

Index

Index of symbols. These are grouped according to the type of object to which they most closely relate.

Groups

\dagger : operation on quasi-invertible elements, 13
 $\prod_{i \in I} G_i$: product of groups, 45
 $g^0 = 1, g^{-i} = (g^{-1})^i$, 194
 $GL_n(R)$: general linear group, 9
 $QU(R)$: quasi-invertible elements, 13
 $U(R)$: units of a ring, 9

Homomorphisms

\cong : isomorphism, 6
 $\alpha_1 \oplus \cdots \oplus \alpha_k$: direct sum of homomorphisms, 42
 χ : characteristic homomorphism, 6
 $F(\lambda)$: induced map on torsion-free quotient, 247
 id_M : identity homomorphism, 19
 ι : a canonical embedding, 47
 $\iota: R \rightarrow M_s(R)$, diagonal map, 62
 $T(\lambda)$: induced map on torsion part, 247
 $T_p(\lambda)$: induced map on primary component, 245
 $\text{Hom}(M, N)$, $\text{Hom}({}_R M, {}_R N)$: homomorphisms of (left) modules, 19
 $\text{Hom}(M_R, N_R)$: homomorphisms of right modules, 18

Ideals

$r \equiv s \pmod{\mathfrak{a}}$: congruence, 7
 $\{\mathfrak{a}\}$: class of \mathfrak{a} , 191
 (x_1, \dots, x_n) : ideal, 5
 $1 + \mathfrak{j}$, \mathfrak{j} an ideal, 176
 $\mathfrak{a}, \mathfrak{b}, \dots$: ideals, 4

$\mathfrak{a} \mid \mathfrak{b}$: divides, 193
 $\mathfrak{a}^2, \mathfrak{a}^k$: powers of ideal, 179
 $\mathfrak{a}\mathfrak{b}$: product ideal, 5, 189
 $\mathfrak{c}(\mathfrak{p})$: complement of \mathfrak{p} , 243
 $\text{Ann}(\mathfrak{x})$: annihilator, 24
 $\text{Fit}_k(A)$: Fitting ideal, 141
 $\text{Frac}(\mathcal{O})$: fractional ideals, 189
 $\text{In}(R)$: invertible ideals, 200
 $\text{Ker } f$: kernel of ring homomorphism, 6
 mR : infinite matrices, almost all entries 0, 171
 \mathbf{P} : nonzero prime ideals, 194
 \mathbf{Pe} : representative nonzero primes (irreducibles), 196
 $\text{Pr}(\mathcal{O})$: principal ideals, 189

Matrices

$(m)^F$: coordinate vector, 65
 ${}_F(m)$: coordinate vector, 56
 A^t : transpose of a matrix, 34
 $A^{FG}(\alpha)$: matrix of a homomorphism, 66
 $A_{GF}(\alpha)$: matrix of a homomorphism, 58
 $M_{t,s}(R)$: set of $t \times s$ matrices, 58
 $\mu_F(\alpha)$: matrix with respect to F , 60
 $\mu^F(\alpha)$: matrix with respect to F , 66

Modules

$\bigoplus_I L_i$: external direct sum, 44
 $\bigoplus_\Lambda L_\lambda$: external direct sum, 46
 $M = \bigoplus_{i \in I} M_i$: (internal) direct sum, 43
 $M = \bigoplus_{\lambda \in \Lambda} M_\lambda$: (internal) direct sum, 46
 $\prod_I L_i$: direct product, 44
 $\sum_{i \in I} M_i$: sum of submodules, 21
 $\alpha M'$: image of submodule, 23
 $\alpha^{-1} N'$: inverse image of a submodule, 23
 α_i : multiplicity of a component, 241
 $\text{Cok } \alpha$: cokernel, 76

- $\text{edt}(M)$: elementary divisor type, 245
 $\text{edt}_p(M)$: elementary divisor type, 241
 $\text{Fr}_R(X)$: standard free module, 47
 $\text{Im } \alpha$: image of module homomorphism, 22
 $\mathcal{I}(R)$: representative irreducible modules, 151
 $\text{Ker } \alpha$: kernel of module homomorphism, 22
 $L^{(\Lambda)}$: direct product, 46
 L^Λ : direct sum, 46
 L^k : direct sum, 53
 $L_1 \odot \cdots \odot L_k$: external direct sum, 40
 $m + M = \bar{m}$: equivalence class, 22
 ${}_R M$: left module, 16
 ${}_R M_S$: bimodule, 17
 M/M' : quotient module, 22
 M° : opposite module, 19
 $\{M\}$: ideal class of a module, 228
 $M_1 + \cdots + M_k$: sum of submodules, 21
 $M_1 \oplus \cdots \oplus M_k$: direct sum, 37
 M_R : right module, 16
 $M\mathfrak{a}$: module times ideal, 24
 $\text{mult}(M)$: multiplicity type, 151
 R^Λ : free module, 47
 R^n : free right module, 17
 ${}^n R$: free left module, 17
 $(\Lambda)R$: direct product, left module, 47
 ${}^\Lambda R$: direct sum, left module, 47
 $\text{rad}(M)$: radical, 173
 $\text{Sp}_R(X) = \sum_{i \in I} x_i R$: span, 21
 $T(M)$: torsion submodule, 238
 $T_p(M)$: p -component, 245
 $T_{\mathfrak{p}}(M)$: \mathfrak{p} -primary component, 243
- Number theory, factorization**
- $\|$: total division, 126
 $\|a\|, \|a\|$: norm, 215
 $[\mathcal{K} : \mathcal{Q}]$: degree, 201
 $C_x(X)$: characteristic polynomial, 204
 $\text{Cl}(\mathcal{O})$: ideal class group, 191
 (d/p) : Legendre symbol, 214
 γx : conjugate of x , 204
 Nx : norm, 204
 $\text{Tr}x$: trace, 204
 \hat{p}, p_1, p_2 : primes above p , 213
 $v(p, x)$: exponent of p in x , 197
 $v(\mathfrak{a})$: valuation, 231
 $v(\mathfrak{p}, \mathfrak{a})$: exponent in \mathfrak{a} , 194
 (I) : inert, 213
 (R) : ramified, 212
 (S) : split, 212
- Rings**
- \mathbb{C} : the complex numbers, 2
 \mathbb{H} : quaternions or Hamiltonians, 67
 \mathbb{N} : the natural numbers, 26
 \mathbb{Q} : the rational numbers, 2
 \mathbb{R} : the real numbers, 2
 \mathbb{Z} : the integers, 2
 $0_R, 1_R$: zero, identity of ring R , 3
 $A[\epsilon]$: ring of dual numbers, 79
 AG : group ring, 106
 $A[T, \alpha]$: skew polynomial ring, 116
 $A\langle X_1, \dots, X_k \rangle$: noncommutative polynomials, 11
 $C(a, \alpha)$: skew centralizer, 172
 CR : cone of R , 70
 \mathcal{D}^α : invariant elements, 124
 $\text{End}(M_R)$: endomorphism ring of M_R , 18
 $\text{End}({}_R M)$: endomorphism ring of ${}_R M$, 19
 H : Hurwitz quaternions, 130
 $\text{Im } f$: image of ring homomorphism, 6
 \mathcal{K} : a field, 9
 $\mathcal{K}(T, \alpha)$: skew rational functions, 118
 $M_\omega^{cc}(E)$: column convergent matrices, 72
 $M_\Lambda^{cf}(R)$: column-finite square matrices, 65
 $M_{\Sigma, \Lambda}^{cf}(R)$: column-finite matrices, 65
 $M_{\Lambda, \Sigma}^{rf}(R)$: row-finite matrices, 66
 $M_\Lambda^{rf}(R)$: row-finite square matrices, 66
 $M_n(R)$: matrix ring, 3
 \mathcal{O} :
 a commutative ring, usually a domain, 9
 ring of integers, 203
 \mathcal{O}_p : valuation ring, 232
 $Q(X)$: rational functions, 118
 R, S, T, \dots : rings, 2
 \bar{R} : unitalization, 13
 R° : opposite ring, 19
 R/\mathfrak{a} : residue ring, 7
 $R[T]$: polynomial ring, 3
 $T(X)$: tiled matrices, 230
 $T_2(R)$: triangular matrix ring, 86
 $Z(R)$: centre, 4
 \mathcal{Z} : principal ideal domain, 196
 $\mathcal{Z}[x]$: x adjoined to \mathcal{Z} , 202
- Sequences**
- \mathbf{E} : short exact sequence, 76
 $\text{Ext}_R^1(M'', M')$: group of extensions, 80
 $\phi_* \mathbf{F}$: push-out of sequence, 83
 $\text{SES}(M'', M')$: set of extensions, 88
 $\theta^* \mathbf{E}$: pull-back of sequence, 83
- Sets**
- \subset : strict inclusion, 4
 $I \setminus \{i\}$: I without i , 43
 ω : ordered natural numbers, 26

Index of names

Armitage, V., 221
 Camillo, V., 124
 Cohn, P. M., 161
 Hamilton, W. R., 131
 Hartshorne, R., 221
 Hurwitz, A., 130
 Kneser, M., 218
 Qin Jiushao, 188
 Stillwell, J., 186
 Sun Zi, 188

Index of terms

abelian group, 2
 as \mathbf{Z} -module, 16
 above, 211
 act(ing) as
 α , 29
 a , 171
 a matrix, 16
 additive on short exact sequences, 152
 algebraic integer, 201
 algebraically closed field, 144
 all except a finite set of indices, 5
 almost all, 5
 annihilates, 24
 annihilator, 24, 33, 141, 239
 anti-automorphism, of a ring, 34, 67
 Artinian
 module, 146
 ring (right, left), 146
 Artinian not Noetherian, 160, 161
 Artinian semisimple ring, 162
 Artinian simple ring, 162
 Artinian Splitting Theorem, 154
 ascending chain, 110
 condition, 111
 finite, 111
 infinite, 111
 proper, 111
 terminates, stationary, 111
 associated
 matrices, 61
 elements, 128, 135
 atom, 128
 automorphism of a module, 22
 Axiom of Choice, 46
 base change, 83
 basis, 55
 of a vector space, 26
 becomes a left module, 20
 below p , 211
 bimodule, 17
 balanced, 20
 symmetric, 20
 block diagonal matrix, 51
 bounded module, 141
 cancellation, 229
 canonical embedding, 47
 canonical form, 61
 canonical homomorphism, 23
 cartesian
 product, 44, 46
 square, 81
 centralizer, 63
 centre
 of a ring, 4
 of a skew polynomial ring, 124
 chain, 26
 ascending, 110
 descending, 146
 change of rings, 24, 177
 characteristic
 homomorphism, 6
 of a ring, 8, 184
 characteristic series, 158, 183
 characteristic submodule, 157
 Chinese Remainder Theorem, 188
 class group, 191
 class number, 218
 class, of an ideal, 191
 clearing denominators, 189, 226
 co-fibre-product, 81
 cobase change, 83
 cocartesian square, 82
 cofibre square, 82
 cokernel, 76
 column vector, 46
 column-convergent matrix, 72
 column-finite matrix, 56, 64
 comaximal, 101
 ideals, 188
 common denominator, 189
 common left divisor, 122
 companion matrix, 142
 complement, 38
 Complete Reducibility Theorem, 154
 completely reducible, 154
 componentwise, 40, 44, 98, 101
 composition factors, 149
 composition series, 148
 cone of a ring, 70, 73, 74, 113, 115, 170
 congruence class, 7
 congruence on natural numbers, 73
 congruent modulo an ideal, 7
 conjugate
 elements, 172
 idempotents, 93
 in quadratic field, 204
 matrices, 32, 61, 62

- quaternion, 67
- conjugation automorphism, 223
- connecting homomorphism, 84
- coordinate ring, 222
 - of circle, 222
- coordinate vector, 30, 57, 65
- coprime elements, 143
- coprime ideals, 188, 243
- cyclic algebra, 172
- cyclic module/submodule, 21

- decomposed, 212
- Dedekind domain, 191
- degree
 - of a polynomial, 116
 - of an extension, 201
- derivation, 128
- derivative of a polynomial, 209
- descending chain, 146
 - condition, 146
 - finite, 146
 - infinite, 146
 - proper, 148
 - terminates, 146
- Diagonal Reduction Theorem, 133
- dimension, 163
- dimension type, 151
- direct factors, 38
- direct product, 44–46, 96
 - of copies of L , 46
 - of rings, 98, 99
- direct sum
 - external, 40
 - internal, 37
 - of copies of L , 46
 - of sequences, 83
- discrete valuation, 232
- division algorithm, 8, 120, 121, 216
- division for ideals, 193
- division ring, 9, 67
 - modules over, 163
 - polynomials over, 119–132, 159, 171
- divisor, 120
- divisors of degree zero, 222
- domain, 9, 119
 - commutative, 238
 - noncommutative, 160
 - not in a division ring, 13
 - principal ideal, right, 135
- dual numbers, 79, 95, 174, 199, 210

- Elementary Divisor Theorem, 160
- elementary divisor type, 241, 245
- elementary divisors, 241, 244
- elementary operations, 133
- elliptic curve, 222
- endomorphism, 18
 - finite rank, 171
 - of a ring, 116
 - proper, 124
 - endomorphism ring, 18
 - block form, 53
 - of direct sum, 103
 - entire ring, 9
 - enveloping ring, 35, 106
 - equivalent matrices, 61
 - Euclid's algorithm, 122
 - Euclidean domain, 121, 128, 133, 216
 - right, 159
 - Euler's Theorem, 130
 - exact at M , 75
 - exact sequence, 75
 - long, 76
 - split, 78
 - standard, 77
 - term, 76
 - extends (to), 49
 - extension
 - field, 201
 - of modules, 76
 - of ζ , 49
 - ring, 4, 112
- external direct sum, 40, 44, 46

- fibre product, 81
- fibre square, 81
- field, 9
 - extension, 201
 - global, 221
 - of fractions, 9
 - of rational functions, 118, 123
 - splitting, 209
- finite, 201
 - group, 184
 - inner order, 124
- finite extension, 201
- finitely generated module, 22
- Fitting ideal, 141
- fractional ideal, 189
 - of an order, 199
 - principal, 189
 - product, 189
- free
 - associative algebra, 11
 - left module, 17
 - presentation, 140
 - right module, 17
 - subset, 48
- full set
 - of idempotents, 93
 - of inclusions and projections, 41
 - standard, 78
 - of orthogonal central idempotents, 99
 - of orthogonal idempotents, 43, 100

- fully invariant
 - series, 158
 - submodule, 157
- function field case, 203
- Galois group, 223
- Galois module, 223
- Gauss' Lemma, 201
- Gaussian integers, 211
- general linear group, 9
- generators of (sub)module, 21
- generators of an ideal, 5
- global field, 221
- greatest common divisor, of ideals, 199
- greatest common left divisor, 122
- group
 - cyclic, 106
 - of quasi-invertible elements, 13
 - of units, 9, 175
 - ring, 106, 113, 236
 - complex, 106
- group algebra, 184
- Hamiltonians, 67
- hereditary order, 237
- Hilbert Basis Theorem, 118
- homological algebra, 104
- homomorphism
 - canonical, 23
 - composition, 18
 - composition product, 18
 - identity, 19
 - induced, 23
 - injective, 22
 - inverse, 22
 - module, 17
 - of a general object, 17
 - product, 18
 - ring, 5
 - surjective, 22
 - zero, 18
- Hopkins–Levitzki Theorem, 179
- Hurwitz quaternions, 130
- ideal
 - class of, 191
 - divisors, 211
 - fractional, 189
 - generated by X , 5
 - integral, 189
 - invertible, 190
 - left, 4
 - maximal, 12
 - maximal right, 25
 - minimal, right, 162
 - nilpotent, 179
 - number, 211
 - prime, 187
 - principal right, 5
 - product, 5
 - proper, 4
 - right, 4
 - zero, 4
- ideal = twosided ideal, 4
- ideal class group, 191
- ideal class, of a module, 228
- idempotent(s), 43, 93
 - central, 99
 - primitive, 105
 - conjugate, 93
 - orthogonal, 43
 - full set, 43, 93, 100
 - orthogonal central
 - full set, 99
 - standard set, 99
- identity homomorphism, 19
- image
 - of module homomorphism, 22
 - of ring homomorphism, 6
- indecomposable module, 38
 - projective, 180
- independent subset, 48
- index of an ideal, 215
- Induced Mapping Theorem for
 - modules, 23
 - rings, 8
- inductive set, 26
- inert, 213
- inner automorphism, 124
- inner order, 124, 131
- integer, 201
- integral domain, 9
- integral quaternions, 130
- integrally closed, 206
- internal direct decomposition, 38
- internal direct sum, 37
 - infinite, 43
- intersection irreducible, indecomposable, 115
- invariant basis number, 69, 112
- invariant factor decomposition, 137
- invariant factor form of matrix, 134
- Invariant Factor Theorem, 136, 137
 - for valuation rings, 235
- invariant factor(s), 235
 - of a matrix, 134
 - of a module, 137
 - uniqueness, 141, 250
- invariant polynomial, 125
- invariant subspace, 30
- inverse image, 23, 197
- inverse, ring homomorphism, 6
- invertible element, 8
- invertible ideal, 190

- of an order, 200
- irreducible element, 128, 188
 - vs.* prime, 197, 223
- irreducible module, 24
- isomorphism
 - of modules, 22
 - of rings, 6
- Isomorphism Theorems, 28
- Jacobson radical, 174, 236
- jokes, 81, 84
- Jordan block matrix, 143
- Jordan normal form, 143
- Jordan–Hölder Theorem, 150
- \mathcal{K} -algebra, 113
 - finite-dimensional, 113
- kernel
 - of module homomorphism, 22
 - of ring homomorphism, 6
- Klein four group, 220
- law of quadratic reciprocity, 215
- leading coefficient of polynomial, 116
- least common multiple, of ideals, 199
- Leavitt ring, 71
- left Artinian ring, 146
- left divisor, 120
- Legendre symbol, 214
- length
 - of $\text{edtp}(M)$, 241
 - of composition series, 148
- linearly independent, 26, 48
- local ring, 181, 233
- localization, 232
- locally finite matrix, 70
- Loewy series, 158
- Maschke's Theorem, 184
- matrix
 - $\Lambda \times 1$, 46
 - column-finite, 56, 64
 - of a linear map, 30
 - represents homomorphism, 61
 - row-finite, 66
 - scalar, 62, 68
 - transpose, 34, 69
 - twosided invertible, 59
- matrix units, standard set, 12
- maximal
 - element, 25
 - ideal, 12
 - order, 209, 237
 - right ideal, 25
 - submodule, 25, 149, 173
- maximum condition, 111
- method of descent, 123
- minimal
 - polynomial, 201
 - right ideal, 162
 - submodule, 152
- minor, 141
- module
 - Artinian, 146
 - becomes left, right, 20
 - bounded, 141
 - completely reducible, 154
 - cyclic, 21, 135
 - decomposable, 38
 - divisible, 97
 - factor, 22
 - finitely generated, 22
 - free (on X), 48
 - free left, 17
 - free right, 17
 - generators, 21
 - in standard form, 228
 - indecomposable, 38
 - injective, 95
 - irreducible, 24, 146
 - isomorphism, 22
 - left, 15
 - Noetherian, 109
 - over nonunital ring, 35
 - primary, 239
 - projective, 89
 - indecomposable, 180
 - quotient, 22
 - rank, 17
 - residue, 22
 - right, 15
 - semisimple, 152
 - simple, 24
 - stably free, 138
 - standard free, 47
 - standard free left, 47, 65
 - standard free right, 47
 - torsion module, 238
 - torsion-free, 28, 113, 238
 - unital, 15
 - zero, 16
- module-finite, 112
- monic, 116
- monoid, 2
 - abelian, 190
- monomial, 10
- multiplicity type, 151, 166
- multiplicity, of an irreducible module, 151
- Nakayama's Lemma, 175
- nilpotent ideal, 179
- Noetherian
 - module, 109
 - ring, 109

- non-cancellation, 94, 138
- noncommutative polynomial ring, 11, 71, 161
- nonunital ring, 3, 13, 106
 - module over, 35
- norm, 204
 - of an element, 215
 - of an ideal, 215
- normal form, 54, 61, 133
- normal polynomial, 125, 127
- number field case, 203

- onesided working, 34
- opposite ring, 19
- opposite the scalars, 17
- order, 113, 209, 236
 - hereditary, 237
 - maximal, 209, 237
 - tiled, 200, 230, 236
- ordered sequence, 46
- ordered set, 25, 45, 55
- ordinary polynomials, 117
- Ore extension, 129
- orthogonal idempotents, 43

- p -adic, p -adic valuation, 231
- pairwise comaximal, 105
- pairwise coprime, 143
- partially ordered set, 26
- p -component, 245
- p -component, 243
- p -elementary divisor type, 241
- permutation matrix, 67
- polynomial
 - left, 117
 - monic, 116
 - right, 116
- p -primary module, 239
- p -primary submodule, 245
- p -primary module, 239, 249
- p -primary submodule, 243
- Prüfer group, 160
- primary component, 243
- primary module, 239
- prime element, 187, 234
 - vs. irreducible element, 197, 223
- prime ideal, 187
- principal ideal, 5
- principal ideal domain, 196, 216, 219, 248
 - right, 97, 122
- principal ideal ring, 195
- principal valuation ring, 234
- product ideal, 5
- projection operator, 93
- proper endomorphism, 124
- pseudoring, 3
- pull-back
 - diagram, 80
 - modules, 81
 - of sequence, 83
 - square, 81
- push-out
 - diagram, 81
 - modules, 81
 - of sequence, 83
 - square, 82

- quadratic extension, 203
- quasi-invertible, 13, 176, 181
- quasicyclic p -group, 160
- quaternion algebra, 67
 - generalized, 172
- quaternions
 - Hurwitz (integral), 130
- quotient
 - in integers, 8
 - module, 22
 - ring, 7

- radical, 173
 - of ideal, 198
- radical series, 183
- ramification index, 211
- ramified, 212
- rank, 56
 - of a matrix, 134
 - of a module, 137, 228, 235, 239, 249
- rank one valuation, 232
- rational canonical form, 143
- refinement of a chain, 148
- remainder, 8
- repeated root, 209
- representative
 - of an ideal class, 217
- representative set
 - of nonzero primes or irreducibles, 196
 - of irreducible modules, 151, 165, 177, 178
- residue, 8
- residue class, 7
- residue ring, 7
- restriction of scalars, 24
- R -free, 48
- right Artinian ring, 146, 162
- right Artinian semisimple ring, 162
- right divisor, 120
- right Euclidean domain, 121
- right module-finite, 113
- right quotient, 120
- right remainder, 120
- R -independent, 48

- ring, 2
 - anti-automorphism, 34, 67
 - Artinian simple, 162

- block matrix, 53
 - centre of, 4
 - centre of matrix $-, 63$
 - characteristic of, 8
 - commutative, 3
 - cone, 94
 - direct product, 98
 - division ring, 9
 - domain, 97
 - entire, 9
 - enveloping, 13
 - extension, 4, 112
 - factor, 7
 - IBN, 69
 - local, 181
 - Noetherian, right, left, 109
 - non-IBN, 69
 - non-Noetherian, 109
 - noncommutative, 3
 - nonunital, 3, 13, 106
 - of algebraic integers, 203
 - of coefficients, 112
 - of column-finite matrices, 170
 - of dual numbers, 79, 95, 174, 199, 210
 - of endomorphisms, 18
 - block form, 53
 - of direct sum, 103
 - of noncommutative polynomials, 11
 - of triangular matrices, 86, 95, 96, 170, 182, 183, 250
 - generalized, 96, 115, 182
 - opposite, 19, 69, 131
 - ordinary polynomial, 117
 - prime, 8
 - principal ideal, 195
 - quotient, 7
 - residue, 7, 112
 - right Artinian, 146, 162
 - right Artinian semisimple, 162
 - semilocal, 180
 - simple, 11
 - skew polynomial, 116
 - trivial, 3
 - type, 106
 - zero, 3
- ring homomorphism, 5
 - canonical, 7
 - induced, 8
 - standard, 7
 - row vector, 47
 - row-finite matrix, 66
 - row-space, 65, 164

 - scalar multiplication, 15
 - Schur's Lemma, 157, 168
 - semigroup, 3
 - semiinvariant polynomial, 125, 127
 - semilocal ring, 180
 - semisimple module, 152
 - separable extension, 205, 208
 - separable polynomial, 205
 - Serre's conjecture, 140
 - set
 - of generators, 22
 - of matrix units, 63, 68
 - short exact sequence, 76
 - short exact sequences
 - equivalent, congruent, 88
 - similar elements, 128, 159
 - similar matrices, 62
 - simple module, 24
 - simple ring, 11, 162
 - skew
 - centralizer, 172
 - field, 9
 - polynomial ring, 116
 - rational functions, 118
 - Smith normal form, 134
 - Snake Lemma, 84
 - socle, 152
 - socle series, 158, 183
 - span, 21
 - split
 - at, over, 78
 - ideal, 212
 - splitting field, 209
 - squarefree, 141, 198, 203
 - stably free module, 138
 - standard
 - basis, 56
 - free left module, 47, 65
 - free module on Λ , 47
 - free right module, 47
 - full set of inclusions and projections, 78
 - inclusions, 41
 - projections, 41
 - scalar matrix, 62
 - unit vectors, 56
 - standard embedding
 - R in $M_s(R)$, 62
 - nonunital ring in ring, 13
 - standard form
 - for a module, 228
 - submodule, 21
 - characteristic, 157
 - cyclic, 21
 - fully invariant, 157, 175
 - generated by, 21
 - maximal, 25, 149, 173
 - minimal, 152
 - proper, 21
 - sum, 21
 - zero, 21
 - subring, 4

- sum of ideals, 5
- summands, 38

- term, of a sequence, 76
- Three Lemma, 84
- tilted matrices, 230
- tilted order, 200, 230, 236
- torsion element, 238
- torsion module, 238
- torsion submodule, 238
- torsion-free module, 28, 113, 238
- total degree, 10
- total divisor, 126
- totally ordered set, 25
- trace, 204
- transition matrix, 60, 65
- transpose matrix, 34, 69
- triangular matrix ring, 86, 95, 96, 115, 170, 182, 183, 250
 - generalized, 96
- trivial ring, 3
- twosided Euclidean domain, 121
- type
 - of a relation, 74
 - of a ring, 71, 185

- underlying subspace, 31
- uniformizer, 234
- uniformizing parameter, 234
- unique factorization, 127, 129
- unit, 8
- unit group, 9
- unitization, 13
- unramified, 212
- upper triangular block matrix, 32

- valuation, 231
- valuation ring, 232, 233
- vector space, 16

- Wedderburn–Artin Theorem, 162, 169

- zero
 - homomorphism, 18
 - ideal, 4
 - module, 16
 - polynomial, 116
 - ring, 3
 - submodule, 21
- \mathcal{Z} -integer, 201
- Zorn's Lemma, 26