CORRESPONDENCE.

FRIENDLY SOCIETY LEVIES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The following problem is sometimes met with in the valuations of small Friendly Societies, namely: Let there be m members in a society, and let there be upon the death of each member a levy of 1 paid by each remaining member. It is required to find the value of all the levies.

The only published solutions that I know of were given by Mr. G. F. Hardy in a letter to the *Insurance Record* of 10 Sept. 1880. He there demonstrated the three formulas

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$$W = \frac{m^2}{2} \bar{A}_{xx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$W = \frac{m(m-1)}{2} \bar{A}_{xx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$W = m^2 \left\{ \frac{a_{x:x-1}}{p_{x-1}} - a_{xx} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where all the members are assumed to be of one age, x, and where the symbol W is used to represent the value of the levies.

Mr. Hardy in his letter also showed that formulas 1 and 3 give only approximate solutions, since they contemplate the possibility of a fraction of an individual dying, and the remaining fraction paying the proportionate part of a levy; but that formula 2 is rigidly accurate.

For Mr. Hardy's solutions your readers may be referred to his letter. My present object is to give an exceedingly simple and elegant demonstration of formula 2, which has been supplied to me by Mr. Hubert Ansell.

Let each of the m members be supposed to have effected several assurances, namely, one upon the life of each remaining member jointly with his own, but so that there are no duplicate pairs. Then it will happen that on the death of every member, since he is assured jointly with every other member of the Society, there will become payable as many sums of 1 as there are surviving members; and this is the same sum as would be paid if a levy of 1 were made upon each surviving member. Therefore the present value of the levies must be the same as the present value of the assurances supposed to have been effected, namely, $\frac{m(m-1)}{2} \bar{A}_{xx}$; the number of assurances being the number of combinations that may be made of m things taken two at a time, or $\frac{m(m-1)}{2}$.

Perhaps Mr. Ansell's solution, as given above, may be made a little clearer by means of a slight modification. Instead of assuming *joint-life* assurances, let each member effect a *contingent* assurance on the life of each other member against his own life, by means of which to pay his share of the levies as they arise. The value of the levies will then be exactly equal to the value of all the contingent assurances. The value of each contingent assurance is $\bar{A}^1_{xx}$, which is equal to $\frac{1}{2}\bar{A}_{xx}$. There are m members, each of whom effects $m-1$ contingent assurances, so that there are $m(m-1)$ such assurances at the outset, and therefore their aggregate value is $\frac{m(m-1)}{2} \bar{A}_{xx}$, as before.

When all the members are not of one age, an average age must be found, and this may conveniently be done by means of Mr. Makeham's formula for the law of human mortality.

Theoretically, formula 2 is the correct solution of the problem, but in practice it is not entirely satisfactory. If members secede from the Society, or if new members join, the value of the levies,

even as regards the original members who remain, is changed; and there is no practical method by which account can be taken of these causes of disturbance.

I am, Sir,

Your obedient servant,

92, *Cheapside, London, E.C.*
25 *May* 1887.

GEORGE KING.

[We hope to place before our readers, in our next issue, a communication from Mr. G. F. Hardy on the above subject.—Ed. *J.I.A.*]

INSURANCES AGAINST ISSUE.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—You have on two occasions, namely in January 1877 and January 1882, given tables showing the amounts of the Issue Insurances in force according to the latest returns; and it may interest your readers to see the following table, in which the information is brought down to the present time. The figures are, in all cases, taken from the five Blue-Books containing the Board of Trade returns of the years 1881 to 1885 inclusive, except in the case of the *Caledonian* and the *Sovereign* Offices, which made no return during those years, in consequence of their investigations being made at intervals of seven and six years respectively. In these cases, therefore, I have used the figures given in the valuation returns to 31 December 1885, which will appear in the Blue-Book of 1886.

There has been a very great development of this class of business during the period embraced by the returns, as will be at once seen from the following summary.

Date of Table	Number of Policies	Net Sums Assured	Net Premiums Received	Average Premium per-cent
		£	£	
1877	261	875,558	62,238	6·75
1882	383	1,264,166	97,495	7·51
1887	539	1,696,747	116,704	6·82

The general result is that the number and amount of the transactions have about doubled in the course of ten years. The average premium is now almost exactly the same as it was in 1877, but in 1882 it was somewhat higher.

I am, Sir,

Your obedient servant,

Edinburgh,
10 *March* 1887.

T. B. SPRAGUE.
