A Magnetic Betelgeuse? Numerical Simulations of Non-linear Dynamo Action

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Abstract. Betelgeuse is an example of a cool supergiant displaying brightness fluctuations and irregular surface structures. Simulations by Freytag, Steffen, & Dorch (2002) of the convective envelope of the star have shown that the fluctuations in the star's luminosity may be caused by giant cell convection. A related question regarding the nature of Betelgeuse and supergiants in general is whether these stars may be magnetically active. If so, that may in turn also contribute to their variability. By performing detailed numerical simulations, I find that both linear kinematic and non-linear dynamo action are possible and that the non-linear magnetic field saturates at a value somewhat below equipartition: in the linear regime there are two modes of dynamo action.

1. Introduction

The cool super-giant star Betelgeuse is one of the the stars with the largest apparent diameters on the sky—corresponding to a radius somewhere in the range 600–800 $R_\odot$. Freytag, Steffen, & Dorch 2002 performed detailed numerical three-dimensional radiation-hydrodynamic simulations of the outer convective envelope and atmosphere of the star under realistic physical assumptions. They tried to determine if its observed brightness variations may be understood as convective motions within the star's atmosphere: the resulting models are largely successful in explaining the observations as a consequence of giant-cell convection on the stellar surface, very dissimilar to solar convection. These detailed simulations bring forth the possibility of solving another question regarding the nature of Betelgeuse and super-giants in general; namely whether such stars may harbor magnetic activity that in turn may also contribute to their variability and other phenomena derived from the presence of a magnetic field (such as dust formation and mass-loss). A possible astrophysical dynamo in Betelgeuse would most likely be very different from those thought to operate in solar type stars, both due to its slow rotation, and to the fact that only a few convection cells are present at its surface at any one time.

2. Model

We solve the full three-dimensional magneto-hydrodynamical (MHD) equations for a fully convective star—a so called "star-in-a-box" simulation—employing

the "Pencil Code" by Brandenburg & Dobler (2003). This code has a specially designed "convective star" module, that allows the solution of the non-linear MHD equations by the numerical pencil scheme in a star with a fixed radius R and mass M. Variables are measured in terms of these latter two parameters so that e.g. the unit of the star's luminosity L becomes $\frac{M}{R}(GM/R)^{\frac{3}{2}}$. In the present case I set R = 640 $R_\odot$ and M = 5 $M_\odot$ yielding a luminosity of L = 46000 $L_\odot$, consistent with current estimates of Betelgeuse' actual size, mass and luminosity. We use a numerical resolution of 128^3 uniformly distributed grid points, a fixed gravitational potential, an inner small heating core, and an outer thin isothermally cooling surface at r = R, with a cooling time scale set to τ_{cool} = 1 year (corresponding to the model of Freytag et al. 2002).

Betelgeuse is only slowly rotating and a rotational frequency was chosen corresponding to a surface rotational velocity of 5 km/s, yielding a small inverse Rossby number meaning that the flows are not very helical.

2.1. Diffusion and magnetic Reynolds number

Dynamo action by flows are often studied in the limit of increasingly large magnetic Reynolds numbers $\mathrm{Re_m} = \ell U/\eta$, where ℓ and U are characteristic length and velocity scales. Most astrophysical systems are highly conducting yielding small magnetic diffusivities η, and their dimensions are huge resulting in huge values of $\mathrm{Re_m}$. Betelgeuse is not an exception; most parts of the star is better conducting than the solar surface layers, which has a magnetic diffusivity of the order of $\eta \approx 10^4$ m^2/s (see Dorch & Freytag 2002).

There is some uncertainty as to defining the most important length scale of the system, but taking ℓ to be 10% of the radial distance from the center (a typical scale of the giant cells), and U = u_{RMS} along the radial direction yields $\mathrm{Re_m} = 10^{10}$–10^{12} in the interior part of the star where R $\leq$ 700 $R_\odot$.

In the present case we cannot use that large values of $\mathrm{Re_m}$ (partly due to the fact that we are employing a uniform fixed diffusivity), but rely on the results from generic dynamo simulations indicating that results converge already at Reynolds numbers of a few hundred (e.g. Archontis, Dorch, & Nordlund 2003a, 2003b). Furthermore, Dorch & Freytag (2002) obtained kinematic dynamo action in their model of a magnetic Betelgeuse at $\mathrm{Re_m} \sim 500$. Here we put $\mathrm{Re_m} \sim 300$ (based on the largest scales). Additionally the magnetic Prantl number is $\mathrm{Pr_m} \sim 80$.

3. Results

There is some disagreement as to what one should require for a system to be an astrophysical dynamo. Several ingredients seems necessary: flows must stretch, twist and fold the magnetic field lines; reconnection must take place to render the processes irreversible; weak magnetic field must be circulated to the locations where flow can do work upon it; and finally, the total volume magnetic energy E_m must increase (kinematic regime) or remain at constant amplitude on a long time scale (non-linear regime). These points are based largely on experience from idealized kinematic and non-linear dynamo models; e.g. Archontis et al. (2003a, 2003b). Here we shall deal mainly with the question of the exponential growth and saturation of of E_{mag}.
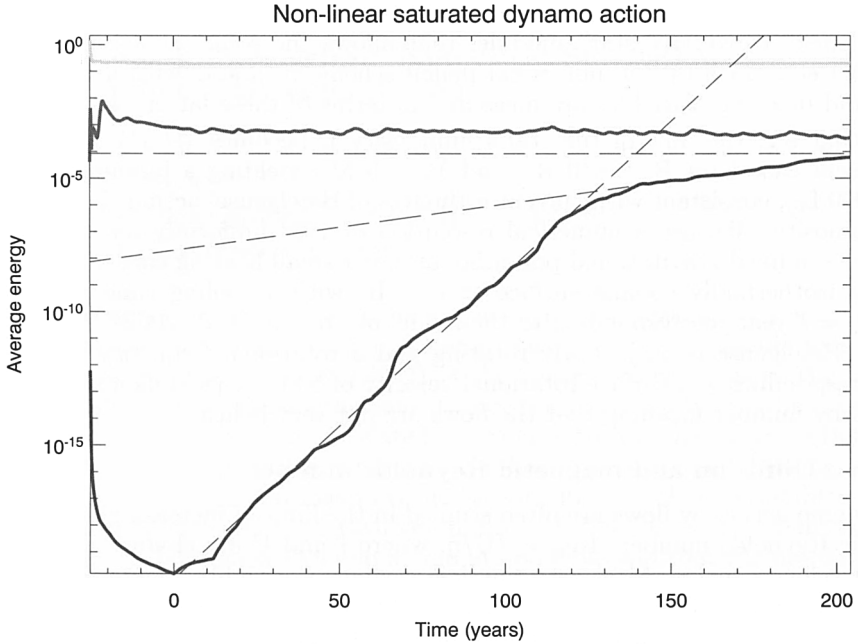
Figure 1.   Linear regime: energy as a function of Betelgeusian time in years. Upper (almost horizontal) line is total thermal energy E_{th} (yellow in color print), middle full horizontal curve with wiggles is total kinetic energy E_{kin} (red in color print), and lower full curve is E_m (blue in color print). The dashed thin lines indicate growth corresponding to growth times of 3.8, 25 and ∞ years.

In an earlier kinematic study of Betelgeuse using a completely different numerical approach (Freytag et al. 2002 and Freytag & Dorch 2002), dynamo action was obtained when the specified minimum value of Re_m was larger than approximately 500 (at lower values of Re_m the total magnetic energy decayed). In the present case with $Re_m \sim 300$, we find an initial clear exponential growth over several turn-over times, and many orders of magnitude in energy. Figure 1 shows the evolution of E_{mag} as a function of time, for the first 200+ years (in Betelgeusian time): initially there is a "short" transient of 25 years, where the field exponentially decays because the fluid motions has not yet attained their final amplitude. Once the giant cell convection has properly begun, however, the magnetic field is amplified and we enter a linear regime (of exponential growth). There are two modes of amplification in the linear regime; the initial mode with a growth rate of about 3.8 years, which in the end gives way to a mode with a smaller growth rate corresponding to 25 years. This is a slightly strange situation, since normally modes with smaller growth rates are overtaken by modes with larger growth rates (cf. Dorch 2000); the explanation is probably that while both modes are growing modes, only the one with the largest growth rate is a purely kinematic mode—the second mode is not kinematic, but the
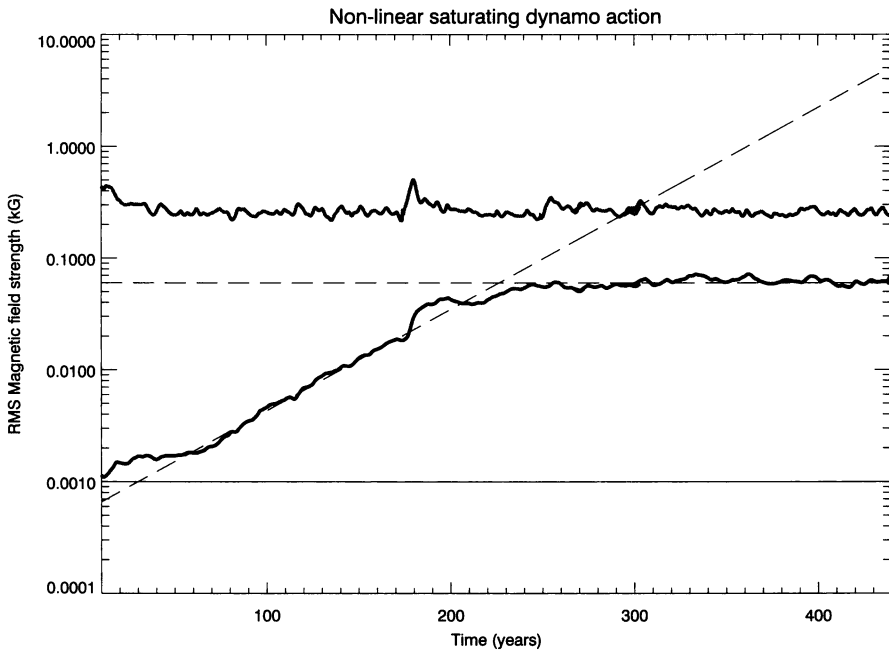
Figure 2.   Transition to the non-linear regime: RMS magnetic field B in kilo-Gauss (kG) as a function of Betelgeusian time in years. Upper full curve is the equipartition field strength B_{eq} corresponding to the fluid motions (red in color print) and the lower full curve is the actual RMS field strength B (blue in color print). The dashed thin lines correspond to growth times of 25 and ∞ years, while the full thin line is the field strength at $t = 10$ year, corresponding approximately to the start of the second linear mode.

presence of the magnetic field is felt by the fluid (through the Lorentz force becoming important), which quenches the growth slightly.

No exponential growth can go on forever and eventually the magnetic energy amplification must saturate. Figure 2 shows E_{mag} as a function of time for 440 Betelgusian years: the second linear mode as well as the mode in the non-linear regime are shown. The magnetic field saturates at an RMS values of about 60 Gauss, corresponding to a total magnetic energy E_{mag} below equipartition with the kinetic energy E_{kin} by a factor of ~ 0.25.

We can study the geometry of the magnetic field that the dynamo generates[1]: the field becomes concentrated into elongated structures much thinner than the scale of the giant convection cells, but perhaps due to the very dynamic nature of the convective flows, no "intergranular network" is formed in the solar sense.

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[1]The CD-ROM contains a volume rendering of isosurfaces of magnetic field structures with a high field strength relative to the maximum: the figure `dorch.color-fig.ps`.

The field is highly intermittent, i.e. only a small fraction of the volume carries the strongest structures. The magnetic structures are well resolved, with maximum power on the largest scales corresponding to wavenumbers of a few. Additionally we observe a slight trend in the topology of the field (see also Freytag & Dorch 2002); fields near the surface of the star are predominantly horizontally aligned, while those in deeper layers are more or less radial.

4.   Concluding remarks

Based on the results presented here, we may not say conclusively if Betelgeuse has a magnetic field; the results are tentative and should be used with caution. But we may say that it seems that it might have a presently unobserved magnetic field. The main highlights are the following:

- In the linear regime two modes are present: an initial kinematic mode with a high growth rate is overtaken by a linear mode with a lower growth rate and thus a longer growth time.

- The growth time of the last occurring mode in the linear regime is about 25 years, i.e. the same value as was found in previous purely kinematic dynamo models (Freytag et al. 2002).

- In the non-linear regime the field strength saturates at an RMS value of about 60 Gauss, corresponding to sub-equipartition at $E_{mag} \approx 0.25 \, E_{kin}$.

- Magnetic structures in the non-linear regime are large by solar standards, but smaller than the giant convection cells, with a typical scale of about 15% of the radius.

References

Archontis, V.D., Dorch, S.B.F., & Nordlund, Å. 2003a, A&A, 397, 393

Archontis, V.D., Dorch, S.B.F., & Nordlund, Å. 2003b, A&A, 410, 759

Brandenburg, A., & Dobler, W. 2003, see http://www.nordita.dk/data/ brandenb/pencil-code/

Dorch, S.B.F. 2000, Physica Scripta, 61, 717

Dorch, S.B.F. & Freytag 2002, in Modelling of Stellar Atmospheres (IAU Col. 188), ESA SP-505,

Freytag, B., Steffen, M., & Dorch, S.B.F. 2002, Astron. Nachr., 323, 213