Second Meeting, December 8th, 1893.

JOHN ALISON, Esq., ex-President, in the Chair.

Note on the number of numbers less than a given number and prime to it.

By Professor STEGGALL.

The following proof of the well-known result, n being any number, a, b, c the different prime factors that singly, or multiply, compose it

$$\phi(n) = n\left(1 - \frac{1}{a}\right)\left(1 - \frac{1}{b}\right)\dots$$

seems worthy of notice.

Consider any number N : let p be any smaller number; consider also the number aN where a is a prime factor of N : then the numbers

$$p, \ p + \mathrm{N}, \ p + 2\mathrm{N}, \ \dots \ p + (a-1)\mathrm{N}$$

are all less than aN.

They are also all prime to N (and to aN) if p is, and not otherwise; for p has no prime factor of N (and of aN) and N, 2N . . etc., have every prime factor of N (and of aN).

Hence if a is a prime factor of N the number of numbers less than aN and prime to it (sometimes called the totient of aN) is a times the totient of N,

or $$\phi(a\mathrm{N}) = a\phi(\mathrm{N}). \qquad (1)$$

Again let b be a prime number not a factor of N; then of the numbers

$$p, \ p + \mathrm{N}, \ p + 2\mathrm{N}, \ p + (b-1)\mathrm{N}$$

one, and one only, is divisible by b.

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Hence as before if p is prime to N, $b-1$ of the above numbers are prime to bN. Thus if b is a prime non-factor of N the totient of bN is $(b-1)$ times the totient of N, or

$$\phi(bN) = (b-1)\phi(N) \tag{2}$$

Now $\phi(a) = a-1$ and we note that the totient of any prime a is $(a-1)$ times that of unity, if we call that of unity one.

We see then that in multiplying the prime factors a p times, b q times, etc., in any order, the totient of the numbers resulting is once (viz., at the first introduction of a new factor a) increased in the ratio $a-1$, and at every other introduction of a in the ratio a; similarly for b, c, etc. Hence

$$\phi(a^p b^q c^r \ldots) = a^{p-1} b^{q-1} c^{r-1} (a-1)(b-1)(c-1)\ldots \times \phi(1)$$
$$= a^{p-1} b^{q-1} c^{r-1} \ldots (a-1)(b-1)(c-1)\ldots$$

or

$$\phi(n) = n\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots$$

Professor Steggall also exhibited two geographical models, and said :—

I had intended to bring before you a simple definition of a ridge line on a surface, and to shortly discuss the equation deduced; but since offering my paper I learnt that Dr M'Cowan had developed very fully the consequences of an equivalent definition; and therefore, lest I should accidentally impair the interest of his paper, I shall leave with him the treatment of the whole subject, a treatment that I believe includes all I had to say.

The models I exhibit were originally made with a view to the presentation in a concrete form of the mathematical conceptions of contour lines, lines of slope, saddle points, indicatrices, and ridges in surfaces. As a minor and secondary object, the educational value of the representation of an actual hill seemed to justify the construction of the model of a real mountain rather than that of any surface derived from merely arbitrary design, or from fixed equation. Besides this, there is a probability that such a model as I show may result in a more lasting impression in the mind of the student; and there seems more than a probability that the application of mathematical terms to a surface that has not been constructed to suit the requirements of the analysis will serve to fix the general theories and their wide application securely in the learner's mind.

For my first model, Ben Cruachan was selected, because of the richness of feature it presents; for the other model, which was left in a state to show the method of construction, Glencoe was selected, because of its steepness, the deep indentations in its hills, and its general interest.

The scale of Ben Cruachan is three inches to the mile horizontal, and four vertical. That of Glencoe is $2\frac{1}{2}$ inches to the mile in both directions.

On Ridge Lines and Lines connected with them.*

By J. M'Cowan, D.Sc.

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