INTRODUCTION

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Abstract. A historical sketch of the nonlinear theory of variable stars is outlined briefly. The main break-through came from the hydrodynamic study of stellar pulsation. From a theoretical point of view, coupling needs to be discussed more carefully. The impact of new opacities on the astrophysical problems is also discussed.

1 Nonlinear Phenomena in Variable Stars

In the classic textbook written by Rosseland, nonlinear phenomena in pulsating stars are described by inharmonic and relaxation oscillations. The former is the deviation of stellar oscillations from sinusoidal curves, and the latter involves the variability of U Gem stars and shock phenomena.

Astrophysical nonlinear phenomena, which will be studied in the present colloquium, differ from classical nonlinear phenomena. Trends in the studies of pulsation theory changed in recent decades. The strongest influence was the development of electronic computers. Hydrodynamic stellar models then succeeded in showing the features of pulsating stars. Hydrodynamic simulations applied to accretion discs also succeeded in showing details of the variability in cataclysmic variables. Even though such remarkable developments in the studies of stellar variability have occurred, several questions, such as the period-ratio of double-mode cepheids, the complex behaviour found in the δ Scuti stars, and the nature of irregularity observed in various giant stars and cataclysmic variables still remain. Irregular features found in hydrodynamic simulations have not yet been explained theoretically.

In the studies of the irregular nature of stellar pulsations, Baker, Moore and Spiegel pointed out the importance of nonlinear studies for one-zone stellar models. They suggested paying attention to recent developments in nonlinear dynamics which stressed the universal importance of deterministic chaos. The application of modern nonlinear dynamics to stellar variability first concentrated on the study of the conservative case. Later, dissipative systems were investigated. This study brings out new aspects on the nonlinear problem. Period-doubling bifurcation and intermittency have been recognized in hydrodynamical models. The limit-cycles of self-exciting systems have been investigated along the lines of Krogdahl, and used to discuss the progression of bumps in classical cepheids. These systems are studied on electronic computers and interesting features such as period-doubling bifurcation into chaos and phase-locking have been found. These numerical experiments remind us of several important characteristics of nonlinear oscillation theory. Progress in the nonlinear studies of stellar variability is to be reviewed and summarized in the colloquium.

Astrophysics and Space Science **210**: 1–5, 1993.

2 Scale Length of Variability

Stellar variability is usually accompanied by irregular features. Real variability consists of two kinds of instability; one is global instabilities such as the pulsations of a star or an accretion disc; and the other is local instability such as solar flares. The timescale of the latter is related indirectly to the dimensions of the star or the disc, so that the activity seems sporadic compared to the overall phenomena. The observed irregularity can be the superposition of these two types of instability. It is natural to suppose that the variability decomposed into a few modes could be caused by the local instability.

Since a deterministic system can yield chaotic motions, apparently sporadic variability can be produced by global phenomena. This is a new idea which comes from nonlinear dynamics. Several studies have been done on nonlinear oscillators which are expected to show deterministic chaos. The nonlinear oscillations of conservative models were studied first. Then, dissipative models such as the Moore-Spiegel type, the Tanaka-Takeuti oscillator, which is a modified version of the Rössler oscillator, and coupled Krogdahl type oscillators were examined. These models succeeded in showing the period-doubling bifurcation into chaos, but are not directly comparable with observational results. The nonlinear oscillations of one-zone models studied by Saitou, Takeuti and Tanaka, who tried to compare their results with the RV Tau stars, are also one of such oscillators. This exercise has been performed on accretion discs, but there is no complete review of the subject.

3 Breakthrough

The first results directly comparable with observations came from a careful hydrodynamic simulation of cool Population II giant stars. These should correspond to the RV Tau stars. The models demonstrate the period-doubling bifurcation into chaos. Strictly speaking, the models are not complete enough to compare with observations because the models do not include the effect of convection. Convection may play an important role for the stars studied here. What is important is the appearance of the period-doubling bifurcation, both in the simplified model oscillators and complicated hydrodynamic models. It thus is confirmed that the nonlinearity of the excitation and dissipation mechanisms in pulsating stars can produce the deterministic chaos. At least, therefore, we may apply the results of nonlinear dynamics for dissipative systems to semi-regular variable stars.

The physical processes at work in the outer layers of pulsating giant stars include the effects of opacity changes on radiation and other dissipative processes such as cooling. The nonlinearity in the opacity first dams up and then releases the flow of radiation, and then the outer layers drive the increasing and decreasing oscillations of the star. The decrease of the surface gravity makes the amplitude of oscillation large there compared to the inner parts. This causes an increase in nonlinearity both in the excitation and dissipation mechanisms.

4 Intermittent Enhancement of Pulsations

There exists the question as to the reality of intermittent enhancement of pulsations in low surface-gravity stars. This phenomenon was found by Tuchman, Sack and Barkatt and then also found in the models of Fadeyev and his collaborators. Nakata demonstrated that it is real in hydrodynamic simulations and appeared with decreasing surface gravity. Aikawa investigated the details of physical processes in the outer layers and has found that the development of a strong shock may play an important role. The occurrence of such a strong shock may be related to the high opacity layers which dam up the radiation very efficiently. Unfortunately, there is no precise numerical investigation of the effect of convection on the intermittency, although convection can weaken the effect of strong opacity.

5 Coupling

Over more than a decade, coupled oscillations of self-exciting systems have been investigated by several authors. The strength of coupling is determined by two different factors; one is resonance and the other is the similarity of wave functions that is indicated by the coupling coefficients. Resonance can be studied by comparing linear periods, so that this possibility is easily discussed. When resonance works efficiently, the largest amplitude will be that of the lightest mode which is usually a higher mode, since the law of equipartition works for the energy of oscillation among the coupled modes.

The calculation of the coupling coefficients, which are essential for the study of non-resonant coupling, is more complicated than the comparison of the period ratio. The formulation of Buchler and Goupil is elegant but unfortunately difficult to apply to numerical evaluation because of its complexity. On the other hand, simplified formulations are not as complete.

In any case, oscillations become either synchronized or non-synchronized, depending on the strength of coupling. The observed period in the synchronized oscillation differs slightly with one of the linear periods. For non-synchronized oscillations, the observed periods can be different to the linear ones. We have to pay attention to the phase-locking where the period-ratio tends to the ratio of small integers close to the linear period-ratio. The phase-locking needs carefull examination in the study of double-period variability. We can see complex variability in δ Scuti stars and white dwarf stars. The light curves sometimes decompose into several sinusoudal curves, but the modes are not so stable. The transition of the energy of oscillations should be investigated to analyze these stars. It seems probable that we can obtain new results on the inner structure and evolutionary aspects by considering the modal coupling.

6 Yellow Supergiant Stars

The semi-irregular variability found in yellow supergiant stars is studied on return maps on which we plot the successive maximum brightness. The maps never show any one definite pattern, but some of them show an egg-shape pattern and others show a horse-shoe-like pattern. In the numerical studies of one-zone stellar models, the egg-shape appears in low surface-gravity models and the horse-shoe pattern, in high surface-gravity models. It should be pointed out, however, that the egg-shape pattern will also appear for the double-mode oscillations.

The return map of UU Her shows the eff-shape pattern. After the variability of UU Her is decomposed into approximately double-period oscillations, there is no reason to suppose the surface-gravity would be low. UU Her should be a massive star.

7 Accretion Discs

The numerical simulation of the variability of accretion discs have been performed to a high precision. The origin of the variability is not in simple oscillation as is the radial stellar pulsation. The oscillation theory is, however, still applicable to more complex variability of accretion discs. The variability of U Gem is explained as a relaxation oscillation, so that the movement will be approximated by Moore-Spiegel type oscillations. They have two divergent fixed points and one saddle point. Such a model may explain the overall light variation of the discs.

The irregular variability of X-ray sources should be another target for nonlinear study. It seems interesting to separate global and local variations. The disc should show the UU Gem-like variability which controls the mass-flux at the inner edge of discs. Moreover, local activity at the inner edge is also expected. X-ray irradiation from the sources near the central body keeps the mass-flux nearly constant, but flare-like activity will occur frequently in such a high temperature domain accompanied by the strong magnetic field. The problem is now to distinguish the type of variability in the observed X-ray variability.

8 Conclusions

Before finishing the Introduction, we have to stress the importance of the examination of past results using new opacities. The difference in the opacities does not affect stellar models so much, but does affect the pulsation properties. We will see these effects later in the colloquium.

Trends in computing systems are towards down-sizing. More compact, easier to handle computers are now available. Work stations and personal computers are the most useful tools to analyze observational data and to try the models. The exchanges of young astronomers among institutions is fruitful for using these tools. They will have a chance to learn the skills of computation and the details of programming codes. Such technological progress may change the style of variable star studies.

Finally, I should stress that the nonlinearity is never connected with only the large amplitude phenomena. The motion near the limit-cycle should be studied as well as nonlinear phenomena. We anticipate that the papers presented in this colloquium will study the various cases of nonlinear phenomena.

https://doi.org/10.1017/S0252921100013890 Published online by Cambridge University Press