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Mapping the Dark Energy Equation of State

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Abstract. The acceleration of the expansion of the universe has deep implications for structure formation, the composition of the universe, and its fate. Roughly 70% of the energy density is in a dark energy, whose nature remains unknown. Mapping the expansion history through supernovae, mapping the geometry of the universe and formation of structure through redshift surveys, and mapping the distance to recombination through the cosmic microwave background provide complementary, precise probes of the equation of state of the dark energy. Together these next generation maps of the cosmos can reveal not only the value today, but the redshift variation, of the equation of state, providing a critical clue to the underlying physics.

1. Introduction

Observations of the distance-redshift relation of Type Ia supernovae have given firm evidence of an accelerated expansion of the universe (Knop et al. 2003; Tonry et al. 2003). As calibratable "standardized candles", these supernovae are excellently suited to map the expansion history $a(t)$ due to the direct relation between the measurements and the cosmological dynamics. The redshift of the supernova, $z = a^{-1} - 1$, measures the scale factor a, the size of the universe when the supernova exploded relative to its current size. The calibrated peak magnitude gives the distance, which translates to the lookback time to the explosion.

The data clearly indicate an acceleration to the expansion rather than the slowing down under gravitational attraction that was previously expected. This gravitational repulsion is generally interpreted in terms of an additional component to the energy density of the universe, and given the name dark energy. Since the effective gravitating mass in general relativity depends on both the energy density ρ and pressure p in the combination $\rho + 3p$, such a repulsion and hence acceleration could be induced by a component with strongly negative pressure. Characterized in terms of the equation of state ratio $w = p/\rho$, the condition for a single component to accelerate the expansion is $w < -1/3$.

In order to achieve the acceleration deduced from the distance-redshift measurements, in a flat universe with (decelerating) matter density as well, the energy density in dark energy must amount to $\sim$70% of the total. Thus the majority of the universe is composed of dark energy, determining the cosmic dynamics and the fate of the universe. Moreover, the equation of state must be substantially negative, $w \approx -1$. The physics underlying the dark energy sets the equation of state, so to understand this new gravitational or high energy physics requires precise and accurate measurement of this quantity. Is the dark energy Einstein's cosmological constant ($w = -1$ exactly), some high energy physics scalar field (often called quintessence), or a sign of modifications to gravity or the presence of extra dimensions? Except for the cosmological constant, almost all theories predict dynamical dark energy, with an equation of state evolving with the cosmic expansion. In fact, this dynamics, in the form of the time variation $w(z)$, contains the main clue to the new physics. Thus the goal is to bring together astrophysics and particle physics to map the dark energy equation of state.

2. Mapping the Expansion History

The dark energy affects both the expansion history of the universe and the growth history of large scale structure. In addition to the supernova measurements of distances to redshifts of order one, the location of the acoustic peaks in the cosmic microwave background power spectrum provides the distance to the last scattering surface at $z = 1089$. Observations of galaxy clustering, the mass power spectrum, and velocity distortions of large scale structure depend on the growth history.

At present, no one method provides tight constraints on the equation of state (EOS) due to degeneracies between cosmological parameters, e.g. the dark energy density Ω_w and the equation of state w. But combining cosmological probes can break these degeneracies and improve the estimation. One must be careful to ensure that the dark energy has been consistently taken into account when using the quantities derived from different methods, and that, for example, a quoted determination of matter density did not assume a cosmological constant universe. In the current state of the art, such a consistent analysis leads to a measure of the *assumed constant* equation of state $w = -1.05^{+0.15}_{-0.20}$ (Knop et al. 2003). Note that many models other than the cosmological constant possess an averaged EOS near -1 for part of their evolution, so these limits do not rule out many physically distinct models.

But the observational situation is rapidly improving. The approximation of a constant w will soon be confronted with large data sets of supernovae to $z > 1$ and deep galaxy redshift surveys, and of course the CMB data probe a quantity $\langle w \rangle$ different from a low redshift, averaged w. Furthermore, the time variation w' is a critical clue to the underlying fundamental physics. Analysis of these data in terms of an *a priori* fixed EOS is insufficient and can both blind and mislead us.

Fortunately there exists a simple parametrization of EOS that incorporates the dynamical aspects but does not require model dependent elements that interfere with comparison of predictions among models. The parametrization

$$w(z) = w_0 + w_a(1-a) = w_0 + w_a\, z/(1+z) \tag{1}$$

is well behaved to high redshift and serves as an excellent approximation (see Figure 1, left panel) to slow roll scalar field models of dark energy (Linder 2003a).

Moreover, modifications of the Friedmann equation for the rate of expansion can be written in terms of an effective $w(z)$. If we admit ignorance of the physical

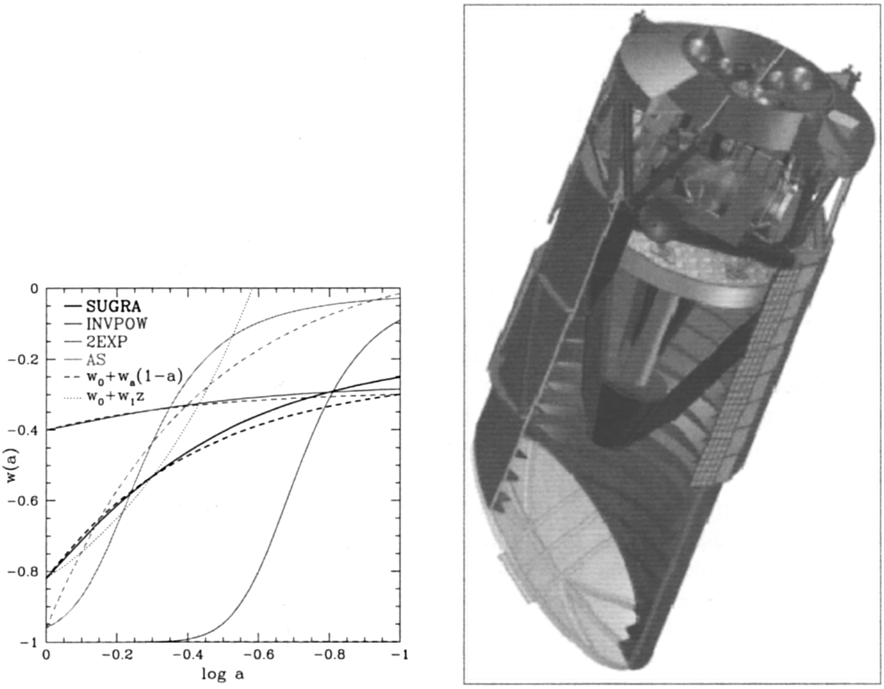

Figure 1. *Left panel* – The equations of state of four dark energy models are plotted as a function of expansion factor. Dashed lines show the reconstruction from the simple parametrization in eq. 1. The dotted line gives the old, linear in redshift, parametrization for the SUGRA case. *Right panel* – Cutaway view of proposed SNAP satellite, designed to measure the equation of state and its variation.

mechanism leading to the observed acceleration, then we would write

$$[H(z)/H_0]^2 = \Omega_M(1+z)^3 + \delta H^2/H_0^2, \tag{2}$$

where we know there exists some matter density Ω_M and allow some additional term δH^2, which may or may not be a real dark energy. But we can still consistently define an effective EOS as

$$w(z) \equiv -1 + \frac{1}{3}\frac{d\ln\delta H^2}{d\ln(1+z)}. \tag{3}$$

So how do we design next generation cosmological probes to uncover the crucial information of $w(z)$? The clearest hope resides in observations that involve simple, well understood physics, with tightly constrained systematic uncertainties. Perhaps the most promising is the technique that first discovered the dark energy – the Type Ia supernova method. Each supernova provides not just a single data point but a rich stream of crosschecking information in the form of its light curve (magnitude vs. time) and energy spectrum.

The Supernova/Acceleration Probe (SNAP; see right panel of Figure 1) is a proposed mission dedicated to studying dark energy, employing the supernova method along with other techniques (see http://snap.lbl.gov). It consists of a 2-meter aperture telescope in space coupled to a 1 degree field of view mosaic camera instrumented with over half a billion pixels, plus a low resolution spectrograph. Nine filters cover the optical and near infrared from 3500-17000 Å. SNAP can discover and follow up over 2000 supernovae in the range $z = 0.1-1.7$, characterizing them precisely in terms of their spectra, and bounding systematic uncertainties below 0.02 mag (1% in distance).

In its deep survey mode, SNAP repeatedly scans 15 square degrees of sky to study supernovae. At the same time these observations can be used to build up a deep weak gravitational lensing map of the sky, detailing the dark matter distribution. Moreover, the data resources cover 9000 times the area of a Hubble Deep Field and reach coadded depth of AB magnitude 30.3. The wide field survey images 300 square degrees or more to AB=28.1 in nine filters, with the weak lensing information providing important constraints on the cosmological parameters complementary to the supernova determinations (Refregier et al. 2004).

Indeed, complementarity between precision methods greatly strengthens the confidence in and leverage of the dark energy parameter estimations. For such a momentous discovery as dark energy, we need to place a premium on accurate observations, where systematics can be well understood and tightly limited. But even so, the use of two or more techniques with distinct sources of systematics should be strongly sought to ensure dependable conclusions.

Figure 2 (left panel) shows the advantages to combining supernova data from SNAP with CMB data from Planck. While SNAP alone does constrain dark energy models, the inclusion of CMB data means that no prior knowledge on the matter density is required. This makes the conclusions much cleaner, and we see that the parameter contours are much tighter as well, equivalent to those with a prior $\sigma(\Omega_M) \approx 0.01$ (Frieman et al. 2003). Together the two data sets can detect the time variation w' from, say, a supergravity inspired dark energy model at the 99% confidence level. This means we will have advanced from originally detecting the mere existence of dark energy (that $\Omega_\Lambda > 0$) at 99% probability, to characterizing its EOS dynamics at the same level.

Weak gravitational lensing offers another important probe. Wide field, deep data such as from SNAP can constrain Ω_M independently and place limits on a combination of the present EOS w_0 and the time variation w'. This provides a good crosscheck on the supernova plus CMB results, or further complementarity in the time varying EOS case. See Linder & Jenkins (2003) for calculations using the linear part of the lensing shear power spectrum and Jain & Taylor (2003) and Bernstein & Jain (2004) for shear crosscorrelation methods.

3. Mapping the Growth History

The expansion history $a(t)$ offers a clear method for mapping the equation of state $w(z)$, with the hope of then revealing the underlying physics, for example the scalar field potential $V(\phi)$. The quantity w' is directly related to the slow roll parameter V'/V. But as we have seen in eq. 3, general modifications of the expansion rate can lead to $w(z)$. So we may not be able to uniquely interpret

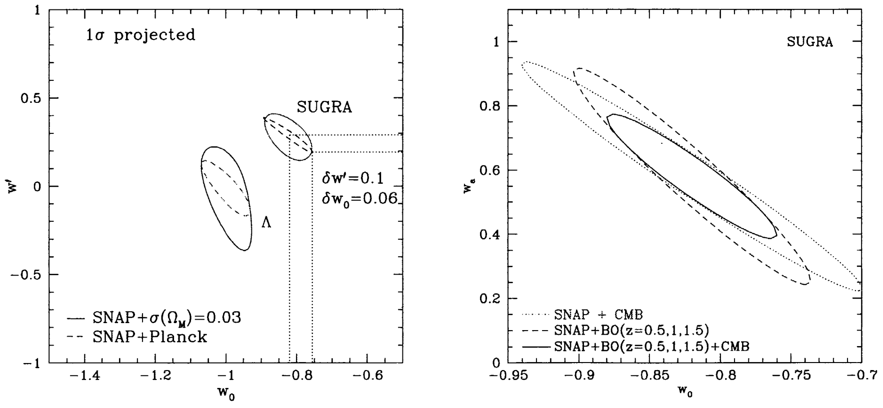

Figure 2. *Left panel* – Complementarity between SNAP supernova data and Planck CMB measurements greatly improves dark energy constraints ($w' = w_a/2$) and removes the need for a prior on Ω_M. *Right panel* – Baryon oscillation measurements also provide good complementarity in the case of time varying equation of state.

even very precise results. Ideally we would like a second avenue to investigate dark energy, where the equation of state enters differently.

Growth of structure in the universe provides such a path in theory. As dark energy begins to become an important fraction of the total energy density, it acts to shut down the growth of density perturbations in the matter. Through the hierarchical process of structure formation this then has implications for, e.g., galaxy mass profiles and the abundance of galaxy clusters. However, these objects also involve hydrodynamics and feedback, nonlinearities, and a host of sources of astrophysical confusion. The requirement of accurate, well understood probes leads us to look at the largest, mostly linear scales.

Data for precise measurements of effects on the matter power spectrum on large scales requires wide field, deep surveys. One example is weak lensing surveys discussed above, which gravitationally detect even the dark matter. Another involves galaxy redshift surveys. For both, one seeks orders of magnitude improvement, taking for example the recent 2dF (two square degree field) survey and enlarging it to, say, 400dF. An exciting prospect is the KAOS (Kilo-Aperture Optical Spectrograph) instrument proposed for the Gemini 8-meter telescope (see http://www.noao.edu/kaos). This would allow simultaneous redshift determination of 4000 galaxies over a 1.5 square degree field of view. Utilization of this facility in a Dark Energy Project (KAOS Purple Book 2003) to observe a million galaxies at redshifts $z \approx 1$ and $z \approx 3$ could probe dark energy by measuring the baryon oscillations in the matter power spectrum.

These baryon oscillations are the analog of the acoustic peaks in the CMB temperature power spectrum. They both arise from the decoupling era, $z \approx 1100$, when density perturbations in both the baryons and photons could only oscillate without growing. While the peaks and troughs left in the photon spectrum are large, the baryons are overwhelmed by cold dark matter and so only leave wiggles in the matter spectrum. Since the scale of these wiggles is set by the physics at the decoupling era between baryons and photons, they act as a standard ruler to determine the ratio of the observed oscillation scale to the sound horizon. This effectively measures both the angular distance to the redshift of the galaxies used, and the Hubble parameter $H(z)$ at that redshift (Blake & Glazebrook 2003; Linder 2003b; Seo & Eisenstein 2003). Because the wiggles depend on well determined physics, measurements in the linear regime, and scales rather than amplitudes, this probe is substantially free from systematic uncertainties. The baryon oscillation method offers good crosschecks and complementarity with the more precise supernova and CMB methods, as illustrated in the right panel of Figure 2.

Another aspect of large scale structure in the linear regime is the growth of perturbations. The growth factor determines the linear part of the matter power spectrum, and enters the nonlinear part in various semianalytic treatments like extended Press-Schechter formalisms or the halo model. The influence of dark energy appears in two areas: the Hubble drag term that slows the linear perturbation growth; and the size and evolution of the matter source term (see Linder & Jenkins 2003 for more discussion). Figure 3 (left panel) shows this influence on the normalized growth $(\delta\rho/\rho)/a$. Note that time varying EOS models can have an appreciable effect at quite high redshifts. Another point of importance is that dark energy models that appear degenerate with respect to the CMB (the constant $w_{\rm eff}$ models and the corresponding time varying model in brackets) can be distinguished via the growth factor.

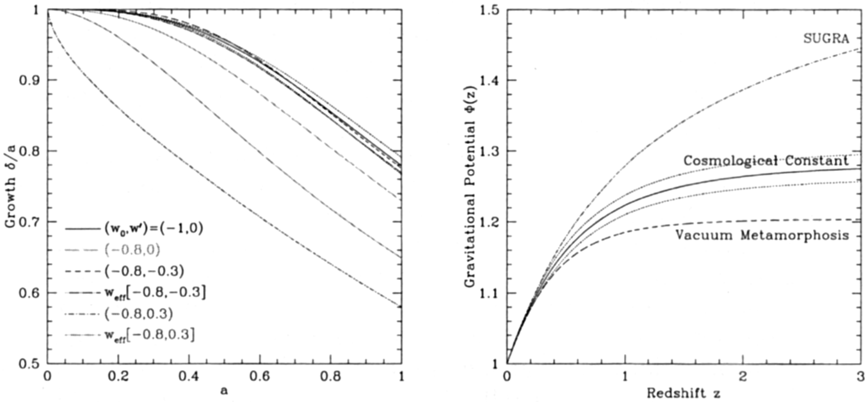

Figure 3. *Left panel* – The growth factor of linear density perturbations can detect time variation in the dark energy equation of state. Moreover, it separates the time varying models in brackets from the constant $w_{\rm eff}$ ones degenerate with respect to the CMB. *Right panel* – Gravitational potential exhibits clear variations in time evolution for different dark energy models. Dotted lines denote variation of Ω_M by ±0.02 in the cosmological constant model.

The growth factor δ/a is directly proportional to the gravitational potential $\Phi(z)$, through the Poisson equation. This not only gives a useful visual representation of the decay of potentials as dark energy begins to dominate over matter (see the right panel of Figure 3), but is central to the integrated Sachs-Wolfe effect on the CMB low multipoles or large angles. Dark energy parameters enter with a different dependence than for supernova distances or the CMB acoustic peak location. Unfortunately the large angular region of the CMB suffers strongly from cosmic variance, so it is not easy to extract dark energy characteristics from the data, though various methods of crosscorrelation are proposed to try. Nevertheless, the growth history in one manifestation or another offers attractive complementarity with the expansion history as a probe, in particular for looking at modifications of gravity or more complicated dark energy models that involve nonminimal couplings or noncanonical sound speeds.

To attempt to use properties of galaxies or clusters as dark energy probes, we must understand enough to cleanly disentangle astrophysics of these objects from the cosmology. This is a challenging prospect, both for theory and observation. For example a shift by 10% in the limiting mass threshold when counting clusters as a function of redshift is degenerate with a systematic bias in the EOS by 10% (M. White, private communication).

On the theoretical side, until recently no calibration of the cluster mass function (numbers of clusters as a function of mass and redshift) existed for models with time varying EOS. The results of Linder & Jenkins (2003) for the highest mass clusters (least subject to nonlinearities and astrophysical effects) offer some hope as the mass function seems determined predominantly by the linear growth factor. Indeed, it is fit to within 20% by the standard Jenkins et al. (2001) mass formula and definite differences in the amount and evolution of large scale structure exist for various dark energy models. But considerable research remains before we can confidently use galaxies and clusters directly for precision cosmology.

4. Conclusion

Dark energy poses a fundamental mystery as to what composes the majority of the universe, dominates its dynamics through a gravitational repulsion, and determines the fate of the universe. Constraints on an averaged equation of state quantity from impressive efforts over the last five years show that it behaves roughly like a cosmological constant. The precision of 15% is already good enough that continuing measurements along these lines are unlikely to be able to detect a deviation from cosmological constant properties at 3σ. But powerful next generation experiments are being designed that are sensitive not to a crude approximation of a constant EOS, but that can map out the dynamical physics of a time varying equation of state $w(z)$. These will also generate extraordinary astronomical data resources.

Mapping the expansion history to $z = 1.7$ through the Type Ia supernova method offers great promise. Even more valuable gains in accuracy and precision come from working together with crosschecking and complementary methods such as CMB measurements, weak gravitational lensing, and baryon oscillations measured in the matter power spectrum by large, deep galaxy surveys. Mapping the dark energy equation of state will give us guideposts to high energy physics, the early universe, extra dimensions, or the theory of gravity, as well as reveal to us the true nature of the universe we live in and a picture of our fate.

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References

Bernstein, G., & Jain, B. 2004, ApJ, 600, 17

Blake, C., & Glazebrook, K. 2003, ApJ 594, 665

Frieman, J. A., Huterer, D., Linder, E. V., & Turner, M. S. 2003, Phys.Rev.D, 67, 083505, astro-ph/0208100

Jain, B., & Taylor, A. 2003, Phys.Rev.Lett, 91, 141302, astro-ph/0306046

Jenkins, A., et al. 2001, MNRAS, 321, 372

KAOS Purple Book 2003, http://www.noao.edu/kaos/KAOS_Final.pdf, ed. A. Dey & B. Boyle

Knop, R., et al. 2003, ApJ, 598, 102

Linder, E. V. 2003a, Phys.Rev.Lett, 90, 091301, astro-ph/0208512

Linder, E. V. 2003b, Phys.Rev.D, 68, 083504, astro-ph/0304001

Linder, E. V., & Jenkins, A. 2003, MNRAS, 346, 573

Refregier, A., et al. 2004, AJ, 127, 3102

Seo, H-J., & Eisenstein, D. J. 2003, ApJ, 598, 720

Tonry, J., et al. 2003, ApJ, 594, 1