

# STRATIFICATION OF ELEMENTS IN A QUIET ATMOSPHERE: DIFFUSION PROCESSES

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## 1. ABUNDANCE ANOMALIES: THE SIGNATURE OF STABILITY

In the absence of turbulence or convection one expects that, in stars, heavy elements would tend to settle gravitationally while light elements would go to the surface. Eddington (1930) however realized that this general tendency could be modified both by the electric field and by differential radiation pressure. In spite of their small mass, electrons do not all float on the surface of stars because an electric field is generated that keeps them from separating from the protons. Instead of settling gravitationally, heavy elements often concentrate on the surface because they absorb relatively much more photons than hydrogen or helium and are dragged to the surface by the radiative flux. Eddington concluded that turbulence was too strong for diffusion to be important in stars since the relation that diffusion predicts between surface abundances and stellar masses did not appear to be realized in most stars.

However, we now know that stars whose outer envelopes are most likely to be stable, thus where diffusion is most important, show surface overabundances of heavy elements and underabundances of helium. These are the Fm, Am, Ap and Bp stars that are slow rotators, and often have strong magnetic fields. Furthermore, the abundance anomalies become larger as the importance of the outer convection zone decreases from Fm to Ap (Smith 1971, 1973, Preston 1974).

In order to derive the diffusion equation (§ 2), I will here start from equilibrium gradients in stellar atmospheres and envelopes. The electric fields and local charge separation in stars will be described. The electric force on protons is half as strong as the gravitational force on protons, and this is true even in convection zones. Surface underabundances appear after  $10^4$  years (§ 3) but overabundances after as little as ten years.

Comparisons of observed abundance anomalies with diffusion calculations (§ 4) show that even the largest observed anomalies can be

explained as can their variation with the effective temperatures of stars; and this without any arbitrary parameter. However such phenomena as isotope anomalies and line asymmetries depend sensitively on the structure of the outer atmosphere. Mainly because of our ignorance of the hydrodynamics involved, their explanation requires arbitrary parameters. This may also lead to a better understanding of stellar hydrodynamics.

No attempt is made here to present a complete review. A more complete list of references may be found in Michaud (1975).

## 2. BASIC PHYSICS

In order to introduce the diffusion equation, we study the equilibrium configurations of gases constituted successively only of hydrogen, only of protons and electrons, and finally of protons, electrons and traces of an element of mass  $A$  and charge  $Z$ .

### 2.1 The Equilibrium Configuration of Hydrogen

The reader is probably familiar with the hydrostatic equilibrium of a gas constituted of pure hydrogen (See Fig. 1). Requesting that the

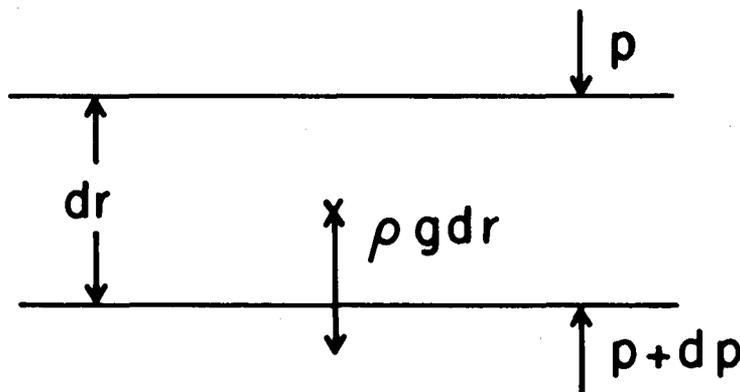


Figure 1. Forces (per  $\text{cm}^2$ ) acting on a slab of material of density  $\rho$ , thickness  $dr$ , in a gravitational field.

sum of the forces acting on a slab material of thickness  $dr$ , density  $\rho$ , submitted to a gravitational acceleration  $g$  be equal to zero, we have

$$dp = - \rho g dr \quad . \quad (1)$$

Or replacing  $\rho$  by the pressure:

$$\frac{d \ln p}{dr} = - \frac{m g}{kT} \quad , \quad (2)$$

where  $m_p$  is the proton mass, the difference between the hydrogen and the proton mass being negligible here.

2.2 The Electric Field of Stars

When hydrogen is completely ionized we now show that an electric field must appear for equilibrium to be maintained.

At equilibrium, collisions must exchange no momentum, nor anything else, between protons and electrons. Equilibrium is reached when transport phenomena have ceased. In the prevailing electric and gravitational fields one must then have equilibrium separately for electrons and protons. One can write separately for protons and electrons an equation similar to eq. (2).

$$\frac{\partial \ln n_p}{\partial r} = - \frac{m_p g}{kT} - \frac{eE}{kT} \quad (3)$$

$$\frac{\partial \ln n_e}{\partial r} = - \frac{m_e g}{kT} + \frac{eE}{kT} \quad (4)$$

Since the electron mass is much smaller than the proton mass, the electrons have a tendency to float and an electric field is needed to keep the matter neutral on the whole. To prevent a large charge separation, one must have

$$\frac{\partial \ln n_p}{\partial r} = \frac{\partial \ln n_e}{\partial r} \quad (5)$$

or, using equations (3) and (4),

$$eE = \frac{1}{2} (m_p - m_e) g \quad (6)$$

In stars, the electric force on protons is equal to half the gravitational force on protons, and the electric force on electrons is much larger than the gravitational force on electrons.

In stellar structure calculations, the electric field is neglected. This is possible since one is generally not interested in the equilibrium configurations of protons and electrons separately. It is trivial to show that equations (3), (4) and (6) lead to the usual hydrostatic equilibrium equation involving the reduced mass [ $\mu \equiv (m_p + m_e)/2$ ]. However the concept of the reduced mass is justified by the presence of the electric field.

We have seen that there must be electric fields in stars, but then the electric field lines must start and end somewhere. There must be some charge separation.

The required charge separation can be estimated at some distance  $r$  from the star center, where the gravitational acceleration is  $g$ . We will compare the total charge to the total mass of the star within that radius. Let  $Z$  be the total charge number (total charge within  $r$

$Q(r) = ze$  and  $a$  the total mass number within radius  $r$  (total mass,  $M(r) = am_p$ ). Eq. (6) requires that:

$$eE = \frac{1}{2} m_p g$$

or

$$\frac{eQ}{r^2} = \frac{e^2 z}{r^2} = \frac{1}{2} m_p \frac{GM}{r^2} = \frac{1}{2} m_p^2 \frac{Ga}{r^2}$$

leading to:

$$\frac{z}{a} = \frac{m_p^2 G}{2e^2} \approx 10^{-37} \quad (7)$$

Since the electric interaction is a much stronger interaction than the gravitational interaction, only a small charge is required for the electric force on a proton to equal half the gravitational force on a proton. Eq. (7) implies a charge separation of the same order:

$$\frac{p_{e^-} - p_p}{p_{e^-} + p_p} \sim 10^{-37} \quad (8)$$

averaged throughout the star. The electron and proton pressure gradients are then equal (eq. [5]) to a very good approximation. (For a more complete discussion, see Milne 1924, Schatzman 1958, Montmerle and Michaud 1976.)

Are stars electrically neutral or where do the electric field lines end? Milne (1924) has shown that, if stars were alone in space and perfectly stable, the field lines would end at  $N \sim 10^{-6} \text{ cm}^{-3}$  where there would be a slight excess of electrons. Since in interstellar space the number density of protons is of order  $1 \text{ cm}^{-3}$ , the treatment of Milne does not apply. Presumably motions occur and there is a slight electron excess where the number density of protons is of order  $1 \text{ cm}^{-3}$ . If a star were not neutral it would accrete a few electrons from space to become neutral. Only  $10^{-3}$  electron per  $\text{cm}^2$  of stellar surface is needed to cancel the charge of the star. This problem has apparently never been studied in detail. The timescale for the establishment of the electric field will be discussed after the introduction of the diffusion equation.

### 2.3 The Diffusion Equation

Diffusion is now presented as the first order process transforming a non-equilibrium into an equilibrium configuration (Eddington 1930). Consider a stellar envelope throughout which element A has the abundance  $c(A)$

$$c(A) \equiv N(A)/[N(H)+N(A)] \quad (9)$$

The equilibrium abundance is given by an equation similar to equation (4):

$$\frac{\partial \ln p_{\text{eq}}(A)}{\partial r} = \frac{Am_p (g_{\text{rad}} - g)}{kT} + \frac{ZeE}{kT} \quad (10)$$

where  $Z$  is the charge of the ion (not that of the nucleus), and  $g_{\text{rad}}$  is the acceleration on element  $A$  due to the absorption of photons by that element, i.e. the "radiative acceleration". In the present context diffusion is the transport phenomenon transforming the actual distribution of abundances of element  $A$  into the equilibrium distribution. To first order the diffusion velocity is then linearly proportional to the difference between the equilibrium gradient and the actual gradient.

$$w = D \left[ \frac{\partial \ln c}{\partial r} \text{eq} - \frac{\partial \ln c}{\partial r} \text{actual} \right] \tag{11}$$

where  $D$ , the proportionality constant, is called the diffusion coefficient. Using equations (4), (9) and (10), one obtains (if  $c(A) \ll 1$ )

$$w = D \left[ - \frac{\partial \ln c}{\partial r} + \frac{m_p g}{kT} (1-A) + \frac{eE}{kT} (Z-1) + \frac{A m_p g_{\text{rad}}}{kT} \right], \tag{12}$$

where  $c$  is used for  $c_{\text{actual}}$ . If the electric field is determined by the protons only, one uses equation (6) to obtain:

$$w = D \left[ - \frac{\partial \ln c}{\partial r} + \frac{m_p g}{2kT} (1-2A+Z) + \frac{A m_p g_{\text{rad}}}{kT} \right]. \tag{13}$$

This diffusion velocity is based on diffusion being a first order process. It is the same as that obtained by Aller and Chapman (1960) from statistical physics considerations except that thermal diffusion is here neglected. Michaud et al. (1976) have shown how thermal diffusion can be taken into account by modifying  $g$ . In the atmosphere, thermal diffusion is generally negligible, but not deeper in the envelope, where  $Z$  becomes large.

In deriving equation (13) we have assumed  $c(A)$  to be small. This is generally an excellent approximation except for helium, whose non-negligible abundance modifies the electric field. Indeed when helium is more than half as abundant as hydrogen, the outward electric force on hydrogen becomes larger than the gravitational pull, and hydrogen is ejected (Montmerle and Michaud 1976) from the region where helium is abundant. The observation of a circumstellar hydrogen shell around  $\sigma$  Ori E (Walborn 1974) is probably related to the electric fields in He-rich stars.

The diffusion coefficient is inversely proportional to the collision probability. It thus depends on the type of interaction between element  $A$  and protons, when diffusion occurs in ionized hydrogen. When element  $A$  is ionized,

$$D \approx 1.5 \times 10^8 T^{5/2} / (NZ^2)$$

where  $N$  is the number density of protons. All quantities are in the cgs system (See Aller and Chapman 1960, Chapman and Cowling 1970, Montmerle and Michaud 1976). When element  $A$  is not ionized, collisions occur mainly through the polarization of element  $A$ , giving

$$D \approx 3.3 \times 10^4 T / (N \alpha^{1/2})$$

where  $\alpha$  is the polarisability (Ratcl 1975). Values of  $\alpha$  for a few elements of interest are given in Table I.

Element	$\alpha(10^{-24} \text{ cm}^3)$
He	0.20
O	0.77
Si	5.5
Mn	14.
Sr	25.

Table I. Polarizabilities (From Teachout and Pack 1971).

In stellar atmosphere the diffusion coefficient of the neutral elements is about two to three orders of magnitude larger than that of the ionized elements. Even if an element has only  $10^{-3}$  of its atoms neutral, the neutral element is sometimes the dominant contributor to the diffusion velocity, the mobility of elements being so much larger in the neutral state. This in particular can enhance the effect of photons absorbed in the neutral state (See Montmerle and Michaud 1976, § VI). In Fig. 2, we see that, for helium, whereas the radiation force transmitted through the lines is always negligible if one neglects the effect just described,

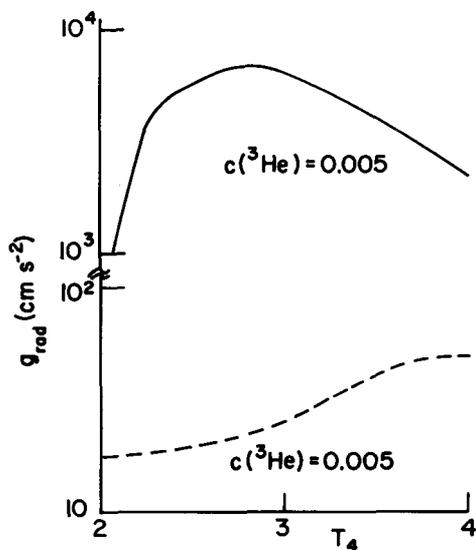


Figure 2. Radiative acceleration transmitted through the lines to  ${}^3\text{He}$ , both taking into account (full line) and neglecting (dashed line) the effect of increased mobility in the neutral state. It leads to a two order of magnitude difference. Only when the effect is taken into account can the lines support helium. (Michaud, Praderie and Montmerle, in preparation.)

this force becomes dominant at  $T = 30,000^\circ\text{K}$  if one includes the effect of the increased mobility. The problem of timescale for the appearance of the helium isotope anomaly probably disappears once this is taken into account (Vauclair et al. 1974a).

### 3. DIFFUSION TIMESCALES

Two very different timescales appear: whereas electrons can always separate rapidly enough from protons to create electric fields whatever turbulence there may be, elements cannot necessarily diffuse rapidly enough to lead to abundance anomalies. So all stars have internal electric fields but only a few have abundance anomalies. However if turbulence is small enough, the timescale to create abundance anomalies is much smaller than the stellar lifetime for stars more massive than  $1.3 M_\odot$  and somewhat smaller than the stellar lifetime for stars more massive than the sun. Whether diffusion can modify surface abundances in the sun sensitively depends on hydrodynamical effects which are poorly understood at the present.

#### 3.1 Electric Fields

The timescale for the establishment of the electric field can be estimated using a diffusion equation similar to eq. (13) for the electron-proton mixture. We assume no charge separation at  $t = 0$  ( $p_{e0} = p_{p0}$ ). Then

$$w_{pe} \approx D_{pe} \left[ \frac{m_p g}{kT} \right] \quad (14)$$

The time for the establishment of the electric field is approximately the time for electrons to move from the point where  $p_{e0} = p_{p0}$  to the point where

$$\frac{p_{e0} - p_{p0}}{p_{e0} + p_{p0}} \approx 10^{-37} \quad .$$

From eq. (3), this is approximately at a distance  $10^{-37}$  of a scale height. Then

$$\Delta t \approx \frac{\Delta r}{w_{pe}} \approx \frac{10^{-37} kT}{m_p g w_{pe}} \approx 10^{-25} \text{ sec.} \quad (15)$$

where a value of  $10^{10}$  cm has been used for the scale height, and a number density of  $10^{15}$  has been used in estimating  $D_{pe}$ . Even in regions where the density is much larger, the timescale for the "creation" of the field,  $\vec{E}$ , is very short. As soon as the separation starts (or  $10^{-25}$  seconds later) it stops because the electric field has been created. No turbulence or convection timescale is close to this one. Electric fields are present whenever there is ionized matter in a gravitational field. More sophisticated calculations of the timescale could be made but it hardly seems worth it. Equation (15) is quite accurate enough!

## 3.2 Timescales for Abundance Anomalies to Appear

There are different diffusion timescales depending on whether an element falls (i.e. settles gravitationally) or is supported by radiation pressure. When an element falls, the downward flux is proportional to the abundance in the convection zone which is assumed completely mixed. The abundance in the atmosphere varies as (See Michaud et al. 1976):

$$c(A)/c_{\odot}(A) = \exp(-t/\tau(A)) \quad (16)$$

Table II, gives the characteristic times for helium diffusion. They were obtained using eq. (13), but taking thermal diffusion into account. If

$M(M_{\odot})$	$\tau(\text{He})$ (years)
2.6	$4.3 \times 10^4$
2.0	$1.3 \times 10^5$
1.55	$1.8 \times 10^6$
1.4	$4.3 \times 10^6$
1.2	$1.1 \times 10^8$
1.07	$1.5 \times 10^9$
1 ( $\alpha = 1.5$ )	$2 \times 10^{10}$
1 ( $\alpha = 1.0$ )	$5.4 \times 10^9$
1 ( $\alpha = 0.7$ )	$5 \times 10^8$

Table II. Diffusion Characteristic Times.

the sun is stable below its convection zone, helium would be underabundant by a factor of two in its atmosphere if  $\alpha = 1$ . ( $\alpha$  is the ratio of mixing length to pressure scale height.) However if  $\alpha = 1.5$ , no anomaly is expected in the solar lifetime and if  $\alpha = 0.7$ , very large underabundances are possible. For the more massive stars, it is clear that very large anomalies are possible in the stellar lifetimes. In stars of  $2.6 M_{\odot}$  or more, the gravitational settling timescales for helium are of  $10^4$  years or so. Helium completely disappears from the surface, except for stars with  $T_{\text{eff}} \geq 18,000$  K where helium begins to be supported by radiation pressure for  $N(\text{He})/N(\text{H}) \approx 10^{-2}$  (See Vauclair et al. 1974a). In stars where helium settles gravitationally, the helium convection zone disappears (Vauclair et al. 1974b).

The timescale for the development of overabundances can be much shorter than the gravitational settling timescale: at a distance  $r$  from the center of the star, the radiative acceleration on un abundant elements ( $c(A) \leq 10^{-9}$ ) is of the order of

$$g_R \approx 1.7 \times 10^8 \frac{T_4^4 R^2}{A T_4 r^2} \text{ cm/sec}^2 \quad (17)$$

where  $R$  is the stellar radius,  $T_e$  the effective temperature in  $10^4$  °K, and  $T_r$  the temperature at  $r$  in  $10^4$  °K. This is up to four orders of magnitude larger than the gravitational acceleration, so that the diffusion timescales for elements pushed upwards can be four orders of magnitude smaller than that for those going downwards. Figure 3 shows the time evolution of abundances for some cases of interest. In stars of  $3 M_\odot$  or more, abundance anomalies start appearing after ten years. Note that the important time here is the time for elements to migrate from the envelope to the surface (Michaud et al. 1976). Watson (1971b) and Cowley and Day (1976) have carried out similar though less detailed calculations and obtained similar results.

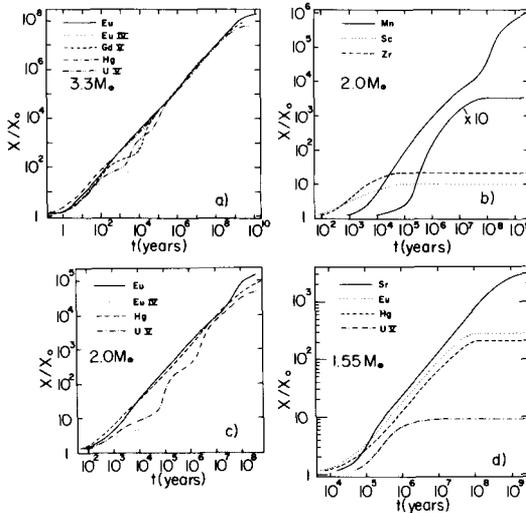


Figure 3. Time evolution of abundances in the atmospheres of main sequence stars. It is here assumed that all elements that get into the atmosphere stay there, and that turbulence is negligible. In general, we thus obtain the maximum over-abundances possible.

#### 4. DIFFUSION AND ABUNDANCE ANOMALIES IN F, A AND B STARS

If stars are stable over periods of  $10^4$  years, diffusion is expected to modify their surface abundances. Since those stars that show abundance anomalies also are the most likely stars to have stable atmospheres or envelopes (Strittmatter and Norris 1971), diffusion appears as a likely explanation of the anomalies.

Indeed the Fm, Am, Ap and Bp stars are, as a group, slow rotators. Meridional circulation is less important in them and can more easily be suppressed by magnetic fields. The hydrogen convection zone is progressively less important (from Fm to Bp) as the effective temperature

increases and carries less and less of the energy flux. Indeed, in the Ap and Bp stars magnetic fields are often observed at the surface and are apparently very stable. As the stars rotate, magnetic fields are observed to vary in a periodic way. All well studied magnetic variables can apparently be explained in this way (Preston 1970, 1971); that is, by the oblique rotator model. In the Ap and Bp stars, the magnetic fields apparently succeed in imposing some order to the atmosphere. The Ap and Bp stars show the largest abundance anomalies. In the Fm and Am stars, however, no magnetic field is generally observed, presumably because the surface convection zone carries relatively more energy flux than in the hotter (Ap and Bp) stars. Diffusion probably goes on below the hydrogen convection zone in the Fm and Am stars. The abundance anomalies are then smaller than in the Ap and Bp stars since a larger mass must be contaminated.

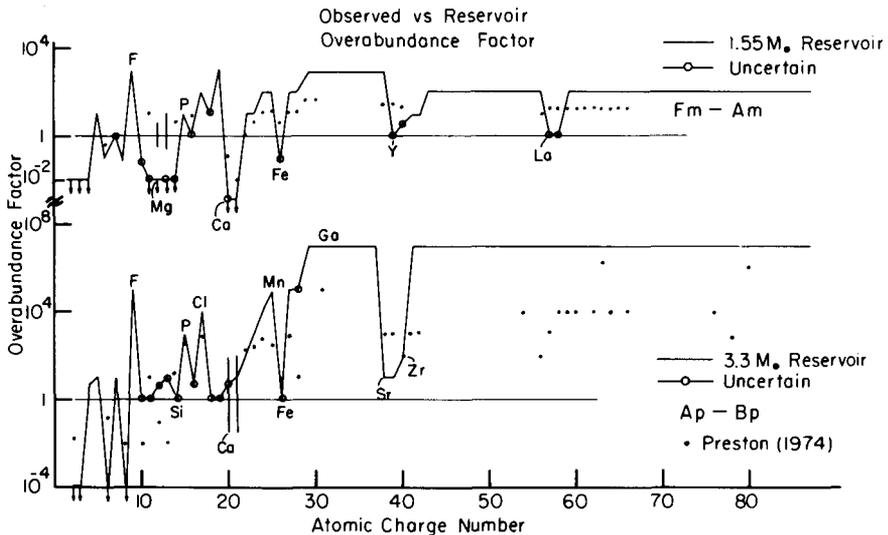


Figure 4. The observed anomalies (fig.7 of Preston, 1974) are compared to the anomalies allowed by the reservoir. No arbitrary parameter is involved. The maximum observed abundance is shown since the reservoir gives us the maximum possible overabundance in a stable envelope. Vertical lines are used when both overabundances and underabundances are observed. Circles indicate calculations in which the radiative and gravitational accelerations are within a factor of three of each other. The result is then uncertain. The only apparent difficulty is with magnesium (Mg).

In Fig.4 the observed abundance anomalies are compared to the abundance anomalies that the diffusion of elements to and from the stable envelope of the stars can lead to. The diffusion equation was here solved for the different elements both in a 1.55 M and in a 3.3 M star. The atmosphere of the 3.3 M star is assumed to be stable but for the

1.55  $M_{\odot}$  only the zone below the hydrogen convection zone is assumed to be stable. All elements that reach the atmosphere (or the convection zone) are assumed to stay there, that is, they are assumed not to leave the stellar atmosphere via a stellar wind. The radiation forces used were accurate enough for  $T_{\text{eff}} \gtrsim 4$  but not at lower temperatures. For the 3.3  $M_{\odot}$  star the region where  $T_{\text{eff}} < 4$  is important and some elements may not be supported in that zone and so may not reach the atmosphere (see Michaud et al. 1976, for details). Turbulence will be discussed in the next paper. It is certainly important for some stars. The calculated abundance anomalies of Fig. 4 are then maximum possible values which should be compared to the envelope of the abundance anomalies observed. On Fig. 4, they are compared to the maximum abundance anomaly of each element from Fig. 7 of Preston (1974). For a few elements both overabundances and underabundances are observed; they are indicated by vertical lines. Stars have been grouped as Am or Ap. Groups 2 and 3 of Preston are both included in "Ap", since the difference between the Mn and the other Ap stars is not expected to be related to the "reservoir" but to the atmosphere (see below and also Michaud 1973). Some of the calculated values are uncertain because the calculated radiative and gravitational accelerations are within a factor of three of each other. The calculated anomaly could then change from an overabundance to an underabundance or vice versa. Even the largest observed abundance anomalies can be explained. Nearly all disagreements appear where the radiative and gravitational accelerations are within a factor of three of each other. The only apparent exception is for Sr, Y, and Zr in the 3.3  $M_{\odot}$  star and for Mg in the 1.55  $M_{\odot}$  star. The case of Sr, Y, and Zr is probably related to turbulence

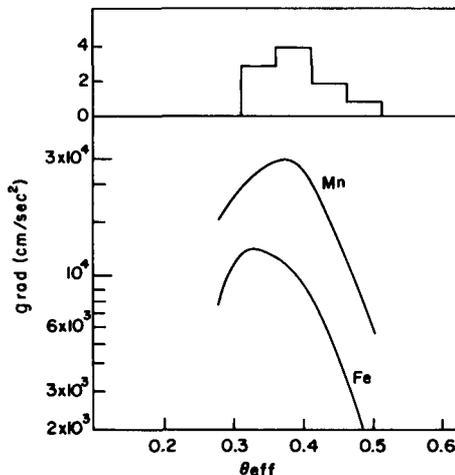


Figure 5. Radiative acceleration transmitted through photo-ionization to Mn and Fe. In the upper part of the figure is shown the number of Mn stars as a function of effective temperature (Sargent and Searle 1967). Most of the Mn stars have the effective temperature for which the radiative acceleration is the largest.

and will be discussed in the next paper. However, if the abundance of Mg is normal in Am stars, it may turn out to be a serious problem for the diffusion model. Watson (1971a) also obtained that Mg should be underabundant in Am stars. Due to the importance of Mg, Ca, Sr, Ba and Hg in Am and Ap stars, Dr Praderie and myself are conducting a detailed NLTE study of elements which have two electrons more than a closed shell when they are neutral (Be, Mg, Ca, Sr, Ba and Hg) (Praderie 1975).

In the Ap and Bp stars, the atmosphere is apparently stable and the radiation force must be larger than the gravitational force in the line forming region for overabundances to appear. The reverse is true for underabundances. Fig. 5 depicts the radiative force as a function of effective temperature for Mn and Fe. Only the radiation forces transferred through photoionization are shown. The contribution of line absorption is not negligible but is not expected to change the shape of the diagram (Alecian 1976). The number of Mn stars at a given effective temperature is also shown. The largest Mn overabundances appear where the radiation forces are the largest. Similar diagrams can be made for He or O with similar agreement (Michaud 1970).

The great success of the diffusion theory is that without any arbitrary parameter, it can explain the largest abundance anomalies in Fm, Am, Ap, and Bp stars and most of the variations with the effective temperature of the stars.

It has not yet been possible however to reproduce the detailed abundances anomalies of individual stars. The needed calculations require a detailed understanding of the relation between stellar atmospheres and the interstellar matter, and NLTE calculations of radiation forces in the outer atmosphere. Such calculations are underway for strontium and atoms with similar atomic configurations, but many more of them are needed. Turbulence will also be important in this comparison. Because of our lack of understanding of the hydrodynamics of meridional circulation, turbulence, convection and of their relation with magnetic fields some arbitrary parameters will unavoidably appear in this comparison. However, it may be that the abundance anomalies will put important constraints on the hydrodynamics of stars.

## 5. STELLAR WINDS, LINE ASYMMETRIES AND ISOTOPE ANOMALIES

The Ap-Bp stars are often separated into two groups, the Mn-Hg stars and the others (hereafter Si-Eu stars). In general the Si-Eu stars have magnetic fields, while the Mn-Hg stars do not. A magnetic field strongly influences diffusion in the outer parts of the atmosphere ( $\tau < 0.1$  at  $\lambda = 5000$  Å, which is also the wave length of the optical depths mentioned below) for ionized elements. Whereas ionized elements can leave completely the atmosphere of a star without a magnetic field, they cannot cross magnetic field lines in the Si-Eu stars. Many more elements are expected to be overabundant in the Si-Eu stars than in the Hg-Mn stars. This is observed to be the case (Preston 1974).

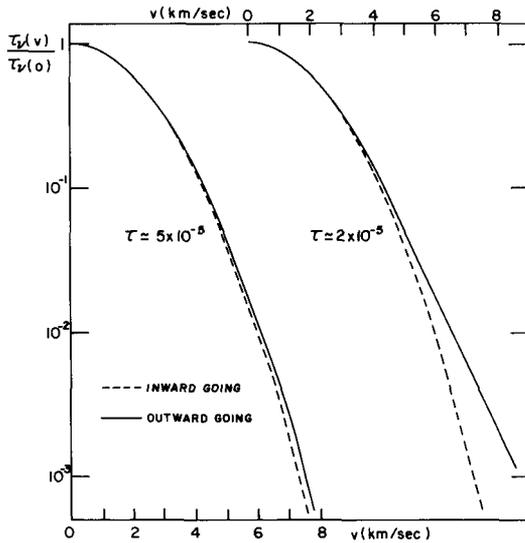


Figure 6. Profile of the optical depth of the line of an element (SrII) in clouds at two different continuum optical depths. The profile is given as a function of the velocity of the absorbing atom. At  $v = 0$ , the absorbing atom is at rest with respect to the star. The line is assumed Doppler broadened. Because of the radiative flux, there are more outward than inward going elements. In a cloud at  $\tau \approx 2 \times 10^{-5}$ , depending on how saturated the line is (how large is  $\tau_\nu(0)$ ) the maximum of the anisotropy will occur somewhere between 4.5 and 7 km/sec (in the observed line).

In the Mn-Hg stars the only elements expected to be overabundant are those whose radiative force decreases as the element tries to leave the star. The element is then trapped. Such an element is pushed from the envelope and migrates to the upper atmosphere but the radiation force decreases, becomes smaller than the gravitational force and the element stays in the atmosphere. It has been shown that the mercury isotope anomaly could be explained in this model (Michaud et al. 1974). Mercury must stand in a cloud at  $\tau \lesssim 10^{-2}$ . The other overabundant elements are, similarly, expected to stand in clouds, somewhere between  $\tau = 0.5$  and the interstellar matter. Some elements will be higher up than others depending on where in the atmosphere their radiative force becomes smaller than the gravitational force.

One important consequence of this model is for line anisotropies. Using equations (13) and (17), it is easy to verify that at  $N \approx 10^{10}$   $\text{cm}^{-3}$ , diffusion velocities of a few kilometers per second become possible. If an element stands in a cloud at that density, anisotropies similarly appear on the line profile. Fig. 6 shows the profile of the optical depth of a Doppler broadened line of an element in such a cloud.

It leads to line anisotropies at 4.5 to 7 km/sec, depending on the strength of the line. Anisotropies at 8 km/sec have been observed by Smith (1976). Whether the difference is unacceptably large is unclear at the moment. This model predicts that the anisotropy should not appear in helium nor in oxygen lines, since those elements are not supported by radiation pressure. Only elements which stand in clouds at  $\tau < 10^{-3}$  should show such an effect.

Whereas diffusion explains without arbitrary parameters the "envelope" of the observed abundance anomalies and their variation with effective temperature, the explanation of such features as isotope anomalies and line anisotropies require a more detailed knowledge of the outer atmosphere than is currently possible, and arbitrary parameters must be introduced in the diffusion calculations.

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