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## 1. INTRODUCTION

The problem of the existence of high-order clustering of galaxies is a basic one in extragalactic astronomy and cosmology. In a conventional formulation this problem consists of indicating the largest configurations in the spatial distribution of galaxies.

All treatments are carried out generally on the material presented in the catalogues of Abell (1958) and Zwicky et al. (1961-1968). However catalogues of the Jagellonian type (Rudnicki et al. 1973) may turn out to be extremely valuable. The nature of the observational material demonstrates that only statistical methods may be applied effectively. It is already clear that the expected effects are not large even if high-order clustering is a basic property in the Universe.

The first important results concerning the existence of secondorder clusters (excluding de Vaucouleurs' work, 1953, on the Local Supercluster) were obtained by Abell (1958, 1961, 1965). There were however objections to the idea of superclustering even earlier (Zwicky 1957). The results of Kiang (1967) and Kiang \& Saslaw (1969) indicated that high-order clusters may exhibit a continuous spectrum of their characteristic sizes as well, i.e. clustering with no preferred size. This was skilfully demonstrated in the excellent regiew by de Vaucouleurs (1971) containing almost all important references on clustering of galaxies up to 1971.

## 2. METHOD AND RESULTS

Various methods for investigating superclustering have been applied to eliminate the basic difficulty that all the observational material must be considered as polluted samples. The methods used most are i) the dispersion-subdivision test (Zwicky 1952) - Zwicky \& Rudnicki (1966), Karpowicz (1967a,b, 1970a,b, 1971c); ii) the analysis of the index of clumpiness (Neyman \& Scott 1952, Neyman et al. 1954) - Kiang (1967). Kalinkov (1974b), Kalinkov \& Tomov (1976);
iii) $x$-squared test - Abell (1958), Abell \& Seligman (1965, 1967), Gusak (1969), Fullerton \& Hoover (1972), Kalinkov (1974b); iv) the correlation and power spectrum analysis - Karachentsev (1966), Kiang (1967), Kiang \& Saslaw (1969), Yu \& Peebles (1969), Fullerton \& Hoover (1972), Peebles (1973, 1974, 1975), Hauser \& Peebles (1973), Peebles \& Hauser (1974), Peebles \& Groth (1975), Kalinkov (1973b, 1974a,b, 1975), Kalinkov \& Janeva (1976), Kalinkov et al. (1975a, c,d); v) the distances to the nearest neighbours - Bogart \& Wagoner (1973), Kalinkov (1974b). Some other methods have also been applied - e.g. the area of the largest clusters (Zwicky \& Rudnicki, 1963, Zwicky \& Berger, 1965, Zwicky \& Karpowicz, 1965, 1966, Karpowicz, 1971a,b), the smoothing and filtering of the cluster fields (Kalinkov, 1973a, 1974a,b, Kalinkov et al., $1975 \mathrm{c}, 1976$ ), the so-called "statistical reduction" of Zieba (1975).

All these methods may be classified as methods using either individual (point) information (e.g. the nearest neighbours distances) or group (in cells) information (e.g. breaking up the area under consideration into subareas). Another division of the methods may be made depending on whether a preliminary model is or is not developed which is to be tested. One may make another division of the methods or rather results according to whether or not the investigator is convinced a priori of the existence or non-existence of high-order clustering of galaxies. At least a quarter of the papers cited above are based on biased ideas.

Let us now review some methods and results.
2.1 Methods of comparison between observed and expected distributions for cluster centres in subareas

The simplest version consists of breaking up the area under investigation into subareas, counting the clusters there and comparing the observed distribution with a Poisson distribution for the mean number of objects per unit area. Another version is the introduction of the ratio of the observed and computed dispersions as a measure of the departure from a randomly distributed population. This analysis has been criticized many times and justly so; for example, Neyman et al. (1954) demonstrated that the behaviour of this ratio is quite different from what was expected by intuition.

Abell (1958) found that for clusters in each different distance group, a particular cell-size can be found for which the probability of departure from randomness is greatest. The angular diameters of these cells was shown to be roughly inversely proportional to the mean redshift of the distance group. If this picture is correct then a very serious objection against the idea of second-order clustering is removed since neither intergalactic nor interstellar absorption is responsible for the nonrandom cluster distribution.

A new method of this type was proposed recently (Kalinkov, 1974b,


Fig. 1. Expected results from the generalized $\chi^{2}$-test. $S$ is the area of the cells compactly covering the whole area. The zone $C$ indicates a well defined second-order clustering.


Fig. 2. The generalized $\chi^{2}$-test for Zwicky ED clusters.

1976, Kalinkov \& Tomov, 1976) which does not contain the shortcomings of the other methods. It is based on the ideas and results from Neyman et al. (1954), Abell (1958), Slakter (1966) and Kiang \& Saslaw (1969). This is in fact a generalized $x$-squared test for searching for characteristic sizes, especially for second-order clusters. This method is applicable for all possible cases of subarea divisions. An idealized and qualitative picture of the expected results is given on Fig. 1. Let $P\left(X^{2}\right)$ stand for the probability that the distribution observed is random and $S$ be the area of the various cells covering the area under consideration completely. If the results of the test lie in zone $A$, this would mean the acceptance of the null hypothesis there are no tendencies for super-clustering. If they lie in zone B there is a dilemma - either the clustering has a continuous spectrum or the characteristic size has not been reached. If the results lie in a region such as zone $C$ this is a strong indication of the existence of super-clustering.

This picture was tested with artificial fields containing some gaussian "clusters of clusters". For a region of about $1300 \mathrm{deg}^{2}$ centred on the NGP the results for Zwicky ED clusters and for all Abell clusters are shown in Fig. 2 and Fig. 3. Now repeating Abell's procedure (Abell, 1958, 1961, 1965) one could test the linearity between the reciprocal angular dimensions of second-order clusters and the distance (Fig. 4). In fact the relation is a straight line. There is an exeeption only for VD clusters. The most plausible explanation is that this is due to a systematic error made by Zwicky in the cataloging of VD clusters.

### 2.2 The method of the nearest neighbours

The possibilities of the previous method are naturally limited in the sense that if third-order clusters exist they can not be recognised by this method. This limitation is not valid for the method of the nearest neighbours, applied successfully by Bogart \& Wagoner (1973) to Abell clusters. They established the existence of second-order clusters containing a fev rich clusters. This method is based on the technique developed by Wagoner (1967) and consists of computing the angular separation between each cluster and its nearest, second nearest and so forth neighbours. According to Bogart \& Wagoner the second nearest and so forth distances give little new information. But let us suppose that we have a two-dimensional random field composed of the cluster centres. Let us denote the theoretical and computed mean distances to the $n$-th nearest neighbour by $\bar{R}_{n}$ and $\hat{R}_{n}$. Then the difference $\bar{R}_{n}-\hat{R}_{n}$ will contain some information on the whole for highorder clustering of galaxies. Each significant difference $\bar{R}_{n}-\hat{R}_{n}=$ max $>0$ will be related to a characteristic high-order (not only second-order) clustering size.

The results for the two-dimensional problem of all Abell clusters having $b>70^{\circ}$ (to avoid edge effects, clusters with $b<70^{\circ}$ are also included) are given in Fig. 5, where $\bar{R}-\hat{R}$ is a function of the nearest-


Fig. 3. The generalized $\chi^{2}$-test for Abell clusters.


Fig. 4. A linear relation between the reciprocals of the characteristic angular size for second-order clusters and the redshifts.


Fig. 5. $\bar{R}_{\mathrm{n}}-\hat{\mathrm{R}}_{\mathrm{n}}$ for two-dimensional nearest neighbours problem as a function of the nearest-neighbour number.


Fig. 6. $\bar{R}_{n}-\hat{R}_{n}$ for three-dimensional nearest neighbours problem.


Fig. 7. (a) Map filtered with [1] - [2] for Abell clusters. All negative densities are removed. The lower isopleth is for $0.1 \mathrm{cl} / \mathrm{sq}$. degr; the step is 0.1 .
(b) Map filtered with [1] - [2] for Zwicky clusters. The lower isopleth is for $0.2 \mathrm{c} 1 / \mathrm{sq}$. degr; the step is 0.2 .


Fig. 8.
(a) Map smoothed with [4] for Abell clusters.
(b) Map smoothed with [4] for Zwicky clusters.
we can not expect so many nev results of such importance in the near future. He established that the covariance function is a power law which might be treated so that the clustering phenomenon is continuous to a first approximation and his results are a solid foundation for checking the results of the theoretical models of all clustering phenomena cosmogony.

### 2.4 Methods of smoothing and filtering of cluster fields

If we have some information at least for the characteristic size for high-order clustering we can represent the apparent as well as the spatial distribution of the clusters of galaxies with the aid of the smoothing and filtering technique. A simple method of utilization of these operations is proposed by Kalinkov (1973a, 1974a) and the construction of complete sets of smoothing functions (SF) and filters or filtering functions (FF) is presented by Kalinkov et al. (1976). Some examples of smoothed and filtered maps for all Abell and Zwicky clusters with $b>0^{\circ}$ are shown on Figs 7 and 8. The observational interval ( $4 \mathrm{deg}^{2}$ ) is hatched. The densities are in cl $\mathrm{deg}^{-2}$. The smoothed and filtered maps exhibit the following peculiarities. It is remarkable that the general features of the maps for Zwicky and Abell clusters are in good agreement - especially Fig. 8. There are areas of $20-100 \mathrm{deg}^{2}$ which indicate convincingly the existence of secondorder clusters. There are about 40 similar objects on both hemispheres. Their density contrast is greater than 3 (Fig. 7). There are not only single but also double and triple configurations. Moreover for both hemispheres there are four areas with density contrast 2 , which may be considered as third-order clusters. Their characteristic sizes are about $300 h^{-1} \mathrm{Mpc}$ (a value, highly uncertain since various systematic effects are introduced perhaps determining these sizes).

## 3. CONCLUSION

There is now evidence demonstrating beyond doubt the existence of second-order clusters having characteristic size of about 50 Mpc . There is also evidence for higher-order clustering with dimension of 100 Mpc . There exists a tendency toward clustering with certain preferred sizes of about 150, 200, 300 (?) Mpc.

At the same time the individuality as well as the density contrast of the clustering decreases with the increase of the order of the system.

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