Say $Z \neq 0$, and take $\lambda = -z/Z$.

Thus $$\phi(x, y, z) \equiv \phi\left(x - \frac{Xz}{Z}, y - \frac{Yz}{Z}, 0\right),$$

that is to say, $\phi(x, y, z)$ is a quadratic function of the two linear functions $x - Xz/Z$ and $y - Yz/Z$.

JOHN DOUGALL.

On the sufficiency of the condition for a limit.— Let $z_1, z_2, z_3, \ldots$ be a sequence of quantities, real or complex, such that, corresponding to any arbitrary small positive quantity ϵ, we can find a positive integer n such that $|z_{n+p} - z_n| < \epsilon$ for all positive integral values of p.

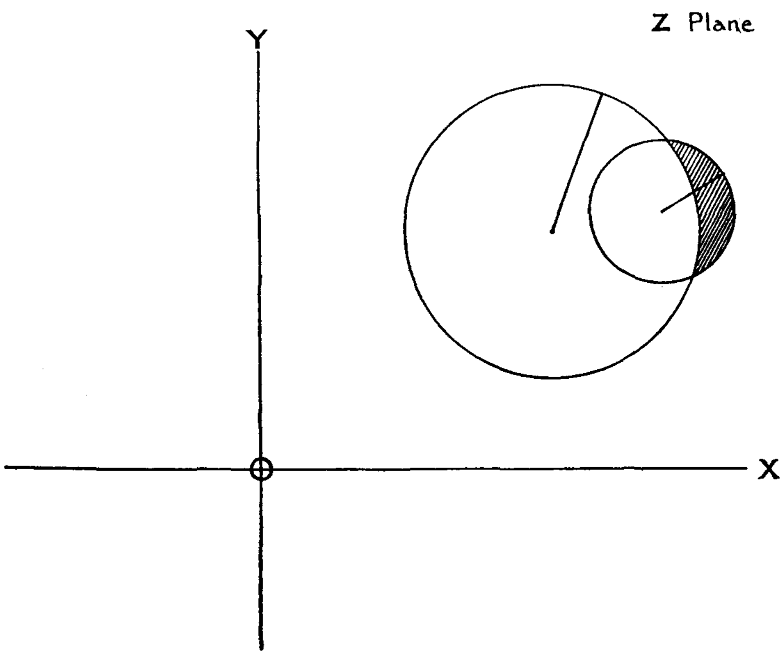
Take a sequence of ϵ's,

$$\epsilon_1, \epsilon_2, \epsilon_3, \ldots$$

which steadily decreases to the limit zero.

Let $n_1, n_2, n_3, \ldots$ be the smallest n's corresponding to $\epsilon_1, \epsilon_2, \epsilon_3, \ldots$ respectively.

We have $n_1 \leqq n_2 \leqq n_3 \leqq \ldots$.



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With centre z_{n_1} and radius ϵ_1 describe a circle on the z-plane.

All the z_{n_1+p}'s lie inside the circle.

With centre z_{n_2} and radius ϵ_2 describe a circle.

All the points z_{n_2+p} lie inside this circle, and, if the circles intersect, in that part of it which lies inside the first circle.

Thus, by considering the ϵ's and n's in succession, we get a sequence of areas tending to the limit zero, and each lying inside all the preceding areas.

Now there must be some point which lies inside every one of these areas, however small, and there cannot be more than one such point. For if there were two such points, we could choose an ϵ less than half the distance between them, and the corresponding area would exclude at least one of them. Hence there is only one such point, and the corresponding z is the limit of the sequence.

THOMAS M. MACROBERT

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