of X takes the form

$$X + \sum_{i=0}^{s_0} X^{pi} b_{pi}$$

where s_0 is an integer such that $ps_0 = n$ if $n \geqq 2$, and $s_0 = 0$ if $n = 1$.

(2) *Let $I = \{b \in B : \delta(b) = 0\}$. Then, the center of $B[X;\delta]$ coincides with $I[X^p]$, the polynomial ring in X^p over the algebra I. Moreover, $B[X^p]$ is the unique maximal commutative subalgebra of $B[X;\delta]$ containing B.*

Proof. Let ϕ_0 be the B-linear map of $B[X;\delta]$ to itself induced by

$$X^k \longrightarrow \left(\sum_{i=0}^{n} X^i b_i\right)^k, \quad k = 0, 1, 2, \ldots.$$

We note that ϕ_0 is a B-endomorphism of $B[X;\delta]$ if and only if

$$b\phi_0(X) = \phi_0(X)b + \delta(b)$$

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for any b in B. From this, it is easily seen that ϕ_0 is a B-endomorphism of $B[X;\delta]$ if and only if the following equalities hold for any b in B (cf. [**2**, (1.1)]).

$$\sum_{k=0}^{n} \delta^k(b)b_k = b_0 b + \delta(b),$$

$$\sum_{k=i-1}^{n} \binom{k}{i-1} \delta^{k-(i-1)}(b)b_k = b_{i-1}b \quad (i \geqq 2).$$

Assume that ϕ_0 is a B-endomorphism of $B[X;\delta]$. Since $\delta(z) = 1$, $\delta^k(z) = 0$ $(k \geqq 2)$. Substitute here z for b in the above equalities. Then, the condition (i) will be easily seen from the first equality. Moreover, from the second equality, we have

$$zb_{i-1} + i\delta(z)b_i = b_{i-1}z.$$

This enables us to see (ii).

Conversely, assume that (i) and (ii) are satisfied. If $k > i-1$ and $p \mid k$ then

$$\binom{k}{i-1} \delta^{k-(i-1)}(b) = 0$$

for any b in B by Lemma 2 and our assumption $\delta^p = 0$. On the other hand, if $k > i-1$ and $p \nmid k$ then $b_k = 0$ by (ii). This shows that the above second equality holds. Moreover, noting $b_1 = 1$ (i), we have the first equality in a similar way. Thus (1) has been proved.

To see the assertion (2), C will denote the center of $B[X : \delta]$. First, we shall prove that $I[X^p] \subset C$. Let

$$f(X) = \sum_{i=0}^{k} X^{pi}\epsilon_{pi} \quad (\epsilon_{pi} \in I)$$

be an arbitrary element in $I[X^p]$. Then, $f(X)$ commutes with every element in B. Indeed, it is easily seen that

$$bX^p - X^p b = \sum_{j=0}^{p-1} X^j \binom{p}{j} \delta^{p-j}(b)$$

for any b in B. Then, it follows from Lemma 2(1) and our assumption $\delta^p = 0$ that $bX^p = X^p b$ for any b in B. Also, since $\delta(\epsilon_{pi}) = 0$ $(0 \leqq i \leqq k)$, $f(X)$ commutes with X and so does with every element of $B[X;\delta]$. This means that $I[X^p] \subset C$.

Conversely, let $g(X) = \sum_{i=0}^{k} X^i c_i$ $(k \geqq 0, c_k \neq 0)$ belong to C. Since

$$0 = g(X)X - Xg(X) = \sum_{i=0}^{k} X^i \delta(c_i),$$

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it follows that $\{c_i\}$ is contained in I. Hence it is enough to show that $\sum_{i\in T} X^i c_i = 0$ where $T = \{i : 0 \leqq i \leqq k \text{ and } p \nmid i\}$. Suppose that $\sum_{i\in T} X^i c_i \neq 0$ and let m be the maximal element in $\{i \in T : c_i \neq 0\}$. Then, since $X^p \in C$, we have

$$bg(X) - g(X)b = b\left(\sum_{i\in T} X^i c_i\right) - \left(\sum_{i\in T} X^i c_i\right) b \quad (b \in B)$$

which is equal to zero. As is easily seen, the coefficient of X^{m-1} is $m\delta(b)c_m$. Hence we have $mc_m = 0$ by taking z as b. Since $p \nmid m$ and $c_m \neq 0$, this is a contradiction.

Moreover, since $bX^p = X^p b$ for any b in B, $B[X^p]$ is a commutative subalgebra of $B[X;\delta]$ containing B. Let S be a commutative subalgebra of $B[X;\delta]$ containing B. Then, for every element $g(X) = \sum_{i=0}^k X^i c_i$ in S, we have $bg(X) = g(X)b$ $(b \in B)$. Hence one can easily see that $g(X) = \sum_{i=0}^h X^{pi} c_{pi}$ for some integer $h \geqq 0$ as in the above argument. This shows that $S \subset B[X^p]$, and hence $B[X^p]$ is the unique maximal commutative subalgebra of $B[X;\delta]$ containing B.

The following lemma is a special case of the formula (31) of [**4**, p. 189].

LEMMA 4. *Let E be an F-algebra. For given a and b in E, define $b^{(k)}$ $(0 \leqq k \leqq p-1)$ inductively as follows:*

$$b^{(0)} = a \text{ and } b^{(k)} = [b^{(k-1)}, b]$$

where $[c, d] = cd - dc$ $(c, d \in E)$. If a commutes with all $b^{(k)}$ $(1 \leqq k \leqq p-2)$, then

$$(a+b)^p = a^p + b^p + b^{(p-1)}.$$

By D^*, we denote the derivation of R induced by D, that is,

$$D^*\left(\sum_{i=0}^k X^i b_i\right) = \sum_{i=0}^k X^i D(b_i)$$

for each $\sum_{i=0}^k X^i b_i$ in R. Then, we can apply the above lemma to $a = \sum_{i=0}^s X^{pi} a_{pi}$ and $b = X$ in the algebra R to obtain

$$(*) \qquad (X+a)^p = X^p + D^{*p-1}(a) + a^p,$$

because

$$b^{(1)} = [a, X] = D^*(a) \quad \text{and}$$

$$b^{(k)} = D^{*k}(a) = \sum_{i=0}^s X^{pi} D^k(a_{pi}) \quad (0 \leqq k \leqq p-1)$$

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which are contained in the maximal commutative subalgebra $A[X^p]$ of R by Lemma 3. Moreover, we have

$$a^p = \sum_{i=0}^{s} X^{p^2 i} a_{pi}^p$$

which is used in the subsequent study.

Proof of Theorem 1. Let ϕ be the A-linear map induced by (#) and C the center of R. Then $C = F[X^p]$ by Lemma 3. Assume that the map ϕ is an A-endomorphism of R. Then, by Lemma 3, (a-i) and (a-ii) are satisfied and we can write

$$\phi(X) = X + \sum_{i=0}^{s} X^{pi} a_{pi}.$$

Hence, by (*), we have

$$\begin{aligned} \phi(X^p) &= \phi(X)^p \\ &= X^p + \sum_{i=0}^{s} X^{pi} D^{p-1}(a_{pi}) + \sum_{i=0}^{s} X^{p^2 i} a_{pi}^p. \end{aligned}$$

We write here

$$\phi(X^p) = \sum_{i=0}^{n} X^{pi} \alpha_{pi}.$$

Obviously there hold the following equalities:

$$(**) \quad \begin{aligned} \alpha_{pi} &= 0 \quad \text{for all } i \in \{j : s < j < ps = n \text{ and } p \nmid j\}; \\ \alpha_p &= D^{p-1}(a_p) + 1; \\ \alpha_{p^2 i} &= D^{p-1}(a_{p^2 i}) + a_{pi}^p \quad \text{for all } i \in \{j : 0 \leqq j \leqq t\}; \\ \alpha_{p^2 i} &= a_{pi}^p \quad \text{for all } i \in \{j : t+1 \leqq j \leqq s\}; \\ \alpha_{pi} &= D^{p-1}(a_{pi}) \quad \text{for all } i \in \{j : 2 \leqq j \leqq s \text{ and } p \nmid j\}. \end{aligned}$$

Then, we note that $D^{p-1}(a_{pi})$ and a_{pi}^p $(0 \leqq i \leqq s)$ are in F. Hence, the α_{pi} are contained in F. This shows that the A-endomorphism ϕ induces an F-endomorphism of C.

Now, assume that the map ϕ is an A-automorphism of R. Then, the A-automorphism ϕ induces uniquely an F-automorphism of C such that

$$X^p \longrightarrow \sum_{i=0}^{n} X^{pi} \alpha_{pi}.$$

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As is well-known (cf. [**3**, p. 331, Theorem 3]),

$$Y \longrightarrow \sum_{i=0}^{n} Y^i \alpha_{pi}$$

induces an F-automorphism of the commutative polynomial ring $F[Y]$ if and only if $\alpha_p \neq 0$ and $\alpha_{pi} = 0$ for all $i \geqq 2$. It follows therefore that (b-i)–(b-iv) are fulfilled.

Next, we shall show the converse. Assume that the conditions (a-i)–(a–ii) and (b-i)–(b-iv) are fulfilled. Then, combining (a-i)–(a-ii) with the result of Lemma 3, we see that the map ϕ is an A-endomorphism of R and

$$\phi(X) = X + \sum_{i=0}^{s} X^{pi} a_{pi}.$$

Hence $\phi(X^p) = \alpha_0 + X^p \alpha_p$ by (b-i)–(b-iv) and (**). Thus, the A-endomorphism ϕ induces an F-automorphism ϕ_c of $C = F[X^p]$ such that

$$X^p \longrightarrow \alpha_0 + X^p \alpha_p,$$

and the inverse map ϕ_c^{-1} of ϕ_c satisfies

$$\phi_c^{-1}(X^p) = -\alpha_0 \alpha_p^{-1} + X^p \alpha_p^{-1}.$$

For the A-endomorphism ϕ, there exists an A-endomorphism ψ of R such that

$$\psi(X) = X + \sum_{i=0}^{s} X^{pi} b_{pi} \quad \text{and} \quad \phi\psi(X) = X.$$

Indeed, by Lemma 3, the map

$$X \longrightarrow X + \sum_{i=0}^{s} X^{pi} b_{pi}$$

induces an A-endomorphism of R for any b_{pi} in A. Putting $Y = \alpha_0 + X^p \alpha_p$, we have

$$\begin{aligned}
\phi\psi(X) - X &= \sum_{i=0}^{s} X^{pi} a_{pi} + \sum_{i=0}^{s} (\alpha_0 + X^p \alpha_p)^i b_{pi} \\
&= \sum_{i=0}^{s} [\{(Y - \alpha_0)\alpha_p^{-1}\}^i a_{pi} + Y^i b_{pi}] \\
&= \sum_{j=0}^{s} Y^j \left\{ \sum_{i=j}^{s} (-1)^{i-j} \binom{i}{j} \alpha_0^{i-j} \alpha_p^{-i} a_{pi} + b_{pj} \right\},
\end{aligned}$$

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because of the commutativity of the center $C = F[X^p]$ of R. Therefore, an A-endomorphism ψ with

$$b_{pj} = \sum_{i=j}^{s}(-1)^{i-j+1}\binom{i}{j}\alpha_0^{i-j}\alpha_p^{-i}a_{pi} \quad (0 \leqq j \leqq s)$$

has the property $\phi\psi(X) = X$.

We shall now prove that $\psi\phi(X) = X$. We define β_{pi} as (**), using b_{pi} in place of a_{pi}. Then, the restriction ψ_c of ψ to $F[X^p]$ maps X^p to $\sum_{i=0}^{n} X^{pi}\beta_{pi}$. Since $\phi\psi(X) = X$, we have $\phi\psi(X^p) = X^p$ and so $\phi_c\psi_c(X^p) = X^p$. Thus,

$$\psi_c(X^p) = \sum_{i=0}^{n} X^{pi}\beta_{pi} = \phi_c^{-1}(X^p) = -\alpha_0\alpha_p^{-1} + X^p\alpha_p^{-1}.$$

Hence, we obtain

$$\beta_0 = -\alpha_0\alpha_p^{-1}, \quad \beta_p = \alpha_p^{-1} \quad \text{and} \quad \psi(X^p) = \beta_0 + X^p\beta_p.$$

Now we are in a position to complete the proof. Indeed, we have

$$\begin{aligned}
\psi\phi(X) - X &= \psi\left(X + \sum_{i=0}^{s} X^{pi}a_{pi}\right) - X \\
&= \sum_{i=0}^{s} X^{pi}b_{pi} + \sum_{i=0}^{s}(\beta_0 + X^p\beta_p)^i a_{pi} \\
&= \sum_{j=0}^{s} X^{pj}\left\{b_{pj} + \sum_{i=j}^{s}\binom{i}{j}\beta_0^{i-j}\beta_p^{j}a_{pi}\right\} \\
&= \sum_{j=0}^{s} X^{pj}\left\{b_{pj} + \sum_{i=j}^{s}(-1)^{i-j}\binom{i}{j}\alpha_0^{i-j}\alpha_p^{-i}a_{pi}\right\} \\
&= 0
\end{aligned}$$

which shows, together with $\phi\psi(X) = X$, that ϕ is an A- automorphism of R and $\psi = \phi^{-1}$.

3. Remarks and examples. In the rest of this paper, let y be the image of x in A by the canonical homomorphism from $F[x]$ to

$$A = F[x]/(x^p - \alpha)F[x].$$

We shall now present some interesting results which are obtained from our theorem.

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Remark 1. Since each a_{pi} is an element in A, we can write

$$a_{pi} = \sum_{k=0}^{p-1} y^k \gamma_{pi,k} \quad (\gamma_{pi,k} \in F).$$

Then

$$a_{pi}^p = \sum_{k=0}^{p-1} \alpha^k \gamma_{pi,k}^p \quad \text{and}$$

$$D^{p-1}(a_{pi}) = (p-1)!\gamma_{pi,p-1} = -\gamma_{pi,p-1}$$

by Wilson's Theorem. Thus, one can replace the condition (b-i)–(b-iv) with the following:

(b-i)′ $\gamma_{p,p-1} \neq 1$;

(b-ii)′ $\gamma_{p^2i,p-1} = \sum_{k=0}^{p-1} \alpha^k \gamma_{pi,k}^p$ for all $i \in \{j : 1 \leqq j \leqq t\}$;

(b-iii)′ $\sum_{k=0}^{p-1} \alpha^k \gamma_{pi,k}^p = 0$ for all $i \in \{j : t+1 \leqq j \leqq s\}$;

(b-iv)′ $\gamma_{pi,p-1} = 0$ for all $i \in \{j : 2 \leqq j \leqq s \text{ and } p \nmid j\}$.

Obviously in case $\alpha = 0$, these relations show that whether or not the A-endomorphism ϕ of R is an A-automorphism depends only on the coefficients $\gamma_{pi,p-1}$ of y^{p-1} and constant terms $\gamma_{pi,0}$ of a_{pi} $(1 \leqq i \leqq s)$. Therefore the coefficients $\gamma_{pi,k}$ of intermediate terms y^k $(1 \leqq k \leqq p-2)$ can be taken freely, and so if p is an odd prime (i.e., $p \neq 2$) then one can easily make different A-automorphisms of R from any given A-automorphism of R. This also means that there exist at least $|F|^{(p-2)s}$ A-automorphisms of R whose image of X is of degree $n = ps$, where $|F|$ is the cardinal number of the field F.

Remark 2. In case $n = 1$, M. Ferrero and K. Kishimoto [**2**, Lemma 2] have shown that if B is a ring and δ is a derivation of B, then the map $X \longrightarrow b_0 + Xb_1$ induces a B-automorphism of $B[X;\delta]$ if and only if b_1 is a central unit and

$$b_0 b - b b_0 = \delta(b)(b_1 - 1) \quad \text{for all } b \in B.$$

Noting $D(y) = 1$, one will easily see the map $X \longrightarrow a_0 + Xa_1$ induces an A-automorphism of R if and only if $a_1 = 1$. Thus, one can consider Theorem 1 to contain the case $n = 1$.

Examples. Let $\alpha = 0$ i.e., $A = F[x]/(x^p)F[x]$.

1. Suppose that $p = 2$. Let maps ϕ_1 and ϕ_2 be A-endomorphisms of R induced by

$$X \longrightarrow X + X^2 y \quad \text{and} \quad X \longrightarrow X + X^2 y\beta \quad (\beta \neq 1 \in F)$$

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respectively. Then, by the condition (b-i) in Theorem 1 (or the condition (b-i)′ in Remark 1), ϕ_1 is not an A-automorphism of R. But ϕ_2 is an A-automorphism of R by Theorem 1. When this is the case, then

$$\phi_1(X^2) = 0 \quad \text{and} \quad \phi_2^{-1}(X) = X - X^2 y(\beta + 1)^{-1}\beta.$$

2. We shall make a note about an interesting property of the coefficients of

$$\phi(X) = \sum_{i=0}^{n} X^i a_i.$$

It is easily seen by the condition (b-ii) (or (b-ii)′) that a_{pi} don't have to be nilpotent for all $i \geqq 2$, though the map ϕ is an A-automorphism of R. Actually, we know by Theorem 1 that the map

$$X \longrightarrow X + X^p + X^{p^2} y^{p-1}$$

induces an A-automorphism of R, though $a_p = 1$ is not nilpotent. This shows that there exists an automorphism whose form is quite different from ones known before now, because all results in [**1**], [**2**], [**3**] and [**5**] have shown that a_i $(i \geqq 2)$ must be nilpotent for the map $X \longrightarrow \sum_{i=0}^{n} X^i a_i$ to induce a B-automorphism of a commutative or skew polynomial ring over a ring B.

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