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THE APPLICATION OF BOUNDARY ELEMENT METHOD TO 3-D SOLAR LINEAR FORCE-FREE MAGNETIC FIELDS

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ABSTRACT In this paper a 3-d formulation is applied to extrapolate the solar magnetic field from the measured magnetogram directly, and no additional data treatment is needed. The validity of the proposed formulation is demonstrated by comparison between calculated and analytical or observed results of some magnetic force-free field problems.

FORMULATION

There have been many research in solar magnetic fields. Recently, Yan, Yu, and Kang (1991) have established a new linear force-free field model with the unique solution as well as a finite energy content, and, for the first time, proposed a boundary element method (BEM) procedure to the problem. Then, constant-α force-free magnetic fields is described as follows (Yan, Yu, and Kang 1991)

$$\begin{cases} \nabla^2\mathbf{B} + \alpha^2\mathbf{B} = 0 & \text{in} \quad \Omega \\ \mathbf{B} = \mathbf{B}_o & \text{on} \quad \Gamma \\ \mathbf{B} = O(\frac{1}{r}) \ \text{and} \ r(\frac{\partial \mathbf{B}}{\partial r} + j\alpha\mathbf{B}) \to 0, & \text{when} \quad r \to \infty \end{cases} \tag{1}$$

where $\mathbf{B}_o$ denotes known boundary values, j is imaginery unit, Ω is the semi-space above the sun, and Γ is the photosphere surface. If $\alpha = 0$, a Laplacian equation is obtained for the current-free (or potential) field problem. The Sommerfeld condition, or third one, was included by Yan, Yu, and Kang (1991) to ensure a unique solution and a finite energy content when $\alpha \neq 0$(Stratton 1941).

The boundary integral equation of the above exterior boundary value problem is expressed as follows

$$c_i\mathbf{B}_i = \int\!\!\!\!-\!\!\!\int_\Gamma (F\frac{\partial \mathbf{B}}{\partial n} - \mathbf{B}\frac{\partial F}{\partial n})d\Gamma \tag{2}$$

where the integral is in a sense of Cauchy principal value integration, c_i is a constant, and F is the fundamental solution of Helmholtz equation in free space.

By the BEM discretization, the following set of simultaneous equations in a matrix form can be finally obtained (Yan, et al 1991)

$$[\mathbf{H}]\{\mathbf{B}\} = [\mathbf{G}]\left\{ \frac{\partial \mathbf{B}}{\partial \mathbf{n}} \right\} \tag{3}$$

where [**H**] and [**G**] are coefficient matrices, $\{\mathbf{B}\}$ and $\{\frac{\partial \mathbf{B}}{\partial \mathbf{n}}\}$ are vectors of nodal values. At each node **B** is known (as given by the boundary condition). Solution of the above equation permits unknown values to be found.

EXAMPLES

The first case is a force-free magnetic field problem which has an analytical solution (Low 1982). The field components are respectively $B_x = -B_0(\cos\phi)/r$, $B_y = B_0[(x-2)(y-2)\cos\phi - (z+1)r\sin\phi]/(rr_1^2)$ and $B_z = B_0[(x-2)(z+1)\cos\phi + (y-2)r\sin\phi]/(rr_1^2)$, in which ϕ is a free generating function, and $\phi = \phi_0 + (r-1)/2(\phi_1 - \phi_0)$ to give a force-free magnetic field with constant $\alpha = \frac{d\phi}{dr}$. In above formula, $r^2 = (x-2)^2 + (y-2)^2 + (z+1)^2$ and $r_1^2 = (y-2)^2 + (z+1)^2$. In the example, we chose $B_0 = 1000$ Gs, $\phi_1 = \pi/2$, and $\phi_0 = \pi/4$. A mesh with 21×21 nodes covering a area of $\Gamma = \{(x,y)|, 0 \le x \le 4, 0 \le y \le 4\}$ is employed. The total nodes and elements are respectively 441 and 100.

Analytical and calculated contours of the longitudinal component, B_z, and the transverse component, $\mathbf{B}_t = (B_x, B_y)$, at the $z = 0.5$ plane are compared in Fig. 1 and Fig. 2 respectively. It can be seen that the calculated and analytical distributions of longitudinal and transverse components are very similiar.

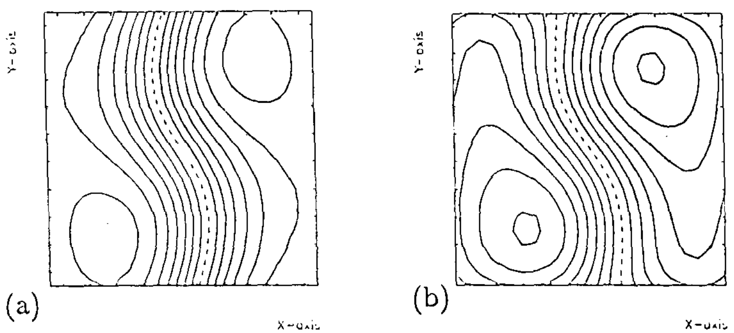

Fig.1 Distribution of (a) analytical and (b) numerical results of B_z ($\max|B_z|_{Ana} = 374.3$ Gs, $\max|B_z|_{BEM} = 356.3$ Gs)

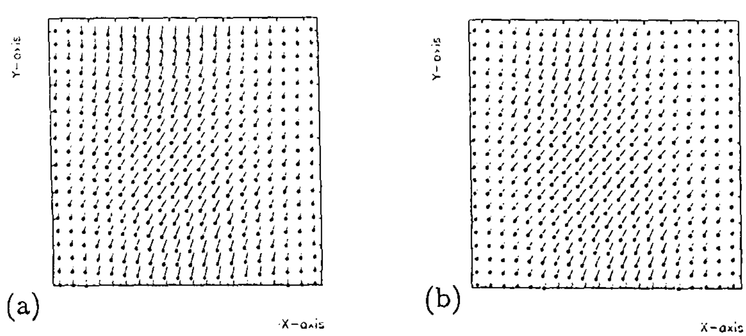

Fig.2 Distribution of (a) analytical and (b) numerical results of $\mathbf{B}_t$ ($\max|\mathbf{B}_t|_{Ana} = 666.7$ Gs, $\max|\mathbf{B}_t|_{BEM} = 669.9$ Gs)

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Fig. 3 shows the CR 1742 heliospheric magnetic field B_l contours at the source surface when $\alpha = 0$ by present method and potential field-source surface model (Hoeksema and Scherrer 1986). It can be seen the distribution of neutral line, or location of current sheets, are fairly alike (The isolines were not with same levels). A 48×96 mesh was used. Other examples with comparison between calculated and measured results could be found elsewhere (e.g., Yan, et al 1991).

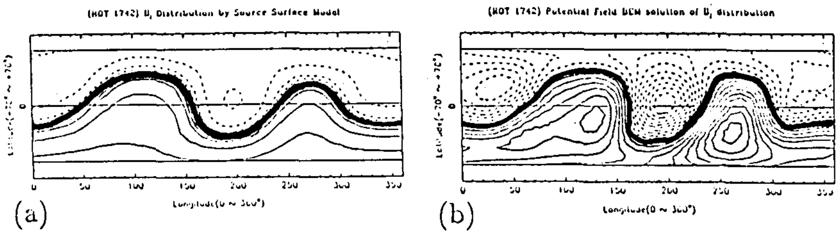

Fig.3 B_l distribution by (a) potential field-source surface model, (b) present method. (thick line: neutral; solid: positive; dash: negative)

CONCLUDING REMARKS

The application of the BEM in this area is a new topic both to the solar physics and to the BEM research. In summary, several points can be concluded: (a) The present model of the force-free magnetic field with constant α ensures a unique solution and a finite energy content by introducing the Sommerfeld condition. (b) By using BEM to numerically solve the model, magentogram data are implemented directly into the fomulation; meanwhile no additional data treatment is needed. In addition, it is not required that the net magnetic flux through a magnetogram area is zero. (c). Moreover, the formulation is particularly suitable for the implementation of parallel processing. Further invesitigation is to apply this new technique to analyze the physical process of practical problems.

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