

# Maximum likelihood estimation for stochastic processes - a martingale approach

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This thesis is primarily concerned with the investigation of asymptotic properties of the maximum likelihood estimate (MLE) of parameters of a stochastic process. These asymptotic properties are related to martingale limit theory by recognizing the (known) fact that, under certain regularity conditions, the derivative of the logarithm of the likelihood function is a martingale. To this end, part of the thesis is devoted to using or developing martingale limit theory to provide conditions for the consistency and/or asymptotic normality of the MLE. Thus, Chapter 1 is concerned with the martingale limit theory, while the remaining chapters look at its application to three broad types of stochastic processes. Chapter 2 extends the classical development of asymptotic theory of MLE's (à la Cramér [1]) to stochastic processes which, basically, behave in a non-explosive way and for which non-random norming sequences can be used. In this chapter we also introduce a generalization of Fisher's measure of information to the stochastic process situation. Chapter 3 deals with the theory for general processes develops the notion of "conditional" exponential families of processes, as well as establishing the importance of using random norming sequences. In Chapter 4 we consider the asymptotic theory of maximum likelihood estimation for continuous time processes and establish results which are analogous to those for discrete time processes. In each of these chapters many applications are considered in an attempt to show how known and new results fit into the general

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Received 10 September 1975. Thesis submitted to the Australian National University, September 1975. Degree approved, December 1975. Supervisors: Professor C.R. Heathcote and Dr C.C. Heyde.

framework of estimation for stochastic processes.

In Appendix B, a report on the use of the empirical characteristic function in inference is included in order to indicate how one might deal with situations where the likelihood is intractable.

#### Reference

- [1] H. Cramér, *Mathematical methods of statistics* (Princeton Mathematical Series, 9. Princeton University Press, Princeton, 1946).