Abstracts of Australasian Ph.D. theses

Fixed points of semigroups in a Choquet simplex and compactification

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We are primarily concerned with the action of a semigroup S of affine continuous maps defined on some compact convex set X , with F denoting those points of X fixed under all mappings in S . Letting $\mathrm{ex}X$ and $\mathrm{ex}F$ be the corresponding sets of extreme points conditions are given which ensure that $\mathrm{ex}X \cap F = \mathrm{ex}F$. As a consequence a characterization of extremely left amenable semigroups is obtained. If X is a Choquet simplex, F is a Choquet simplex whenever $s(\mathrm{ex}X) \subseteq \mathrm{ex}X$ for all $s \in S$. If the action of S is weakly almost periodic, conditions are given ensuring that $\mathrm{ex}F$ is K-analytic whenever $\mathrm{ex}X$ is K-Borel. This discussion depends upon the fact that if X is a simplex and $\mathrm{ex}X$ is invariant under S , each maximal measure representing a point in F is S-invariant.

We then consider a family $\left(E_\alpha\right)_{\alpha \in I}$ of sets and a corresponding family $\left(B_\alpha\right)_{\alpha \in I}$ of function algebras. We show how to associate with the set $E = \underset{\alpha \in I}{\times} E_\alpha$ a function algebra B so that $E^\wedge = \underset{\alpha \in I}{\times} E_\alpha^\wedge$ where $E_\alpha^\wedge$ is the compactification of E_α using the algebra B_α and $E^\wedge$ is the compactification of E using B . Applications are given to the problem of extending group and semigroup structure from a set to its compactification.

Returning to fixed point problems, we show that if G is a σ-compact group of affine homeomorphisms of the simplex X then G has at most one fixed point in X if G acts transitively on $\mathrm{ex}X$ and if the associated

map $G \times X \to X$ is continuous or Baire measurable. Our compactification results are applied to deduce the uniqueness of invariant means on certain algebras. Finally we discuss some further results on extremely amenable semigroups relating to fixed point properties.

References

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