mation, then the second extreme case given by $k = 1$ seems to be the most satisfactory. It agrees fairly closely with experience and it gives at the same time a simple rule, at least when only one string is involved. It also gives the maximum total energy with which the given impulse can provide the system, the minimum being given by $k = 0$; this follows from (3). The rule may be stated thus :—

If impulsive forces are applied to a system which includes one inextensible string, first calculate the initial motion and the energy of the system under the same forces but ignoring the string. Now return to the given system with the string and calculate the motion with the given amount of energy reckoned above. This energy equation is the additional information required to account for an unknown impulsive tension in the string.

It is evident that the problem becomes complicated algebraically when more than one string is involved. In principle, the results will depend upon the ratios of the (large) moduli and upon the values of products such as $\tau\lambda^{\frac{1}{2}}$. If such products are taken to be zero, as suggested above, then the total energy can be calculated as before, but it would still be necessary in general to consider the differential equations of motion in order to solve the problem completely. There is a need for more rules to deal adequately with such problems.

A proof of the "Theorem of the Means."

By C. E. WALSH.

Numerous proofs have been given of this familiar theorem,[1] which states that if $a_1, a_2, \ldots, a_n$ are positive, and not all equal, then

$$a_1^n + a_2^n + \ldots + a_n^n > n a_1 a_2 \ldots a_n.$$

The following is an elementary proof by induction, which I

[1] See *e.g.* Hardy, Littlewood & Pölya *Inequalities*, where many references will be found.

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have not seen used before. It is, of course, not claimed to be novel, and not likely to be so.

We have, easily,

$$n\,(a_1^n + a_2^n + \ldots. + a_n^n) - (a_1^{n-1} + a_2^{n-1} + \ldots. + a_n^{n-1})\,(a_1 + a_2 + \ldots. + a_n)$$
$$= \tfrac{1}{2} \sum_{p=1}^{n} \sum_{q=1}^{n} (a_p^{n-1} - a_q^{n-1})\,(a_p - a_q) > 0$$

since any term $(a_p^{n-1} - a_q^{n-1})\,(a_p - a_q)$ on the right which does not vanish, is positive, being composed of two factors with the same sign. There must be at least one positive, as the a's are not all equal.

Hence,

$$\frac{a_1^n + a_2^n + \ldots. + a_n^n}{a_1^{n-1} + a_2^{n-1} + \ldots. + a_n^{n-1}} > \frac{a_1 + a_2 + \ldots. + a_n}{n}. \qquad (1)$$

Let us suppose, now, that the theorem of the means holds good for $n - 1$. Then, omitting $a_1, a_2, \ldots a_n$ in turn, there are n inequalities of the form

$$a_2^{n-1} + a_3^{n-1} + \ldots. + a_n^{n-1} > (n - 1)\, a_2 a_3 \ldots. a_n.$$

As $n - 1$ of the a's can be equal, there may be equality in one, but not more than one, of these. Adding them all, and dividing by $n - 1$, we get

$$a_1^{n-1} + a_2^{n-1} + \ldots. + a_n^{n-1} > a_1 a_2 \ldots. a_n\,(1/a_1 + 1/a_2 + \ldots. + 1/a_n). \quad (2)$$

Finally, multiplying (1) and (2),

$$a_1^n + a_2^n + \ldots. + a_n^n > \frac{a_1 a_2 \ldots. a_n\,(a_1 + a_2 + \ldots. + a_n)\,(1/a_1 + 1/a_2 + \ldots. + 1/a_n)}{n}$$

$> n a_1 a_2 \ldots. a_n$, since, as is easily shewn

$$(a_1 + a_2 + \ldots. + a_n)\,(1/a_1 + 1/a_2 + \ldots. + 1/a_n) > n^2.$$

The theorem thus holds for n. Being true when $n = 2$, it is therefore, by induction, true for all n.

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