PROBLEMS FOR SOLUTION

P 75. In a certain isolated community the marriage contract is for one year only. So great is the satisfaction with this arrangement that each January 1st the entire population, consisting of an equal number of men and women, gathers together and marries (in pairs) for the coming year. It may happen that a couple marry who have been married to each other in the past, there being no stigma attached to this. A "marriage Graph" may be defined as a bipartite graph whose two vertex sets correspond to the men and women and in which two vertices are joined by an edge if and only if the corresponding people have been married to each other at least once.

What are necessary and sufficient conditions for a bipartite graph G to be a "marriage graph" of some such community for a period of n years during which the population remains fixed?

J. W. Moon, University College, London

P 76. If H is a normal subgroup of a group G then, in particular,

(1) H commutes with every subgroup K of G, i.e. $HK = KH$;

(2) H is subnormal in G, i.e. there exists a normal series from G to H.

Thus these two properties are each generalisations of the property of being normal. Show that for any finite group G, any subgroup H which has property (1) also has property (2).

J. D. Dixon, California Inst. of Technology

P 77. Prove that $n > 3$ lines in the projective plane, no three concurrent, determine at least n triangles.

Leo Moser, University of Alberta, Edmonton

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P 78. Prove that a field is formally real if and only if -1 is not a sum of fourth powers.

I. G. Connell, McGill University

SOLUTIONS

P 64. Find all solutions of

$$\tan^{-1} 1 + \tan^{-1} 2 + \ldots + \tan^{-1} n = \frac{k\pi}{2} .$$

Leo Moser, University of Alberta

(A partial solution was published in vol. 6, no. 3.)

Solution by Robert Breusch, Amherst College.

If $\prod_{s=1}^{n} (1+is) = a + ib$, then a and b are clearly integers. Since $\prod_{s=1}^{n} (1+is) = \prod_{s=1}^{n} (1+s^2)^{1/2} \cdot \exp(i. \sum_{s=1}^{n} \tan^{-1} s)$, the given condition implies that $\prod_{s=1}^{n} (1+is)$ is either real, or purely imaginary, and in any case that its absolute value is an integer. It follows that $R = \prod_{s=1}^{n} (1+s^2)$ is a square, and thus that R contains each one of its distinct prime factors at least twice. Any prime whose square divides one of the factors of R, must be $\leq n$, and any prime which divides two distinct factors, $1 + s_1^2$ and $1 + s_2^2$, must divide either $s_1 - s_2$ or $s_1 + s_2$, and thus must be $\leq 2n$. It follows:

(1) R contains no primes $> 2n$.

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