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Mathematical Theory of Motion of Revolving Axes on the Surface of Planets

T.S. Kozhanov, Nizyarov N.

The Kazakh State Agrarian university, Almaty, Kazakhstan

1. The statement of problem.

Let a planet perform translational and rotational motions in the field of solar attraction. Let's assume that the observer on the surface of the planet, knows (even approximately) an orbit and variations of orientation. It is necessary to clarify the motion of the instanteous rotation axis on the planet's surface from the observer's point of view on the planet's surface.

1. The coordinate system. to describe the translational and rotational motions of planets around the Sun we shall take into account the properties of orbits of solar system planets, namely:

1) All planets move in the same direction as the Sun revolves.

2) At the present time, from June until December the Earth's inhabitants see the north pole of the Sun and during the second half of year the southern one (Beleckei 1975, Menzel 1959).

Let's choose the following systems of reference in space around the Sun $C\xi\eta\zeta$, $CA_0P_0W_0$, $OR^0Q^0W^0$, $Oxyz$ (Fig. 1 and Fig. 2). The system of coordinates $C\zeta\eta\xi$ is connected with Sun. The axis $O\zeta$ is the rotation axis of the Sun. The axes $O\xi$, $O\eta$ lie in the Sun's equatorial plane. The system of coordinates $CA_0P_0W_0$ is connected with the plane (EE) of the orbit of the planet. The axes CA_0, CP_0 lie in the plane of the orbit. The axis CA_0 is connected to the axis of the orbit. The system of coordinates $OR^0Q^0W^0$ is named the orbital system of coordinates. The direction of axis OR^0 is the prolongation of the radius-vector from the center of the Sun ($\overline{CO} = \overline{R}$), and axis OQ^0 is located in the plane (EE). The system of coordinates $Oxyz$ is connected to the planet. Axis Oz coincides with the axis of rotation of the planet. Axes Ox, Oy lie in the plane of the planet's equator (e_1e_1). The lines (DCJ), $(\gamma_1 O\gamma)$ are the lines of the intersection of the planes (ee), (EE) and (e_1e_1), (EE).

The angles $l(u_j)$, I and ψ, θ determine the mutual position of the planes (ee), (EE) and (e_1e_1), (EE). Herewith u_j, ψ determine the position of straight lines CJ, $O\gamma$, respectively, of the fixed direction $C\gamma_0 || O\gamma_0$ in the plane (ee). The angle u_j determines the position of the major axis of the orbit relative to the line CJ. The angle ν determines the position of the planet itself relative to the axis CA_0. The angle φ determines the character of the rotational motion of the planet around axis Oz relative to $O\gamma$.

Their derivatives with respect to time determine the angular velocities of the revolving planet relative to the axes $C\zeta$, CJ, OW^0, CW_0, $O\gamma$, Oz. For an

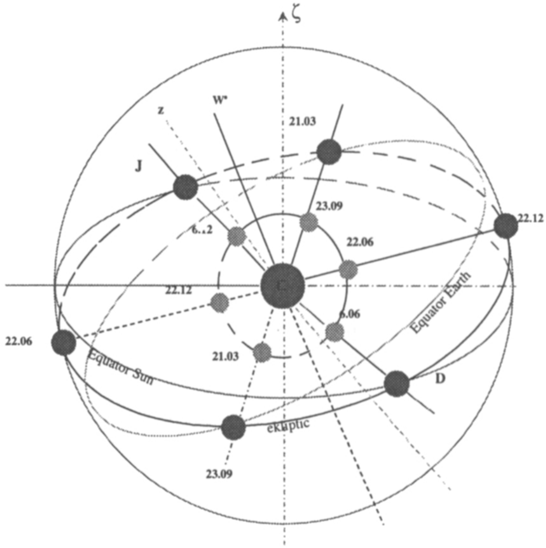

Figure 1.

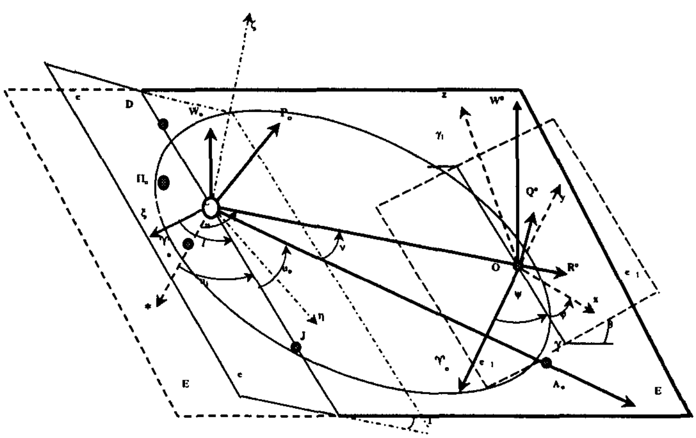

Figure 2.

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observer located on the surface of planet, the regularities of the variations of these angles periods is supposed to be known.

2. The equations describing the variations of the components of angular velocity Ω of planets. Let's assume, that the axes $Oxyz$ coincide with the main axes of the ellipsoid of inertia of the planet at the point O - the center of rotation. The variation of angular momententum $\overline{G}$ in the system $Oxyz$ is described by the Euler's equation

$$\frac{d'\overline{G}}{dt} + \overline{\Omega} \times \overline{G} = \frac{\overline{\partial U}}{\partial R^0} \times \overline{R^0}, \tag{1}$$

where

$$p = a(1 - \varepsilon^2), \tag{2}$$

$$U = \frac{\mu M}{R} + \frac{\mu(A+B+C)}{2R^3} - \frac{\mu M(x_{0_1}\gamma_1 + y_{0_1}\gamma_2 + \gamma_3)}{R^2} - \frac{3\mu(A\gamma_1^2 + B\gamma_2^2 + C\gamma_3^2)}{2R^3}, \tag{3}$$

$$R = \frac{a(1-\varepsilon^2)}{1 - \varepsilon\cos(u - u_{A_0})} \quad \frac{d(u-u_{A_0})}{dt} = \sqrt{\frac{\mu}{p^3}}(1 - \varepsilon\cos(u - u_{A_0}))^2, \tag{4}$$

$$\begin{aligned} \gamma_1 &= \cos(Ox \wedge OR^0) = \cos\varphi\cos(u-\psi) + \sin\varphi\sin(u-\psi)\cos\theta, \\ \gamma_2 &= \cos(Oy \wedge OR^0) = -\sin\varphi\cos(u-\psi) + \cos\varphi\sin(u-\psi)\cos\theta, \\ \gamma_3 &= \cos(Oz \wedge OR^0) = -\sin(u-\psi)\sin\theta, \quad u_{A_0} = u_J + u_0, \quad u = u_{A_0} + \nu. \end{aligned} \tag{5}$$

A, B, C are the main moments of inertia relative to Ox, Oy, Oz; x_{0_1}, y_{0_1}, z_{0_1} are the coordinates of the center of mass in the system of coordinates $Oxyz$; a is the semi-major axis of the ellipse, described by a planet in motion around the Sun; ε - the eccentricity of this ellipse.

The instantaneous angular velocity Ω, defining the position of the pole on the surface of Earth, is equal to the sum of vectors representing the various oscillatory motions. The major axes of these ellipses coincide with the axis Oy, and the minor one - the axis Ox of the ellipsoid of inertia corresponding to the point O.

2. Conclusion

1. From the point of view of the observer on the surface of the Earth, the instantaneous rotation axis performs interconnected oscillatory motions on its surface.

2. The trajectory of free oscillation of the instantaneous axis on the surface of the dynamically asymmetrical Earth (planet) will be an ellipse, whose major axis coincides with the mean axis of the ellipsoid of inertia. The rotational period of the instantaneous axis on this ellipse is determined by the dynamic characteristics of the Earth (planet), and their change is the reason for a change of period and the damping of this oscillation. The larger value of the period of free oscillation corresponds to the greater amplitude.

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3. The incongruity of the center of mass with the point, around which the rotational motion of the Earth happens, is the reason for the forced oscillations of the instantaneous axis. Their periods are determined by periods of the Earth (planet) around the Sun, and around its own axis, instead of the dynamic characteristics of the Earth (planet). The oscillation amplitudes, whose periods are close to the period of rotation are proportional to the deviation of the center of rotation on the meridional plane, and the oscillation amplitudes, whose periods are determined by periods around the Sun, to the distance from center of mass to the center of rotation on the equatorial plane.

4. On the equatorial plane of the ellipsoid of inertia, the center of mass of the dynamically asymmetrical Earth will describe an ellipse around the rotation center. The motion of the center of mass of the Earth on this ellipse is opposite to the Earth rotation. The center of mass of the Earth performs forced oscillations also on the meridianal plane of the ellipsoid of inertia.

5. The periods of the amplitude variations of an instantaneous rotation axis on the surface of the Earth (planet) can be determined.

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