STABILITY OF ACCRETING WHITE DWARFS IN CLOSE BINARY SYSTEMS

R. Sienkiewicz and W. Dziembowski

N. Copernicus Astronomical Center
Polish Academy of Sciences
Aleje Ujazdowskie 4
00-478 Warszawa
POLAND

ABSTRACT

We present results of thermal and vibrational stability analysis for 1 $M_\odot$ white dwarf models corresponding to various accretion rates $\gtrsim 10^{-11}$ $M_\odot$ /y. Accretion is assumed to be spherically symmetric and stationary. Thermal instability due to nuclear burning of hydrogen (at lower accretion rates) and helium (at higher rates) was found. At medium rates two growing thermal modes are simultaneously present. Vibrational instability was found for all models except those corresponding to highest accretion rates. The excitation rates for some nonradial g-modes are at least 3 orders of magnitude higher than those for radial pulsations. These rates are also higher than the excitation rates for thermal modes in certain range of accretion rates corresponding to high luminosities and effective temperatures. Among objects in which these instabilities may be important are symbiotic stars and nuclei of planetary nebulae.

1. Introduction

It has been known for over two decades that white dwarfs accreting hydrogen rich material may be both pulsationally and thermally unstable. These instabilities have been investigated by a number of authors mainly in connection with nova phenomena. Thermal instability due to hydrogen burning as the cause of the eruption was first investigated by Mestel (1952). In Schatzmann's (1953) and Rose's (1968) theories the eruption was due to pulsational instability. In the latter, pulsational instability developed as a consequence of thermal instability.

The discovery of rapid oscillations of the light curves in many cataclysmic binaries gave a new stimulus for studying the pulsational stability of accreting white dwarfs. Warner and Robinson (1972) suggested that the oscillations are due to the excitation of g-modes in the white dwarf component. Such oscillation modes characterized by longer periods than radial pulsations seemed the only ones consistent with the observed oscillation periods.

In the present investigation we studied both thermal and vibrational (pulsational) stability of white dwarfs with mass accretion. In the latter case we considered oscillations corresponding to arbitrary spherical harmonics looking for modes which are most rapidly excited. We believe that our stability calculations are rigorous, provided that dynamical effects of the infalling matter are unimportant.

Serious limitations of our work are connected with the assumptions concerning unperturbed models. The accretion is assumed to be time independent and spherically symmetric. Hydrogen and, in general, helium burning occurs in the outer region of the star. An important property of such models is that for a given mass and surface chemical composition, the accretion rate determines the luminosity and the chemical structure of the star.

The ignorance of the temperature distribution in highly degenerate interiors introduces a negligible uncertainty in the outer layers' structure.

Stability properties of models with the same mass are practically only determined by the accretion rate. Our conclusions concerning violent phenomena that may be expected at various accretion rates are very preliminary due to our simplifying assumptions about the accretion and since only a linear stability analysis was used.

2. Equilibrium Models

Our stability calculations concern the sequence of a 1 $M_\odot$ white dwarf accreting matter with hydrogen content X=0.7 and heavy elements content Z=0.03 in stationary manner. Positions of the models on the H-R diagram are shown in Fig.1.

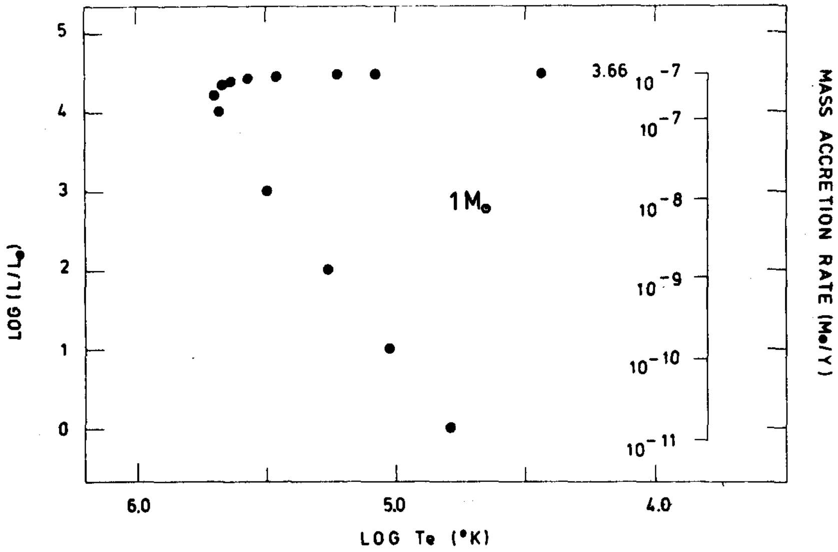

Fig.1. Positions on the H-R diagram of the stationary models corresponding to various accretion rates.

Models lying on the horizontal part of the sequence correspond to accretion rates close to the maximum possible value of $3.66 \cdot 10^{-7}$ $M_\odot$ /y. Their structure is similar to that of the models of nuclei of planetary nebulae calculated by Paczyński (1970, 1971). With the luminosity decrease our models become more and more different from models of the white dwarfs with no external accretion occupying the same positions on the H-R diagram.

In contrast to single white dwarfs, nuclear burning remains the main source of stellar luminosity. In all models except the least luminous one, helium burning occurs and contributes about 13% of the total luminosity. The stationary entropy flow contributes less than about 0.5% of the total luminosity. The rest is due to hydrogen burning.

The sequence was terminated at an accretion rate of $1.46 \cdot 10^{-11}$ $M_\odot$/yr and luminosity 1 $L_\odot$, because at lower rates nonstationary effects connected with cooling are expected to be important. In low luminosity models, the degeneracy in the helium burning shell source was high, but in the hydrogen shell source it was only marginal. A more detailed description of the models and the method of their calculations is given elsewhere (Sienkiewicz, 1975).

3. Stability Analysis

We limited ourselves to the linear stability of the models. Since we assumed that the models are time-nndependent, the temporal dependence of perturbations could be taken in the exponential form $\exp(\omega t)$. For growing modes we have $\mathrm{Re}(\omega) > 0$ and we define the excitation time scale as $1/\mathrm{Re}(\omega)$.

In the linearized energy equation we accounted for the effect of the presence of the entropy flow in the equilibrium model. Resulting terms, however, do not play an important role in the determination of stability properties.

The method and computer program used for vibrational stability calculations is described elsewhere (Dziembowski, 1977). The calculations were done with no a priori assumptions about the size of nonadiabatic effects or gravitational potential perturbations. Non-equilibrium effects in the CNO cycle during oscillations were taken into account.

We did not include such dissipative effects as gravitational wave emission or energy losses due to neutrinos,which are discussed by Osaki and Hansen (1973). These effects are quite unimportant for g-modes, the only modes which may be rapidly excited in white dwarfs. Also, the unknown temperature distribution in the deep interior is unimportant for these modes, as they have their negligibly small perturbation amplitudes.

A separate program was used for thermal stability studies. The method adopted is new and will hopefully be described in the future. Although all instabilities found were aperiodic, the method allows for complex ω. To make sure that no unstable mode was missed, large areas of complex ω-plains were inspected by calculating variations of the argument of the characteristic determinant along the area boundary. Also in this case, the uncertainty of the temperature distribution in the deep interior turned out to be unimportant, because the instabilities found are so rapid that the interior adjusts itself adiabatically. In the conditions of high degeneracy, such adjustment is quite independent of temperature.

We have so far not studied thermal instabilities against non-radial perturbations.

4. Thermal Instabilities

All the models were found to be thermally unstable. The time scale characterizing the growth of the instability as a function of the accretion rate is shown in Fig. 2. At low accretion rates the instability is due to shell hydrogen burning and at sufficiently low rates this is the most violent instability. Nonlinear development of such instability in a 0.8 $M_\odot$ accreting white dwarf was investigated by Paczyński and Żytkow (1977). According to those authors it leads to cyclic flashes characterized by a short-lived high state and a long-lived low state. Practically all hydrogen burning takes place in the high state and later, during the low state, the star resembles a cooling white dwarf. The authors argue that their models are relevant to symbiotic stars but not to novae or other types of erupting binaries. Initial models which led to successful numerical modeling of nova explosions (Starrfield et al., 1974a, b) differ from our models through a higher degeneraca and in a CNO element enhancement.

Thermal instability due to shell helium burning was found at high accretion rates. It exists most likely also at lower accretion rates but is considerably less violent than the hydrogen shell source instability. Nonlinear development of such instability resulting in helium flashes has been intensively investigated in the case of red supergiants. Iben (1975) and Paczyński (1977) demonstrated how such flashes may lead to contamination of the envelope with 3α reaction products, explaining in this way the origin of carbon stars.

It remains to be studied what effect helium flashes may have on the accretion process. Paczyński (private communication suggests that mixing of 3α products into hydrogen rich material in the phase following the flash may be the ultimate cause of CNO elements'

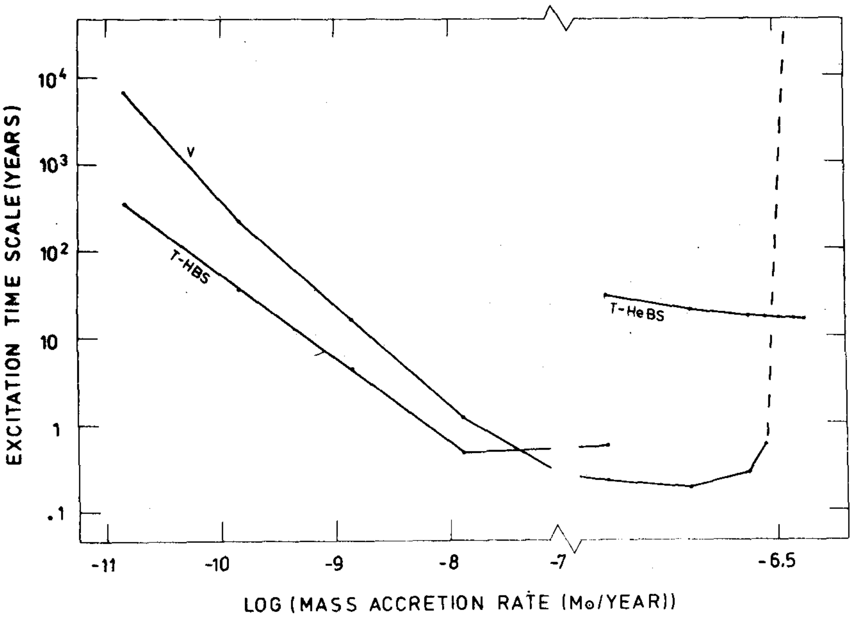

Fig.2. Excitation time scales for the instabilities as functions of the accretion rate. Separate lines correspond to: vibrational instability (V) - the time scale refers to the most rapidly growing g-mode; thermal instability of the hydrogen burning shell (T-HBS); and thermal instability of the helium burning shell (T-HeBS).

enhancement needed for theoretical models of the nova explosion and the overabundance indeed seen in the spectra.

5. Excitation of G-modes

Since the work by Ledoux and Sauvenier-Goffin (1950) it is known that radial pulsations may be excited in white dwarfs if there is a hydrogen burning zone in their outer layers. Our models, except the few most luminous ones, were also found unstable with respect to radial modes but the time scale of their excitation is very long, at least three orders of magnitude longer than that of the most rapidly excited g-modes.

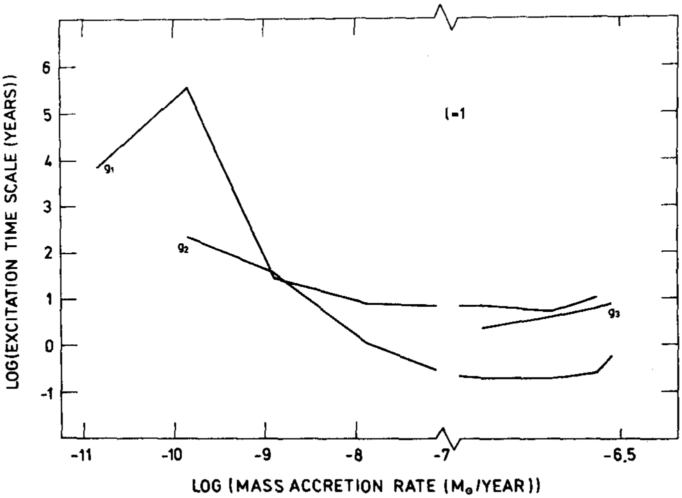

Fig.3. Excitation time scales for low order g-modes corresponding to the first order spherical harmonic (l=1), as functions of the accretion rate.

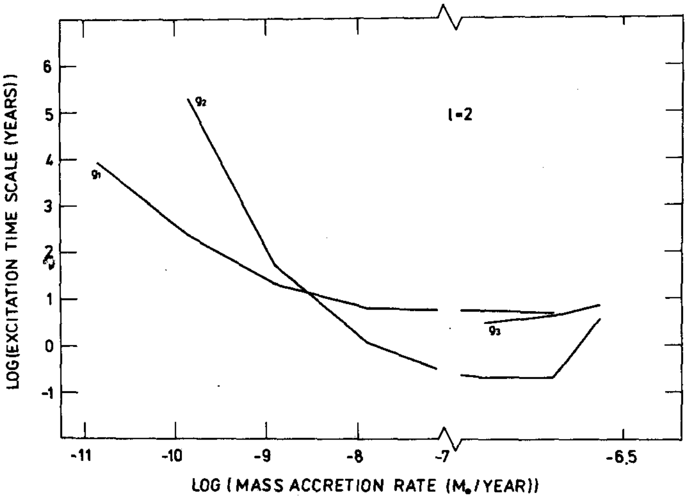

Fig.4. The same as Fig.3 but for l=2.

This is a consequence of the fact that the amplitudes of perturbations for the radial modes in deep interior of the white dwarf are of the same order as in the outer layers. The same is true for non-radial acoustic modes (the so-called f- and p-modes) but not for gravity modes. The latter have large amplitudes only in the superficial layers where driving due to nuclear reactions occurs. Thus, g-modes in white dwarfs similarl to radial pulsations in giants may be excited in the time scale considerably shorter than the Kelvin-Helmholtz time scale for the whole configuration.

In Fig. 2 we show the excitation time scale of the most unstable g-modes as a function of the accretion rate. The excitation occurs for all models except the most luminous ones and becomes more violent than the excitation of hydrogen shell flashes for the rates greater than about $4 \cdot 10^{-8}$ $M_{\odot}$ /Y.

In Figs.3 - 5 the excitation time scales are given for g-modes associated with spherical harmonics of the order, 1, 1,2 and 5. For the majority of the models, g_2-mode corresponding to l=1 is most violently excited, but time scales for g_1 and g_2-modes for all low--order harmonics are of the same order. Instability against g_3-modes was found only for the highest accretion rates. High-order g-modes **cannot be excited due to the predominant role of radiative dissipation for modes having short radial wave-length. Instability disappears also for the modes corresponding to high-order spherical harmonics (l $\gtrsim$ 10) because for such oscillation the amplitude becomes small already in the hydrogen burning zone.**

The role of radiative dissipation in the region above the shell source for vibrational stability varies considerably with the accretion rate. For its largest value the envelope is extended and, just as in giants, stability is determined by the sign of radiative dissipation. At this high effective temperature and, let us not here, using usual Cox-Stewart opacities we find no driving effect. With decreasing accretion flux the role of radiative dissipation becomes smaller because of: 1° envelope contraction, 2° increase of effective temperature causing an increase of radiation contribution in pressure and scattering contribution in opacity, both resulting in small radiation flux perturbation. It is of interest to notice that similar circumstances are quite important for vibrational instability of massive stars.

The helium burning shell plays a minor role in the total excitation of g-modes.

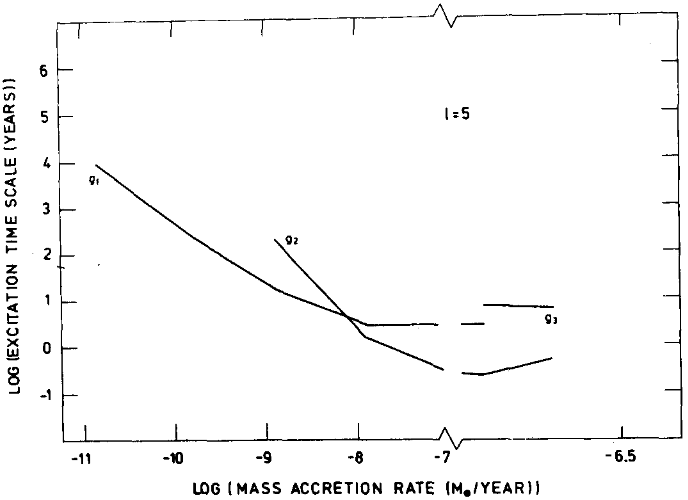

Fig.5. The same as Fig.3 but for l=5.

Periods of unstable g-modes as a function of the accretion are shown in Fig.6. For l $\gtrsim$ 5 (higher spherical harmonics are not likely to be observable) periods are contained in the interval of 10-50 seconds. Because of competition with hydrogen shell flashes, we do not expect g-modes excitation for the accretion rates smaller than about $4 \cdot 10^{-8} M_\odot$ /y. Thus, the objects in which we can expect to detect variability connected with the presence of g-mode oscillations are very luminous, L $\gtrsim$ 10000 $L_\odot$, and hot, log $T_e \gtrsim$ 5. Resulting variability may be quite complex because of possible multi-modal excitation implied by similar excitation rates in many modes and very low excitation rates per period.

As indicated by our preliminary calculations, changes of white dwarf mass do not alter qualitatively the picture. For higher masses, the luminosities and effective temperatures corresponding to the models with expected g-mode excitation are higher.

Periods of the g-modes which may be excited by the action of nuclear reactions in white dwarf models cover the range of periods characterizing rapid variability of the dwarf novae. In spite of this, we do not think that we provided a plausible explanation of the observed variability.

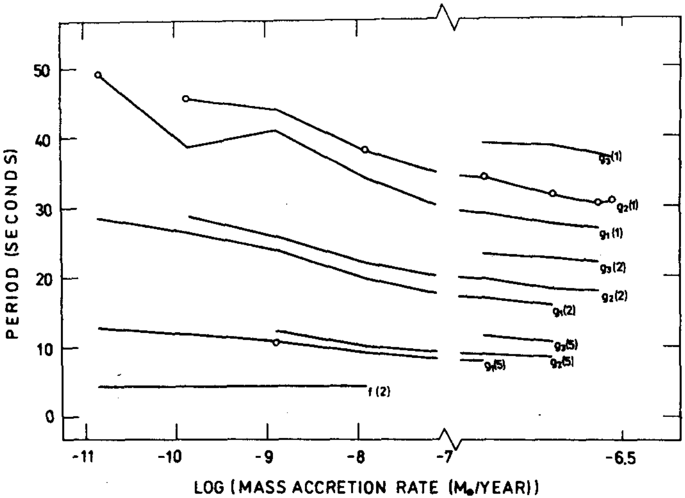

Fig.6. Periods of some unstable g-modes and f-modes as functions of the accretion rate. The points corresponding to the most rapidly growing modes at given accretion rate are denoted with the open circles.

The main difficulty is connected with the time scale of excitation which in our models is of the order of months and this is considerably longer than the time scale characterizing the outburst when the rapid variability is seen. Moreover, time scales of period variations at least in some of the objects (eg. CN Ori) are uncomfortably short for any interpretation involving stellar oscillations. The fact that the rapid variability is a single periodic phenomenon favors the broadly understood rotation as the source of periodicity.

Objects in which our mechanism of g-modes excitation may work are white dwarfs in binary systems in which the second component is a supergiant loosing large amount of hydrogen rich matter in a stationary way. Thus, the hot components of the symbiotic stars are candidates to exhibit this sort of oscillations. We may also expect to detect nuclear reaction driven oscillations in the nuclei of planetary nebulae if they indeed have so high effective temperatures.

6. Conclusions

Direct conclusion of our work is that the stationary accretion on the white dwarf, with rates implying large contribution of nuclear burning to the luminosity, is impossible.

For low accretion rates, thermal instability due to hydrogen burning is the most violent. As suggested by nonlinear calculations of Paczyński and Żytkow (1977), the instability results in hydrogen shell flashes of very large amplitude but not in explosion. Oscillations are probably not excited in such a case, as their excitation rates are lower and in the final development of thermal instability the star spends very little time in conditions favorable for vibrational instability.

At high accretion rates, the hydrogen burning shell is thermally stable and then the excitation of g-modes becomes the most violent instability. We expect that this may lead to observable variability characterized by periods of the order of 10-50 seconds.

We do not think, however, that this is the cause of short period variability of Z Cam type eruptive binaries. Objects in which g-modes excitation can be expected are very hot white dwarf components in the binary, and conceivably the nuclei of planetary nebulae. The observed variability is likely to be multiperiodic.

The thermal instability due to helium burning found in models with relatively high accretion rates is considerably less violent than the previous instabilities. However, onset of helium flashes seems unavoidable because development of the instabilities due to hydrogen burning has little effect on the helium burning shell. On the other hand, helium flashes may affect instabilities due to hydrogen burning. It seems therefore of interest to carry a similar stability analysis for the models representing various stages of the helium flash.

Acknowledgements: We thank Professor B. Paczyński for numerous enlightening discussions on problems relating to this investigation. It is a pleasure to acknowledge that the numerical computations were made with the PDP 11/45 computer donated to the Copernicus Astronomical Center by the US National Academy of Sciences following the initiative and through the action of Dr. R. C. O'Dell and Mr. A. M. Baer.

The research was supported in part by the US National Science Foundation, through the M. Sklodowska-Curie Fund, under Grant No. INT-76-80588.

References

Dziembowski,W. 1977, Acta Astron. 27, 95
Iben,I.Jr. 1975, Astrophys. J. 196, 525
Ledoux,P., Sauvenier-Goffin,E. 1950, Astrophys. J. 111, 611
Mestel,L. 1952, Monthly Notices Roy. Astron. Soc. 112, 598
Osaki,Y., Hansen,C.J. 1973, Astrophys. J. 185, 277
Paczyński,B. 1970, Acta Astron. 20, 47
Paczyński,B. 1971, Acta Astron. 21, 417
Paczyński,B. 1977, Astrophys. J. 214, 812
Paczyński,B., Żytkow,A.N. 1977, Astrophys. J. in press
Rose,W.K. 1968, Astrophys. J. 142, 855
Schatzman,E. 1953, Ann. d'Astrophys. 16, 162
Sienkiewicz,R. 1975, Astron. Astrophys. 45, 411
Starrfield,S., Sparks,W.M., Truran,J.W. 1974a, Astrophys. J. Suppl. 28, 247
Starrfield,S., Sparks,W.M., Truran,J.W. 1974b, Astrophys. J. 192, 647
Warner,B., Robinson,E.L. 1972, Nature Phys. Sci. 239, 2

DISCUSSION of paper by SIENKIEWICZ and DZIEMBOWSKI:

KIPPENHAHN: I have not understood why the g-modes which have higher amplitudes in the outer regions (where the damping occurs) than radial modes favour more your vibrational instability. Don't you need sufficiently high amplitudes in the deeper regions, where the shell sources are in order to drive the ε-mechanism?

DZIEMBOWSKI: In our models hydrogen and helium burning zones occur in superficial layers. Oscillation amplitudes of both g-modes and radial modes are large in this region. The difference occurs in the massive isothermal core where only the radial modes maintain large amplitudes. Because of much larger inertia they are excited with much longer time scales.

KIPPENHAHN: From the evolutionary tracks of stars with surface shell flashes one can estimate the time the star spends in the Cepheid strip. Can one estimate how mass can be expelled? Can one go so far as to assume that planetary nebulae are formed that way?

DZIEMBOWSKI: In any case it must be very little because the envelopes contain only 10^{-3}-10^{-4}.

MIRZOYAN: I would like to make a short comment on FG Sge. Dr. Chalange, Divan and me carried out a spectral study of FG Sge based on the observations made by Chalange's spectrograph at the Haut-Provence observatory during 1969-1974. This study showed continuous change of MK-spectral type of FG Sge and a sharp change in the period when Dr. Kraft and his collaborators mentioned an "s-process". The spectral observations of FG Sge gave also an evidence of the existence around this star of a circumstellar cloud which was changing as well. (Astrofiziks, No. 3, 1977).

SHAVIV: I understand that the stability analysis is a linear one.

If this is so, how can you say something about mass-loss? Or amplitude of variation, etc.?

DZIEMBOWSKI: This is just a speculation based on extremely large excitation rates.

KRAFT: It's amazing that none of the Lick Coudé plates we took between 1969 and 1973, showed velocity variations greater than 5 to 10 km/sec.

SCHOEMBS: Do you consider it to be possible that the period region extends up to about 100 s?

DZIEMBOWSKI: Yes, g-modes excited by ε-mechanisms may have periods of about 100 s for small white dwarf masses.

SMAK: After Dr. Schoemb's question, it is my impression that your oscillations in VW Hyi with $P \sim 90^s$ may be related to the longer-period oscillations reported by McDonald people for RV Peg, and perhaps in both cases we are not dealing with pulsations.