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Innate Abilities, One or Many?* A New Method and Some Results

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Many psychologists today prefer the view that there are a number of independent cognitive factors of about equal importance, rather than one factor of general intelligence (Loevinger 1951, French 1951). Guilford has proposed a general model for the classification of as many as 120 independent cognitive factors (1956). If this concept of a number of separate cognitive factors is accepted, the next question might be: are these different abilities the result of compartmentalized educational experiences or do they represent separate innate potentialities?

A study of the correlations between twin *differences* rather than between the original scores can throw light on this question. The within pair differences of identical twins are due solely to environmental factors, where the environment is conceptualized broadly to include intra-uterine conditions and events during delivery as well as all postnatal events. On the other hand the within pair differences of fraternal twins are due to hereditary differences as well as these environmental ones. (While only *within* family differences are obtained, so that *between* family differences are not represented, it is difficult to see how hereditary factors could differ fundamentally between families but not within families, although variance estimates might be reduced).

It seems reasonable to suppose as a first approximation that the prenatal events will affect later performance on most cognitive tasks equally. That the postnatal events have differential effects on the performance on different cognitive tasks seems rather obvious especially when we think of formal academic education, although we know little about the specific development of such separate abilities. Whether or not hereditary differences exist which affect the performance on different cognitive tests

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differentially is quite a new question, which we can only begin to ask now that we have some tests of independent abilities.

As part of the Michigan Twin Study, Thurstone's Primary Mental Abilities Test (1941) was administered to 45 pairs of MZ and 37 pair of DZ twins of high school age. The overall design of this study has been described by Sutton, Vandenberg and Clark (1962) and the psychological part by Vandenberg (1962). The zygosity diagnosis was based on a decision function derived from the results of tests of blood group systems (for details about this procedure see Sutton, Clark and Schull, 1955).

The intercorrelation of the within pair differences for the 6 PMA scores are shown in tab. 1. All but one of these PMA scores are obtained from 3 tests each, which differ somewhat in format but which measure the same ability factor. The Memory

Tab. 1. Correlations of within pair differences

	MZ pairs (45)						DZ pairs (37)					
	N	V	S	W	R	M	N	V	S	W	R	M
Number	*	−124	−187	−183	179	052	*	272^1	454^2	393^2	461^2	189
Verbal	−124	*	137	394^2	289^1	065	272^1	*	215	521^2	515^2	239
Space	−187	137	*	281^1	−102	-353^2	454^2	215	*	420^2	472^2	359^1
Word	−183	394^2	281^1	*	438^2	123	393^2	521^2	420^2	*	429^2	345^1
Reasoning	179	289^1	−102	438^2	*	317^1	461^2	515^2	472^2	429^2	*	308^1
Memory	052	065	-353^2	123	317^1	*	189	239	359^1	345^1	308^1	*
Significant correlations	0	2	2	3	3	2	4	3	4	5	5	3

[1] correlation significantly different from zero at 5% probability
[2] correlation significantly different from zero at 1% probability

score is obtained from 2 tests only. The 6 PMA scores are obtained by simple summations of these separate test scores.

In these correlations the effect of age is largely eliminated when twin differences are studied, unless older twins differ more than do young ones, in which case higher correlations between twin differences on two tests might be expected for older than for younger pairs, where the errors-of-measurement may obscure the true differences. In this age range, 12-18 years, this was probably not the case.

Looking first at the results for the identical twins in which differences are only due to the environment as defined above, we see that of the 15 different correlations only 3 are significantly different from zero at the 5 per cent level of significance and 3 additional ones at the 1 per cent level. Differences in Number ability are not correlated with differences in the other five scores. Verbal ability, i. e., vocabulary knowledge

demonstrated in multiple choice tests is correlated with word fluency scores which are based on the number of words with a given first letter or ending which are written in a few minutes. The latter score is also correlated with the Reasoning score. The spatial visualization or Space score correlates negatively with the Memory score. The three 5 per cent level correlations will not be considered. It appears then that for these 6 variables the environmental influences produce largely independent differences within the MZ twin pairs. It may be worthwhile to employ this method with more variables to study within family difference producing influences.

Wen we now turn to the results for DZ twins, the story is quite different. Twelve of the fifteen correlations are significant, 8 at the 1 per cent and 4 at the 5 per cent level of significance. The only correlations which are not significant are between Number ability and Verbal abilities; and between Memory and Space as well as between Memory and Word fluency. The correlations still are not high, accounting at most for one fourth of the variance, but they are, with two exceptions, higher than the correlations for MZ twin differences. If we simplify matters grossly and assume that the environmental contribution to the within pair variance is the same for MZ and DZ, and subtract the MZ difference correlations from the DZ ones (after Fisher's z-transformation) we obtain the values shown in tab. 2 for the hypothetical correlations for DZ within pair differences due to heredity only.

Tab. 2. Hypothetical correlations of twin differences due to heredity only

	N	V	S	W	R	M
Number	—	38	59*	54*	31	19
Verbal	38	—	08	16	27	18
Space	59*	08	—	16	55*	63*
Word fluency	54*	16	16	—	-01	23
Reasoning	31*	27	55*	-01	—	-01
Memory	19	18	63*	23	-01	—

While many objections can be raised to this procedure it makes possible the simultaneous grasp of the 2 sets of correlations and thus facilitates some interpretation. Actually only four of the 15 correlations differ at a statistically significant level from one matrix to the other. These four are starred in tab. 2.

Let us look now at the obtained values and interpret them as estimates of correlations such as would be obtained between that part of DZ twin differences which is due to hereditary factors, if they could be measured directly. There appears to be some relation between Number ability on the one hand and Space and Word fluency on the other (but not between these two) and also between Space on the one hand and Reasoning and Memory on the other hand (but not between these two either). The hypothetical correlations are low and would indicate near independence of all 6 abilities. But enough has been made of this crude analysis.

The following method suggested itself while searching for a more acceptable method to answer the question whether or not these 6 abilities were genetically related. Consider the original F-tests for the statistical significance of the increase in within pair variance of fraternal twins as compared with identical twins. For each separate variable we calculate the ratio:

$$F = S^2_{wf} \,/\, S^2_{wi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [1]$$

and evaluate this for N_f and N_i degrees of freedom, where S^2_w is the within pair variance of N_f and N_i pairs of twins A and B

$$S^2_w = \frac{1}{N}\sum x^2_A + \frac{1}{N}\sum x^2_B - \frac{1}{2N}\sum (x^A + x^B)^2 \quad \ldots\ldots\ldots\ldots \quad [2]$$

which conveniently [1] reduces to:

$$S^2_w = \frac{1}{2N}\sum (x_A - x_B)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [3]$$

Now we wish to generalize the F test to a multivariate comparison. The within pair co-variances for scores i and j

$$Cov_{wij} = \frac{1}{N}\sum x_{iA}\, x_{jA} + \frac{1}{N}\sum x_{iB}x_{jB} - \frac{1}{2N}\sum (x_{iA} - x_{iB})\,(x_{jA} - x_{jB}) \quad [4]$$

similarly reduce to:

$$Cov_{wij} = \frac{1}{2N}\sum (x_{iA} - x_{iB})\,(x_{jA} - x_{jB}) \quad \ldots\ldots\ldots\ldots\ldots \quad [5]$$

Let these C_{ij}'s form the elements of the co-variance matrices D (for dizygous) and M (for monozygous). Then the generalization of formula [1] would be: how many significant roots has the characteristic equation.

$$| D - \lambda M | = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [6]$$

A possible interpretation of this equation might be: given M, the within pair co-variance matrix of scores for the monozygous twins, find the (asymmetric) matrix of eigen vectors which will transform M (the within pair co-variance matrix of the score differences for the monozygous twins) into D (the dizygous within pair co-variance matrix). This asymmetric transformation matrix might then be thought of as representing hereditary influences.

[1] I am indebted to Dr. Lawley for having pointed this out to me.

Equation [6] was solved for the 6 PMA scores and the 6 roots shown in tab. 3 were obtained. To test the significance of these roots Bartlett's chi squared test of the homogeneity of the remaining roots after extraction of 1, 2...k roots was applied (1950). The results are shown in tab. 4 and indicate that at least 4 roots are significant, or in other words, that in this sample there are at least 4 independently significant hereditary components.

Tab. 3.
Roots of $| D - \lambda M | = 0$

1	3.99972
2	2.23172
3	1.58655
4	1.00141
5	.64598
6	.38206

Tab. 4. Chi squared values for the homogeneity of the remaining roots after extraction of k roots

k	χ^2	d. f.	p
1	30.698	20	betweeen .10 and .05
2	17.483	6	less than .01
3	7.250	2	less than .05
4	2.147	·5	n. s.
5	0	0	

It is of interest to compare these roots with the F-ratios for the DZ over the MZ within pair variances for the 6 original scores. Tab. 5 shows these F-ratios: four of them are significant beyond the 1 per cent level.

Tab. 5. F-ratios of DZ over MZ within pair variances of 6 PMA scores for 37 and 45 degrees of freedom

Number	2.583	.01
Verbal	2.651	.01
Space	2.419	.01
Word fluency	2.479	.01
Reasoning	1.401	n. s.
Memory	1.213	n. s.

We concluded that there are at least 4 independent hereditary components in the 6 PMA scores of the subjects in this sample. Since the analysis of the single scores led to the conclusion that Number, Verbal, Space and Word fluency were under a statistically significant degree of genetic control, it may be warranted to conclude that the 4 independent components represented by the significant roots of $| D - \lambda M | = 0$ are rather similar to the Number, Verbal, Space and Word fluency abilities.

In an effort to obtain more information on this, the eigen vectors associated with the 6 roots were inspected. Tab. 6 shows these eigen vectors.

The inspection yielded no insight except that this transformation matrix was drastically different from an identity matrix, in other words this transformation is having a differential effect on the columns of monozygous co-variance matrix.

Next this transformation matrix was squared. The matrix TT^1 is shown in tab. 7. This matrix may perhaps be regarded and interpreted as equivalent to a ma-

Tab. 6. Eigen vectors of $| D - \lambda M | = 0$

	I	II	III	IV	V	VI
	4.000	2.232	1.587	1.001	.646	.382
N	.035	−.030	.354	−.181	−.112	−.173
V	−.183	.701	.186	−.239	−.288	207
S	−.219	−.011	.346	.029	.318	058
W	.004	−.401	.237	.289	−.547	048
R	.504	.385	−.099	.584	.505	−.248
M	.815	−.436	.809	−.696	.500	927

Tab. 7. The matrix TT^1 showing the correlations between the 6 eigen vectors

	I	II	III	IV	V	VI
I	1.001	−.290	.513	−.241	.639	.574
II	−.290	.992	−.370	.250	−.004	−.369
III	.513	−.370	1.000	−.651	.242	.783
IV	−.241	.250	−.651	1.000	−.113	−.793
V	.639	−.004	.242	−.113	1.001	.290
VI	.574	−.369	.783	−.793	.290	.999

trix of correlations between the hereditary components only. With only 6 variables in the matrix, it would be pointless to try to seek a rotation to simple structure for these variables, but work is in progress on a matrix with more variables.

Summary and concluding remarks

A generalization of the F test is proposed to see how many independent components are statistically significant when the co-variances between DZ twin differences on several psychological tests are compared with similar co-variances for MZ twins. When applied to the 6 scores of Thurstone's Primary Mental Abilities test battery, at least 4 separate components were found to be significant independently. Then 4 components are tentatively identified as similar to, although not identical

with, the Number, Verbal, Space and Word fluency scores. While the procedure bears some resemblance to factor analysis, it may lead to quite different results since it is based on the comparison of two co-variance matrices of twin differences.

In conclusion, it should be kept in mind that these results are based on small samples of MZ and DZ twins, so that a repetition of the study seems indicated. This replication is now in progress.

References

BARTLETT M. S.: Tests of significance in factor analysis. Brit. J. Stat. Psychol., *3*: 77-85, 1950.

FRENCH J. W.: The description of aptitude and achievement tests in terms of rotated factors. Psychometric Monograph 5, University of Chicago Press, 1951.

GUILFORD J. P.: The structure of intellect. Psychol. Bull., *55*: 287-293, 1956.

LOEVINGER, JANE: Theories of intelligence. In Helson: H. Ed. Theoretical foundations of psychology, New York, Van Nostrand, 1951.

SUTTON H. E., CLARK P. J., SCHULL W. J.: The use of multi-allele genetic characters in the diagnosis of twin zygosity. Amer. J. Hum. Genet., *7*: 180-188, 1955.

— VANDENBERG S. C., CLARK P. J.: The hereditary abilities study: selection of twins, diagnosis of zygosity and program of measurements. Amer. J. Hum. Genet., *14*: 52-63, 1962.

THURSTONE L. L.: The primary mental abilities tests. Science Research Associates, Chicago, 1941.

VANDENBERG S. G.: The hereditary abilities study: hereditary components in a psychological test battery. Amer. J. Hum. Genet., *14*: 220-237

RIASSUNTO

Viene suggerita una generalizzazione del test F, per vedere quante componenti indipendenti siano statisticamente significative quando le covarianze di differenze fra gemelli DZ in svariati tests psicologici vengano raffrontate con covarianze simili per gemelli MZ. I risultati ottenuti sono, tuttavia, basati su campioni ristretti di gemelli MZ e DZ, per cui una replica di tale studio è attualmente in corso.

RÉSUMÉ

Une généralisation du test F est suggérée, afin de voir combien de composantes indépendantes soient statistiquement significatives, lorsque l'on compare les covariances des différences entre jumeaux DZ en plusieurs tests psychologiques avec des covariances semblables pour les jumeaux MZ. Les résultats obtenus se basent, toutefois, sur de petits échantillons de jumeaux MZ et DZ. Par conséquent, la même étude est en train d'être répétée.

ZUSAMMENFASSUNG

Man schlägt hier eine Generalisierung der F-Tests vor, um zu sehen, wieviele unabhängige Komponenten statistisch bedeutungsvoll sind, wenn man die unterschiedlichen Begleitvarianzen in verschiedensten psychologischen Tests bei ZZ mit den ähnlichen Begleitvarianzen bei EZ vergleicht.

Die Ergebnisse beruhen jedoch auf beschränkten EZ— und ZZ—Mustern, weshalb man die Untersuchung zur Zeit noch einmal wiederholen will.