J. Austral. Math. Soc. 23 (Series A) (1977), 467–475.

NON-TALL COMPACT GROUPS ADMIT INFINITE SIDON SETS

M. F. HUTCHINSON

For George Szekeres on his sixty-fifth birthday

(Received 9 September 1975; revised 23 July 1976)

Abstract

Riesz polynomials are employed to give a sufficient condition for a non-abelian compact group G to have an infinite uniformly approximable Sidon set. This condition is satisfied if G admits infinitely many pairwise inequivalent continuous irreducible unitary representations of the same degree. Consequently a compact Lie group admits an infinite Sidon set if and only if it is not semi-simple.

1. Introduction

General properties of infinite Sidon sets for a compact group G are well-known (see §37 of Hewitt and Ross (1970)), but the question of the existence of such sets if G is not abelian has until now been rarely answered in the affirmative. The result enunciated in the title of this paper exhibits a wide class of compact groups which admit infinite Sidon sets. It also answers a question of Parker (1972).

Our proof, which uses Riesz polynomials, is modelled on the proofs of similar results by Rider (1966) and Parker (1972). It yields Sidon sets which enjoy the additional property of being uniformly approximable.

The main theorem is proved in §2 with the result of the title occurring as Corollary 2.5. In §3 the implications of our result for compact Lie groups are presented and we conclude in §4 with two counter-examples to a partial converse of Corollary 2.5.

This research was carried out under the supervision of J. R. McMullen. We also wish to thank A. H. Dooley for his contribution to the final form of this paper.

NOTATION. Let G be a compact group. Normalized Haar measure on G

https://doi.org/10.1017/S1446788700019613 Published online by Cambridge University Press

is denoted by λ_G and the Banach space of all complex Radon measures on G is denoted by $M(G)$.

Let $\hat{G}$ be a maximal set of pairwise inequivalent continuous irreducible unitary representations of G. For each $\sigma \in \hat{G}$ let H_σ denote its corresponding representation space of dimension d_σ, χ_σ its trace and σ_{ij} $(i, j = 1, \cdots, d_\sigma)$ the coordinate functions of σ with respect to a fixed orthonormal basis of H_σ. For any subset P of $\hat{G}$ the Banach spaces $\mathfrak{E}_p(P)$ $(1 \leqq p \leqq \infty)$ and their respective norms are as defined in (28.24), (28.34), (D.36(e)) and (D.37) of Hewitt and Ross (1970). The *characteristic section* of P is the section $I_p \in \mathfrak{E}_\infty(\hat{G})$ defined by

$$I_P(\sigma) = \begin{cases} I_\sigma & \sigma \in P \\ 0_\sigma & \sigma \in \hat{G} \backslash P \end{cases}$$

where I_σ and 0_σ denote the identity and zero transformations on H_σ respectively.

The Fourier–Stieltjes transform of $\mu \in M(G)$ is the section $\hat{\mu} \in \mathfrak{E}_\infty(\hat{G})$ defined by

$$\hat{\mu}(\sigma) = \int_G \sigma(x^{-1})\, d\mu(x) \qquad (\sigma \in \hat{G})$$

and the Fourier transform of $f \in L^1(G, \lambda_G)$ is the section $\hat{f} \in \mathfrak{E}_\infty(\hat{G})$ defined by

$$\hat{f}(\sigma) = \int_G \sigma(x^{-1}) f(x)\, d\lambda_G(x) \qquad (\sigma \in \hat{G}).$$

The closure in $\mathfrak{E}_\infty(\hat{G})$ of the set of Fourier–Stieltjes transforms is denoted by $M(G)^{\wedge -}$.

A subset E of $\hat{G}$ is said to be a *Sidon set* if $C_E(G)$, the set of all continuous functions on G whose Fourier transforms vanish off E, is contained in the Fourier algebra of G (see §37 of Hewitt and Ross (1970)). This is equivalent to saying that for each $\Phi \in \mathfrak{E}_\infty(E)$ there exists $\mu \in M(G)$ such that $\hat{\mu}(\sigma) = \Phi(\sigma)$ for all $\sigma \in E$. A Sidon set E is said to be *uniformly approximable* if the characteristic section I_E belongs to $M(G)^{\wedge -}$. Finite unions of uniformly approximable Sidon sets are uniformly approximable Sidon, this fact following most readily from the characterization of uniformly approximable Sidon sets as those sets $E \subset \hat{G}$ for which $\mathfrak{E}_\infty(E) \subset M(G)^{\wedge -}$ (see Dunkl and Ramirez (1971a)).

Finally, we adopt the convention of McMullen and Price (1976) and say that G is *tall* if for each positive integer n there are only finitely many elements of $\hat{G}$ of degree n.

2. The main theorem

Our use of Riesz polynomials in the proof of Theorem 2.4 requires that the following numbers, first described for general compact groups by Parker (1972), be defined.

DEFINITION 2.1. Let $E = \{\sigma_1, \sigma_2, \cdots\}$ be a countable subset of $\hat{G}$. For $\sigma \in \hat{G}$ and positive integers m, s, N let $R_s(E, \sigma, N, m)$ be the number of subsets $\{\tau_1, \cdots, \tau_s\}$ of cardinality s of $\hat{G}$ satisfying τ_k equals σ_{n_k} or $\bar{\sigma}_{n_k}$ for some σ_{n_k} in E $(k = 1, 2, \cdots, s)$, $n_1 < n_2 < \cdots < n_s \leqq N$, and

$$\int_G \chi_{\tau_1} \cdots \chi_{\tau_s} \bar{\chi}_\sigma \, d\lambda_G = m.$$

Let

$$R_s(E, \sigma, N) = \sum_{m=1}^{\infty} m \, . \, R_s(E, \sigma, N, m)$$

and

$$R_s(E, \sigma) = \lim_{N \to \infty} R_s(E, \sigma, N).$$

We also require two simple lemmas.

LEMMA 2.2. *Let A be a linear operator on a finite dimensional Hilbert space H of dimension n. Then we have*

$$\| A \|_{\phi_\infty} \leqq n \, . \max\{| a_{ij} | : i, j = 1, \cdots, n\}$$

where the a_{ij} are the matrix coefficients of A with respect to any orthonormal basis of H.

PROOF. Write $A = |A| U$ where $|A|$ is positive definite and U is unitary. Let $\{\lambda_1, \cdots, \lambda_n\}$ be the eigenvalues of $|A|$. Since $\sum_{i=1}^{n} \lambda_i^2 = \sum_{i,j=1}^{n} |a_{ij}|^2$ we have that $\| A \|_{\phi_\infty} = \max \lambda_i \leqq n \, . \max | a_{ij} |$.

LEMMA 2.3. *Let $\sigma^{(1)}, \sigma^{(2)}, \cdots, \sigma^{(m)} \in \hat{G}$ each have degree n and let $\tau \in \hat{G}$. Then we have*

$$\left| \int_G \sigma_{ij}^{(1)} \cdots \sigma_{ij}^{(m)} \overline{\tau_{rs}} \, d\lambda_G \right| \leqq d_\tau^{-1} S(\tau)$$

$(i, j = 1, \cdots, n; r, s = 1, \cdots, d_\tau)$ where $S(\tau)$ is the multiplicity of τ in the tensor product representation $\sigma^{(1)} \otimes \cdots \otimes \sigma^{(m)}$.

PROOF. Observe that $\sigma_{ij}^{(1)} \cdots \sigma_{ij}^{(m)}$ is a coordinate function of $\sigma^{(1)} \otimes \cdots \otimes \sigma^{(m)}$ with respect to the obvious orthonormal basis of $H_{\sigma^{(1)}} \otimes \cdots \otimes H_{\sigma^{(m)}}$. Let

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$$T\colon \bigoplus_{\tau\in\hat{G}} S(\tau)\,.\,H_\tau \to H_{\sigma^{(1)}}\otimes\cdots\otimes H_{\sigma^{(m)}}$$

be a linear isometry intertwining $\bigoplus_{\tau\in\hat{G}} S(\tau)\,.\,\tau$ and $\sigma^{(1)}\otimes\cdots\otimes\sigma^{(m)}$. Now if v_{ij}, w_{pq} are the coordinate functions of $\bigoplus_{\tau\in\hat{G}} S(\tau)\,.\,\tau$ and $\sigma^{(1)}\otimes\cdots\otimes\sigma^{(m)}$ respectively, we have

$$w_{pq} = (T(v_{ij})T^*)_{pq} = \sum_{i,j=1}^{mn} t_{pi}v_{ij}\overline{t_{qj}}$$

$(p, q = 1,\cdots, mn)$. Since the entries of T are bounded in absolute value by 1, the orthogonality relations for the coordinate functions on G give that

$$\left|\int_G w_{pq}\overline{\tau_{rs}}\,d\lambda_G\right| \leqq S(\tau)\int_G \tau_{rs}\overline{\tau_{rs}}\,d\lambda_G = d_\tau^{-1}S(\tau).$$

THEOREM 2.4. *Let $E = \{\sigma_1, \sigma_2, \cdots\}$ be a countable subset of $\hat{G}$ such that $d_\sigma = n$ for all $\sigma \in E$ and let B be a positive integer such that $R_s(E, 1) \leqq B^s$ $(s = 1, 2, \cdots)$. Then E is a uniformly approximable Sidon set.*

PROOF. We verify that E is a uniformly approximable Sidon set by showing that for each $\varepsilon > 0$ and each $\Phi \in \mathfrak{E}_\infty(E)$ with

$$\sup\{|\Phi(\sigma)_{ij}|\colon \sigma \in E,\ i, j = 1,\cdots, n\} \leqq 1$$

there exists $\mu \in M(G)$ with $\|\Phi - \hat{\mu}\|_\infty < \varepsilon$ (see (37.2(viii)) of Hewitt and Ross (1970)). We may suppose that, with respect to our fixed basis $\Phi(\sigma)$ has real coefficients for all $\sigma \in E$, that $1 \notin E$ and that $\hat{G}$ is countable.

We may also assume that if $\sigma \in E$, $\sigma \neq \bar{\sigma}$ then $\bar{\sigma} \notin E$ since we can always express E as the union of two sets satisfying this additional requirement. Therefore, by Lemma 3.1 of Parker (1972) we may suppose that $B \geqq n$ and that $R_s(E, \sigma) \leqq B^s$ for all $\sigma \in \hat{G}$.

Choose a positive integer m such that $1/m < \varepsilon/(2n)$ and a positive integer $q \geqq 2$ such that $1/(q-1) < \varepsilon/(8mn^3)$. Let $\beta = 1/(qB^2)$.

Partition the closed interval $[-1, 1]$ into $2m$ disjoint intervals $A_{-m+1}, A_{-m+2},\cdots, A_m$ by setting

$$A_{-m+1} = [-1, (-m+1)/m],$$
$$A_k =](k-1)/m, k/m] \quad (-m+2 \leqq k \leqq m).$$

For each positive integer N let $E_N = \{\sigma_1,\cdots,\sigma_N\}$ and then set

$$E_{Nijk} = \{\sigma \in E_N\colon \Phi(\sigma)_{ij} \in A_k\}$$

$(i, j = 1,\cdots, n;\ k = -m+1,\cdots, m)$. For fixed N, i and j, $\{E_{Nijk}\}_{k=-m+1}^{m}$ is a partition of E_N.

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For each N, i, j, k define a non-negative trigonometric polynomial f_{Nijk} on G by

$$f_{Nijk} = \prod_{\sigma \in E_{Nijk}} (1 + \beta \mathscr{R}(\sigma_{ji})) = 1 + \beta \sum_{\sigma \in E_{Nijk}} \mathscr{R}(\sigma_{ji}) + \sum_{\tau, r, s} C_{rs}(\tau, N, i, j, k) \tau_{rs}$$

where $\mathscr{R}(\sigma_{ji})$ denotes the real part of σ_{ji} and the second sum runs over all $\tau \in \hat{G}$, r, $s = 1, \cdots, d_\tau$. Using Lemma 2.3 we see that

$$|C_{rs}(\tau, N, i, j, k)| \leqq \sum_{p=2}^{N} R_p(E, \tau, N)\beta^p \leqq \sum_{p=2}^{\infty} R_p(E, \tau)\beta^p$$

$$\leqq \sum_{p=2}^{\infty} B^p \beta^p \leqq \beta/(q-1) < \beta\varepsilon/(8mn^3).$$

Similarly let

$$g_{Nijk} = \prod_{\sigma \in E_{Nijk}} (1 + \beta \mathscr{I}(\sigma_{ji})) = 1 + \beta \sum_{\sigma \in E_{Nijk}} \mathscr{I}(\sigma_{ji}) + \sum_{\tau, r, s} D_{rs}(\tau, N, i, j, k) \tau_{rs}$$

where $\mathscr{I}(\sigma_{ji})$ denotes the imaginary part of σ_{ji} and

$$|D_{rs}(\tau, N, i, j, k)| < \beta\varepsilon/(8mn^3).$$

Now set

$$h_N = \sum_{i,j=1}^{n} \sum_{k=-m+1}^{m} nkm^{-1}\beta^{-1}(f_{Nijk} + \sqrt{-1}\, g_{Nijk})$$

$$= (1 + \sqrt{-1}) n^3 m^{-1} \beta^{-1} \sum_{k=-m+1}^{m} k + \sum_{\sigma \in E_N} \sum_{i,j=1}^{n} (k^\delta/m) n \sigma_{ji} + \sum_{\tau, r, s} K_{rs}(\tau)\tau_{rs}$$

where $|K_{rs}(\tau)| < \varepsilon/2$ and k^δ is the integer k such that $\Phi(\sigma)_{ij} \in A_k$.

Define $\mu_N \in M(G)$ by $d\mu_N = h_N d\lambda_G$. Then for all N we have

$$\|\mu_N\| = \|h_N\|_1 \leqq \sum_{i,j=1}^{n} \sum_{k=-m+1}^{m} n|k|m^{-1}\beta^{-1}(\|f_{Nijk}\|_1 + \|g_{Nijk}\|_1)$$

$$\leqq \sum_{i,j=1}^{n} \sum_{k=-m+1}^{m} n|k|m^{-1}\beta^{-1}(2 + \beta\varepsilon/(4mn^3))$$

$$= 2n^3 m \beta^{-1} + \varepsilon/4$$

and for each $\sigma \in E_N$ we have

$$|\Phi(\sigma)_{ij} - \hat{\mu}_N(\sigma)_{ij}| = |\Phi(\sigma)_{ij} - \int_G h_N \overline{\sigma_{ji}}\, d\lambda_G|$$

$$\leqq |\Phi(\sigma)_{ij} - k^\delta/m| + |k^\delta/m - \int_G h_N \overline{\sigma_{ji}}\, d\lambda_G|$$

$$\leqq 1/m + |K_{ji}(\sigma)|/n < \varepsilon/(2n) + \varepsilon/(2n) = \varepsilon/n.$$

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For each $\sigma \notin E_N$, $\sigma \neq 1$ we have

$$|\hat{\mu}_N(\sigma)_{ij}| < \varepsilon/(2d_\sigma).$$

Since $\{\mu \in M(G): \|\mu\| \leqq 2n^3 m\beta^{-1} + \varepsilon/4\}$ is sequentially weak-*compact, we may choose a subsequence of $\{\mu_N\}_{N=1}^\infty$ which converges in the weak-*topology to some $\mu \in M(G)$. Then, by Lemma 2.2, we have

$$\|\Phi(\sigma) - \hat{\mu}(\sigma)\|_{\phi_\infty} < \varepsilon \quad \text{for all} \quad \sigma \neq 1.$$

Now choose a multiple α of the trigonometric polynomial 1 so that $(\mu + \alpha 1)\hat{}(1) = 0$. Then $\|(\mu + \alpha 1)\hat{} - \Phi\|_\infty < \varepsilon$ as required.

COROLLARY 2.5. *Let G be non-tall compact group. Then G admits an infinite uniformly approximable Sidon set.*

PROOF. By definition, for some positive integer n there is an infinite subset E of $\hat{G}$ with $d_\sigma = n$ for all $\sigma \in E$. As in Corollary 3.4 of Parker (1972) we may construct by induction a sequence of subsets $\{F_m\}$ of E such that for each m, the cardinality of F_m is m, $F_m \subset F_{m+1}$ and $R_s(F_m, 1) = 0$ $(s = 1, 2, \cdots)$. For we may choose any element of E to obtain F_1 and then, for each m, construct F_{m+1} by adjoining to F_m any element of E not occurring as a constituent of a tensor product of s distinct elements of $F_m \cup \bar{F}_m$ $(s = 1, \cdots, m)$. This is possible because there are only finitely many such tensor products and each tensor product, being finite dimensional, has only finitely many constituents in E. Now let $F = \bigcup_{m=1}^\infty F_m$. Then $F \subset E$ and $R_s(F, 1) = 0$ $(s = 1, 2, \cdots)$ so F is an infinite uniformly approximable Sidon set.

COROLLARY 2.6. *Let G be the direct product of a countable family of compact groups G_n $(n = 1, 2, \cdots)$ and let B be fixed positive integer. For each n let $\sigma_n \neq 1$ be an irreducible unitary representation of G_n of degree no greater than B. Then $E = \{\sigma_1, \sigma_2, \cdots\}$ regarded as a subset of $\hat{G}$ is a uniformly approximable Sidon set.*

PROOF. Write $E = \bigcup_{k=1}^B E_k$ where $E_k = \{\sigma \in E: d_\sigma = k\}$. Clearly $R_s(E_k, 1) = 0$ $(s = 1, 2, \cdots)$ so E is a finite union of the uniformly approximable Sidon sets E_k $(k = 1, \cdots, B)$.

REMARKS. (1). Dunkl and Ramirez (1971b) have shown that central Sidon sets of bounded representation type are uniformly approximable so the fact that the Sidon sets which we produce are uniformly approximable follows from the fact that they are Sidon.
(2). The example given in §3 of Figà-Talamanca (1967) (see also (37.23) of Hewitt and Ross (1970)) shows that the bound on the degrees of the representations cannot be omitted from the statement of Corollary 2.6.

3. Compact Lie groups

If G is a compact Lie group then a theorem of Cecchini shows that the converse to Corollary 2.5 also holds. Moreover, the (equivalent) conditions of Corollary 2.5 are satisfied precisely when G is not semi-simple. To prove this for possibly disconnected G we require a small lemma.

LEMMA 3.1. *Let G be a compact group with a closed normal subgroup H of finite index in G. Then G is tall if and only if H is tall.*

PROOF. Suppose that H is tall. By the Frobenius reciprocity theorem, each $\sigma \in \hat{G}$ of degree n is a component of an induced representation τ^G where $\tau \in \hat{H}$ is some irreducible component of $\sigma|_H$. Clearly we have $d_\tau \leqq n$. Therefore, since $\{\tau \in \hat{H}: d_\tau \leqq n\}$ is a finite set and each $\tau \in \hat{H}$ induces up to only finitely many $\sigma \in \hat{G}$, we have that $\{\sigma \in \hat{G}: d_\sigma = n\}$ is a finite set.

Conversely, suppose that there are infinitely many $\tau \in \hat{H}$ of degree n. Since each τ has at most $[G/H]$ inequivalent conjugates, Mackey's intertwining number theorem shows that the induced representations τ^G give rise to infinitely many $\sigma \in \hat{G}$ such that $d_\sigma \leqq n\,|G/H|$.

THEOREM 3.2. *Let G be a compact Lie group. The following are equivalent*:

(i) *G is semi-simple*[†];
(ii) *G is tall*;
(iii) *G admits no infinite Λ_4 sets*;
(iv) *G admits no infinite Sidon sets.*

PROOF. We first show that (ii) $\Rightarrow$ (iii) $\Rightarrow$ (iv) $\Rightarrow$ (ii). Theorem 3 of Cecchini (1972) shows that (ii) $\Rightarrow$ (iii) and the implication (iii) $\Rightarrow$ (iv) is given by Theorem (37.10) of Hewitt and Ross (1970). The implication (iv) $\Rightarrow$ (ii) is contained in our Corollary 2.5.

We complete the proof of the theorem by showing that (i) $\Leftrightarrow$ (ii). Note that G_0, the connected component of the identity in G, has finite index in G.

Assume (i). Then G_0 is connected and semi-simple so by Theorem 9 of Rider (1972), G_0 admits no infinite central Sidon sets. Thus G_0 admits no infinite Sidon sets. Corollary 2.5 and Lemma 3.1 then show that (ii) holds.

Conversely, suppose that G is not semi-simple. Then by Theorem 87 of Pontryagin (1939), we may write $G_0 = G^*/N$ where G^* is the direct product of a non-trivial torus T and a semi-simple group S and N is a finite normal

[†]D. Rider (private communication) has given a direct proof that a compact connected semi-simple Lie group is tall.

subgroup of G^*. Clearly S has finite index in NS so $G^*/(NS)$ is an infinite abelian group. Therefore G_0 has infinitely many pairwise inequivalent irreducible unitary representations of degree 1 so, by Lemma 3.1, G cannot be tall.

REMARK. Let G be a compact group with a closed normal subgroup H of finite index in G. If $E \subset \hat{H}$ is a Sidon set for H which is closed under conjugation by elements of G then a rather intricate calculation shows that $F = \{\sigma \in \hat{G}: \sigma \text{ a component of } \tau^G \text{ for some } \tau \in E\}$ is a Sidon set for G. In particular, if H has an infinite abelian continuous homomorphic image then $\hat{H}$ contains an infinite set E satisfying this requirement, and so G admits an infinite Sidon set. This result may be used to give a proof of Theorem 3.2 which does not use Corollary 2.5.

4. Examples

We give two examples of tall compact groups which admit infinite Sidon sets. They are similar to the example given in (37.5) of Hewitt and Ross (1970).

EXAMPLE 4.1. Let $G = \Pi_{n=2}^{\infty} SU(n)$ where, for each positive integer $n \geqq 2$, $SU(n)$ denotes the group of $n \times n$ complex unitary matrices with determinant 1. Since $SU(n)$ is simple, Theorem 3.2 shows that $SU(n)$ is tall. Moreover, if $\pi \in SU(n)^\wedge$ is not the representation which is identically 1, then $\pi(SU(n))$ is a compact Lie subgroup of $U(d_\pi)$ of dimension $n^2 - 1$ (see Helgason (1962), p 346). Since $U(d_\pi)$ has dimension d_π^2 we must have that $d_\pi \geqq n$. It follows that G is tall.

For each n let π_n be the projection of G onto $SU(n)$. Corollary 4.2 of Parker (1972) shows that $E = \{\pi_2, \pi_3, \cdots\}$ is a central Sidon set for G. We show in fact that E is Sidon.

Accordingly, let $f \in C_E(G)$. For each n we may write $\hat{f}(\pi_n) = A_n W_n$ where A_n is positive definite and W_n is unitary. We may then choose a complex number α_n of absolute value 1 and a matrix $X_n \in SU(n)$ such that $W_n = \alpha_n X_n$. Since E is central Sidon there are central measures λ and μ on G such that $\hat{\lambda}(\pi_n) = \alpha_n I$ and $\hat{\mu}(\pi_n) = \overline{\alpha_n} I$ for all n (Parker 1972, Theorem 2.1). Let δ_X be Dirac measure at $X = (X_2, X_3, \cdots) \in G$. Then $g = \mu * \delta_X * f \in C_E(G)$ and $\hat{g}(\pi_n) = A_n$ for each n so by Theorem (34.12) of Hewitt and Ross (1970) we have that g belongs to the Fourier algebra. Thus $f = \delta_{X^{-1}} * \lambda * g$ belongs to the Fourier algebra and E is a Sidon set.

EXAMPLE 4.2. Let $G = \Pi_{n=2}^{\infty} SO(n)$ where for each n, $SO(n)$ is the group of $n \times n$ real orthogonal matrices with determinant 1. As in the previous example G is tall and $E = \{\pi_2, \pi_3, \cdots\}$, where π_n is the projection of G onto $SO(n)$, is a central Sidon set.

Now let $f \in C_E(G)$. Since the π_n are real we have that $\mathcal{R}f, \mathcal{I}f \in C_E(G)$ and that $(\mathcal{R}f)^\wedge(\pi_n)$, $(\mathcal{I}f)^\wedge(\pi_n)$ each have real coefficients. Thus we may suppose that f is real-valued. Since real matrices also have a polar decomposition (Gantmacher (1958), p. 263), a similar argument to that of the previous example shows that f belongs to the Fourier algebra and so E is Sidon.

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Department of Pure Mathematics,
University of Sydney,
Sydney, N.S.W. 2006,
Australia.