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## INTRODUCTION

Starting with the initial understanding that pulsation in variable stars is caused by the heat engine of Hydrogen and Helium ionization in their atmospheres (A.S. Eddington in Cox 1980) it was soon realized that non-linear effects were responsible for the detailed features on their light and velocity curves. With the advent of the computer we were able to solve the coupled set of hydrodynamics and radiation diffusion equations to model these non-linear features (Christy 1968, Cox et. al. 1966). Calculations including the effects of multi-frequency radiative transfer (Davis 1975) showed that grey diffusion was adequate for modeling Cepheids but not for the RR Lyrae or W Virginis type variables. In 1977, in collaboration with J. Castor and D. Davidson (1977), we developed a non-Lagrangian method to resolve the region of the ionization front and remove the zoning effects previously found in theoretical light curves (Keller and Mutschlecner 1972). The new code (DYN) has been used in recent studies by Takeuti (1983), with Simon (1983), in an attempt to understand modal coupling, and with Moffett and Barnes (1981) in a detailed study of X Cygni. A minor success for DYN was obtained by comparing "phase lags", as determined from the first order modal expansions between maximum light and maximum outward velocity, with some low period Cepheid observations (Simon 1984). Another support for the "dips" as observed by Moffett and Barnes (1984) is the observation by Schmidt and Parsons (1982) of a "dip" in the IUE spectra of  $\delta$  Cep near the phase of 0.85.

In this paper we want to describe some recent model results for long period (LP) Cepheids in an attempt to understand these "dips" and possible get another handle on Cepheid masses. In section II we discuss these results and in section III we consider the implications of these model results on the problem of the Cepheid mass discrepancy.

## LONG PERIOD CEPHEID MODELS

It appears that a classification of Cepheids could be: low amplitude low period sinusoidal Cepheids, "bump" Cepheids with periods from 7 to 10 days, and long period Cepheids with periods longer than 10 days. In this paper we are considering this latter class of variables. The models we have selected have effective temperatures near the middle of the instability strip and nearly constant at 5300 or 5500 K. We are mainly interested in seeing the effects of mass on the light curves so we selected near evolutionary masses and 60% or so of the evolutionary masses. The period range was picked in the range from 12 to 22 days with selected stars from the Moffett and Barnes list for comparisons (Table I).

TABLE T						
PERIOD (PROTOTYPE)	LC Lum	DNG PERIC LOW MASS TEFF	D CEPHEID P2/P0	MODELS LUM	HIGH MASS TEFF	P2/P0
12.9 (Z SCT)	1.5	4.5 M .54	.499	?.5	7.5 M .55	.536
14.8 (RW CAS)	2.1	5.0 M .551°	.497	3.0	9.5 M .54	.527
16.4 (X CYG)	2.45	5.7 M .54	.486	3.34	9.0 M .53	.520
22.0 (WZ SGR)	3.5	6.0 M .54	.475	5.5	10.0 M .54	.507
	LUM (x	10 <sup>37</sup> er g/o	em <sup>2</sup> -S) TE	FF(x10 <sup>4</sup>	°K)	

First we note, from Table I, that the low and high mass models stradle the linear theory resonance boundary parameter of  $P_2/P_0=0.5$ . Near 0.5 the resonance interaction is the strongest and the Christy "bump" should appear near maximum light. A reasonable explanation of the connection between resonance theory and the "echo" is given by Whitney (1983). In Fig. 1 we illustrate the DYN results of this phenomena for model of a 10 day Cepheid with evolutionary mass and 60% of evolutionary mass. In the low mass model the "echo", which causes the "bump" at the surface, can clearly be seen near the inner boundary.



a) low mass (4.0 Mg) and b) high mass (7.4 Mg) using the DYN code and KinglVA opacities.

Now it is clear that simply eyeballing the resulting light curves for comparison to observations is highly subjective (Simon and Davis 1983) but for these long period Cepheids, the spiked peak makes Fourier analysis difficult. It is also possible that the so called "bumps" or shoulders on the rising branch of LP Cepheid light curves could be the "dips" now resolved by Moffett and Barnes. These "dips" appear to be due to a surface phenomena that is more easily described by the methods of dynamic zoning used in DYN. Even though the shock is treated by the use of "Pseudo-viscosity" it is well resolved by the fine zones included in the shock forming region. In the atmosphere of the star the strongest shock that develops is the one caused by the stopping of the infall of the envelope near the phase of maximum inward velocity. This shock gives rise to the "artificial viscosity dip" discussed by Davis (1975) and others. In most models another shock develops at the time of rapid expansion. This shock is associated with the brief transition of the ionization front from a "D" to a "R" type (Kahn's notation see Castor and Adams 1974) as it moves rapidly inward in mass. This inward shock remains below optical depth unity and therefore should not effect the light curves. Without the capability to follow line transport in a moving media we must rely on the photospheric continuum results for our comparisons to the observations.



## THE RESULTS AND CEPHEID MASSES

In modeling the non-linear hydrodynamics that occurs in pulsating stars we have considered the effects of radiative transfer but not the effect of convection. It is believed that convection causes the limit to the red edge of the instability strip but is not very important to the light output otherwise. For long period Cepheids (observe Fig. 2) we generally find a "dip" in the rising part of the light curve which signals "shock dissipation" and possibly the limit on the amplitude of the pulsation. These "dips" may have previously been mistaken for "bumps" or shoulders and therefore the resonance effect. As discussed in our paper on X Cygni to remove the bumps on the light curves we need to use masses near the evolutionary masses. More and detailed velocity measurements, in synchronism with the light measurements, may help answer the question, "Are there "bumps" as well as "dips" in the long period Cepheid light curves?"

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