PRECOLLAPSE EVOLUTION OF GLOBULAR CLUSTERS

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ABSTRACT: Density profiles of most globular clusters are well fitted by a King (1966) model. The evolution of a King model in the tidal gravitational field of the Galaxy is first discussed. If the concentration parameter c (= $\log(r_t/r_c)$) is small enough, the evolution is nearly along the King model sequence and c becomes larger. When c becomes large enough (about 2.1), gravothermal instability sets in. The basic properties of gravothermal instability is next discussed. The stability criterion and its interpretation are given. Globular clusters consist of stars with disparate masses, so that finally the evolution of multi-component clusters is discussed. Acceleration of evolution in multi-component clusters and equipartition of the kinetic energies among components are discussed, and conclusions and future problems are given.

1. INTRODUCTION

Spitzer (1985) made an excellent review of this field at IAU Symposium No. 113 at Princeton and relatively little progress has been made in this field since then. In this paper I will review precollapse evolution from a somewhat different point of view.

Globular clusters are bounded by the gravitational field of the galaxy. Their radii are determined by the balance of the gravitational force of the Galaxy and their own gravitational force. Most clusters are well fitted by a King (1966) model. The model is characterized by a parameter c (= $\log(r_t/r_c)$), where r_t is the tidal radius and r_c is the core radius.

If c is smaller than the critical value (about 2.1), clusters evolve due to evaporation of stars from the tidal radii and if c is larger than the critical value, clusters evolve due to gravothermal instability (Katz 1980, Wiyanto et al. 1985). In the former case the central density becomes higher and higher and c becomes larger. In this stage the evolution is nearly along the King sequence. At late-time epochs the evolution is due to gravothermal instability.

2. EVOLUTION DUE TO EVAPORATION OF STARS FROM THE TIDAL RADIUS

If c is smaller than the critical value, the evolution is

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governed by the escape of stars from the cluster. Wiyanto et al. (1985) calculated the evolution of a King model by numerical integration of isotropized Fokker-Planck equation and found that evolution is nearly along a King sequence if c is small enough (Fig. 1). When c becomes about 2.1, gravothermal instability takes place and evolution is accelerated. They also show that the rate of evaporation of stars is constant in time and in good agreement with King's (1966) prediction.

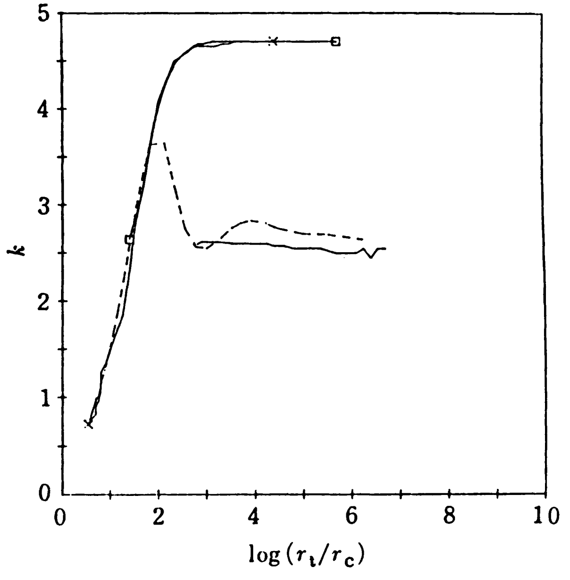

Fig. 1. Comparison of the k-(r_t/r_c) relation of the simulations of Wiyanto et al. (1985) to that of King's (1966) sequence, where k is defined by $E = -(k+1/2)GM^2/r_t$ and E is the total energy of the cluster. The broken curve denotes King's sequence. Solid curves denote the evolutionary sequence obtained by the simulations. The curve between the two cross marks is for model A, which is started from a stable configuration. The curve between the two square marks is for model B, which is started from a nearly marginally stable configuration. A nearly horizontal curve in the middle of the figure is for model C, which is started from an unstable configuration.

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Chernoff, Kochanek and Shapiro (1986) investigated the effect of tidal heating due to giant molecular clouds and to passage through the galactic plane. They showed that passage through the galactic plane is dominant and that small concentrated clusters are disrupted and those with large concentrations are accelerated toward a gravothermal instability. Both processes may occur within the Hubble time for any concentration parameter. For further details, see Chernoff and Shapiro (1987).

3. EVOLUTION DUE TO GRAVOTHERMAL INSTABILITY

If c is larger than 2.1 (Katz 1980), globular clusters evolve due to gravothermal instability. Originally gravothermal instability was investigated under the assumption that clusters are strictly isothermal and therefore bounded by spherical walls (Antonov 1962, Lynden-Bell and Wood 1968, Hachisu and Sugimoto 1978). Lynden-Bell and Wood (1968) examined the gravothermal instability by considering linear series (Fig. 2). The character of instability changes at the turning point of E-v-diagram, where v is the dimensionless central potential. The validity of a linear series to investigate stability is verified by Inagaki and Hachisu (1978), Yoshizawa et al. (1978) and Katz (1978). Hachisu and Sugimoto (1978) formulated the problem of stability by using a Green's function. They expressed the variation of the temperature δT in terms of the variation of the entropy:

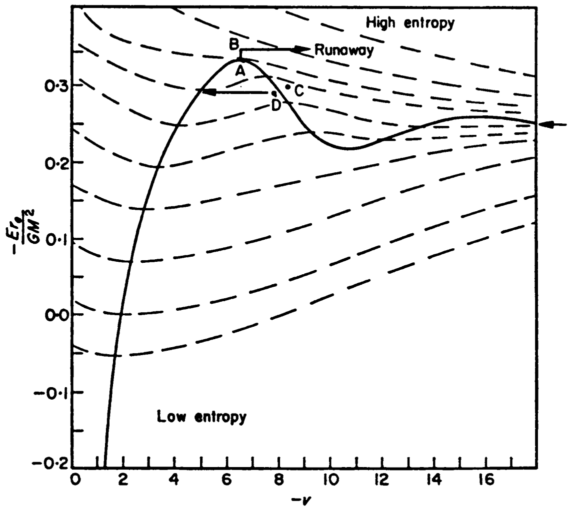

Fig. 2. Energy-v relation for isothermal clusters. The central potential v has a one-to-one relation with the density contrast. The first turning point corresponds to the density contrast of 709. The cluster is unstable for larger values of v.

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$$\delta \ln T(M_r) = \int_0^M F(M_r, M_r')\,\delta s(M_r')\,dM_r' \qquad (1)$$

where δs is the variation of the specific entropy. Therefore F corresponds to the inverse specific heat. The meaning of equation (1) is as follows: If we transport heat inside a cluster, it is represented by the variation of the entropy distribution. The cluster may reach then a new hydrostatic equilibrium and the temperature distribution is changed, and F may be calculated. The second order variation of the entropy is expressed as

$$\delta^2 S = -\int_0^M dM_r \int_0^M dM_r' F(M_r, M_r')\,\delta s(M_r)\,\delta s(M_r') \qquad (2)$$

Equation (2) shows that if the inverse specific heat tensor F is negative, the second order variation of the entropy is positive, i.e., instability. The Green's function F is shown for the stable case, marginally stable case and for the unstable case in Fig. 3. In the stable case the region of negative specific heat is small but it grows as the cluster becomes more unstable. Hachisu and Sugimoto (1978) also calculated the eigenvalues and eigenfunctions which maximize $\delta^2 S$. They are shown in Figs. 4 and 5, respectively. Fig. 4 shows that the fundamental mode becomes unstable at the first turning point of Fig. 2 and the second mode becomes unstable at the second turning point and so on. Thus the cluster is unstable if the density contrast is larger than 709. Fig. 5 shows that if the cluster is unstable, δS and δT take the opposite sign, which is consistent with equations (1) and (2).

Evolution after gravothermal instability was first examined by Larson (1970) by using moment equations of the Fokker-Planck equation. Detailed numerical calculation by numerical integration of the isotropized Fokker-Planck equation was done by Cohn (1980), who showed that evolution takes place homologously (Fig. 6). Homologous evolution after gravothermal instability was examined in detail by Lynden-Bell and Eggleton (1980). They adopted a conductive gas model following Hachisu et al. (1978), by using modified conductivity:

$$K = 6GC \log N \rho/v$$

The usual expression for conductivity for plasmas is inadequate for stellar systems because it increases as the relaxation time becomes larger. Thus Lynden-Bell and Eggleton adopted a expression for conductivity such that the conductivity decreases as the relaxation time increases. According to their results, the central quantities of the cluster change as follows:

$$\rho_c \propto (1 - t_{cc}/t)^{-1.17} \qquad (3)$$

$$v_c \propto (1 - t_{cc}/t)^{-0.055} \qquad (4)$$

$$R_c \propto (1 - t_{cc}/t)^{0.52} \qquad (5)$$

$$M_c \propto (1 - t_{cc}/t)^{0.42} \qquad (6)$$

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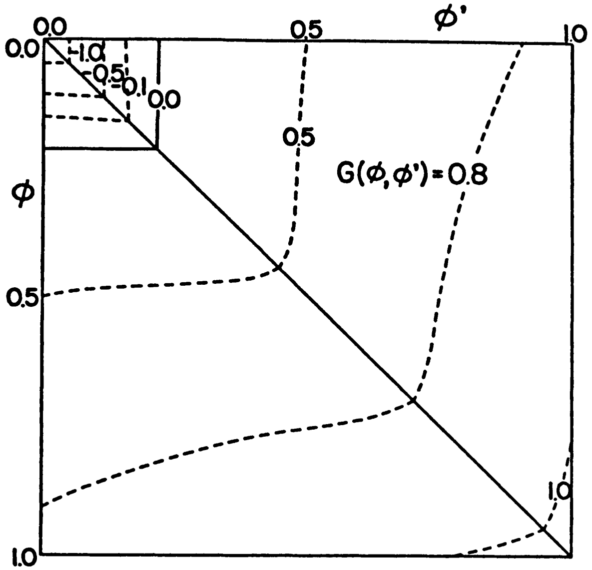

Fig. 3a. Contour map of the off-diagonal part of the inverse tensor specific heat $F(\phi,\phi')$ where $\phi = M_r/M$. This is for a stable system with a small density contrast, 2.5.

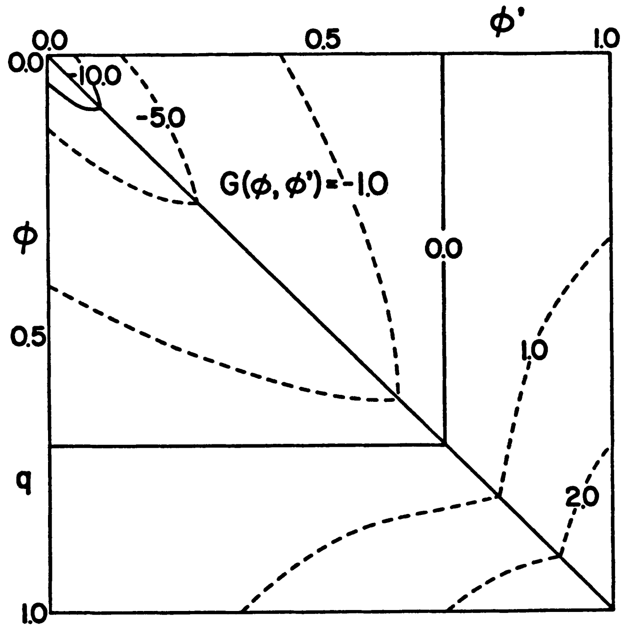

Fig. 3b. The same as Fig. 3a but for a marginally stable system with a density contrast of 709.

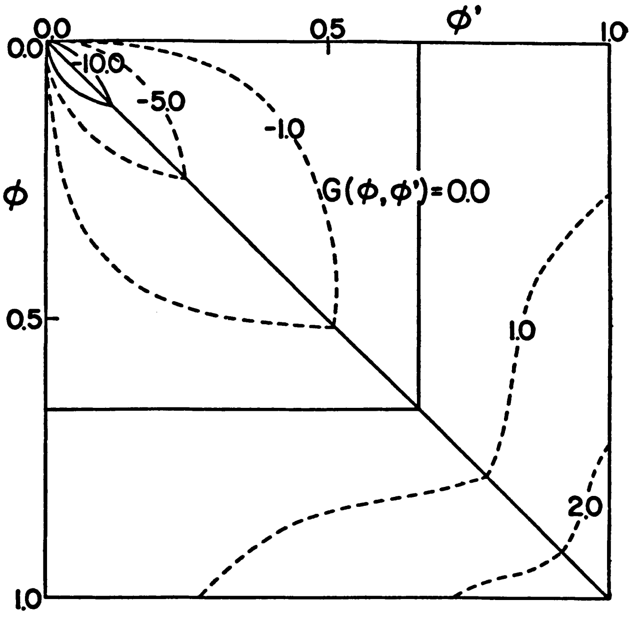

Fig. 3c. The same as Fig. 3a but for a strongly unstable system with a density contrast of 3.16×10^7.

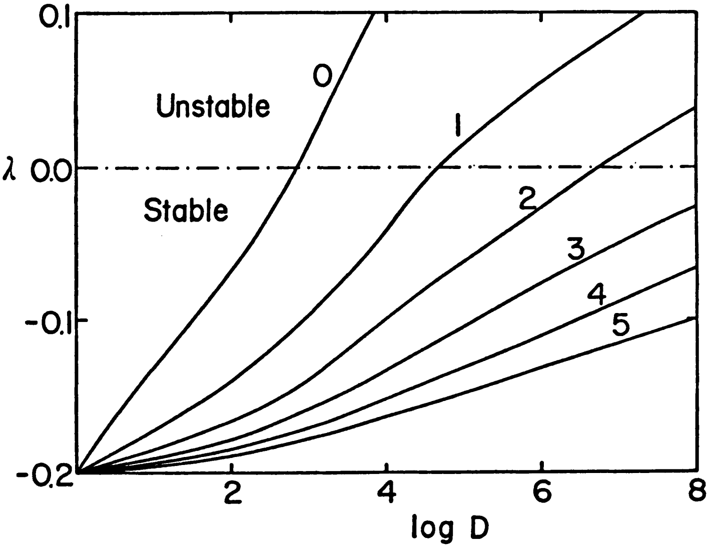

Fig 4. The eigen value λ is shown against the density contrast D for the fundamental mode (0) as well as higher harmonics (1, 2, ...). The condition for gravothermal instability is $\lambda > 0$.

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where ρ_c is the central density, v_c the central velocity dispersion, r_c the core radius and M_c is the core mass. From equation (3) we see that the central density diverges when $t=t_{cc}$. Another remarkable point is that the logarithmic density gradient dln ρr/dln r is -2.21 in Lynden-Bell and Eggleton's model, which is very close to Cohn's (1980) value -2.23, though they adopted a conductive gas model.

4. EVOLUTION OF MULTI-COMPONENT CLUSTERS

Globular clusters consist of stars with different masses. In this subsection, we consider the effect of disparate masses in globular clusters. The main extent of disparate masses is to greatly accelerate the cluster evolution. Let us consider the simplest multi-component cluster, i.e., two-component cluster. Inagaki (1985) carried out numerical integrations of the one-dimensional Fokker-Planck equation and showed that the evolution time scale is about ten times faster than the single component cluster (Table I).

Another feature of the two-component cluster is destabilization due to mass-segregation instability (Spitzer 1969).

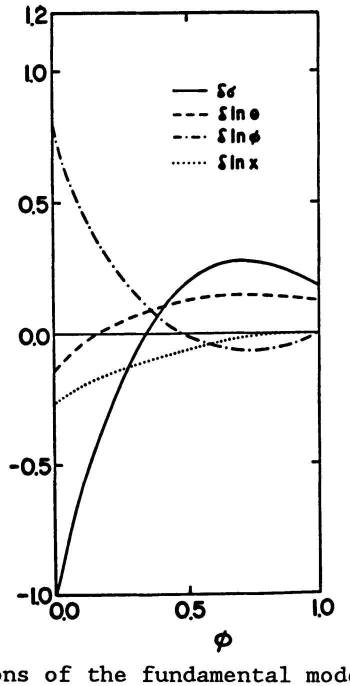

Fig. 5a. Eigenfunctions of the fundamental mode are shown in arbitrary units against the Lagrangian mass coordinate $\phi = M_r/M$ for the case of a stable system with D = 24.2 and λ = -0.11. σ, Θ, and x are dimensionless entropy, temperature and radius, respectively.

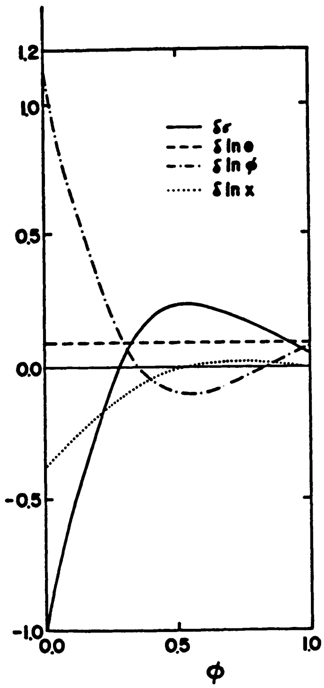

Fig. 5b. The same as Fig. 5a but for the marginally stable system with D = 709 and $\lambda = 0$

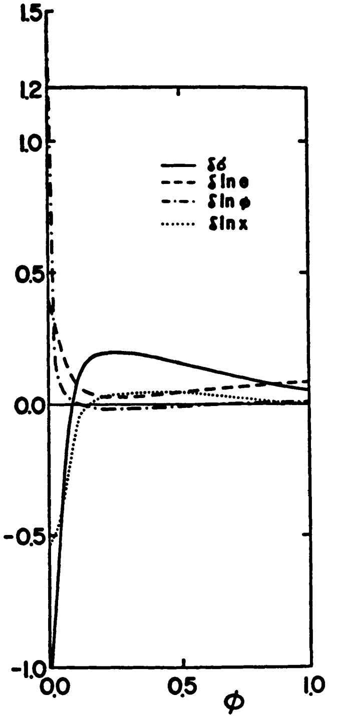

Fig 5c. The same as Fig. 5a but for the case of an unstable system with D = 1.41 x 10^6 and $\lambda = 0.2$.

TABLE I
The time (in the unit of t_{rh}) required for the complete collapse in two-component clusters with $m_1/m_2 = 5$.

m_1/m_2	0.001	0.005	0.014	0.072	0.30
t_{cc}	14.8	10.8	4.2	1.7	1.9

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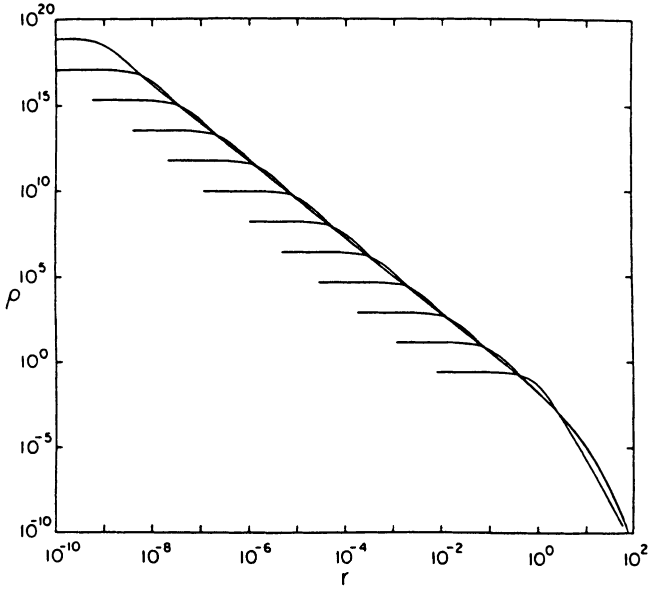

Fig.6. Cohn's (1980) calculation of a single component cluster. Homologous evolution of the cluster is seen from the figure.

Spitzer (1969) showed that if

$$S = (M_2/M_1)(m_2/m_1)^{-1.5} \qquad (7)$$

is larger than 0.16, equipartition between the more massive component and the less massive component is impossible. The stability of isothermal two-component clusters was analyzed by Yoshizawa et al. (1978). The curve of marginal stability is shown in Fig. 7. They showed that the states of marginal stability lie for large range of the density contrast: the cluster can be unstable even if the density contrast is 19 which is much smaller than 709 for a single component system. In Fig. 7 some stabilization effect is seen, i.e., the isothermal cluster with $m_2 = 10$ and $M_2/M_1 = 0.003$ is stable if $\rho_c/\rho_b <$ 5012. This stabilization can be understood as follows. Fig. 7 shows that $\rho_2(0)/\rho_1(0)$ is constant ($\simeq$ 8) along the marginally stable states near this model. This means that the development of the halo does not affect the stability or that the stability is determined by the state of the core. In other words, the stability is caused by the exchange of energy between components in the core. However, if the halo becomes too extended (for example, density contrast of the less massive massive component exceeds 709), the less massive component becomes unstable as a single component cluster. The density contrast at this stage is about $\rho_2(0)/\rho_1(0)$ times 709 so that it is significantly larger than 709. In this sense, the stabilization in a two-component cluster is deceptive.

Another interesting point is whether equipartition of kinetic energies between different components may be achieved. Inagaki and Wiyanto (1984) and Inagaki (1985) made numerical integration of isotropized Fokker-Planck equation and confirmed Spitzer's prediction. Inagaki and Saslaw (1985) further made simulations of fifteen component clusters and found that equipartition is impossible if the mass spectrum is shallower than $dM \propto m^{-6}\,dm$. This conclusion should be compared with Vishniac's (1978) result that equipartition is possible if the mass spectrum is steeper than $dM \propto m^{-2.5}\,dm$, although this assumed a homologous density profile for each mass component, which is never realized.

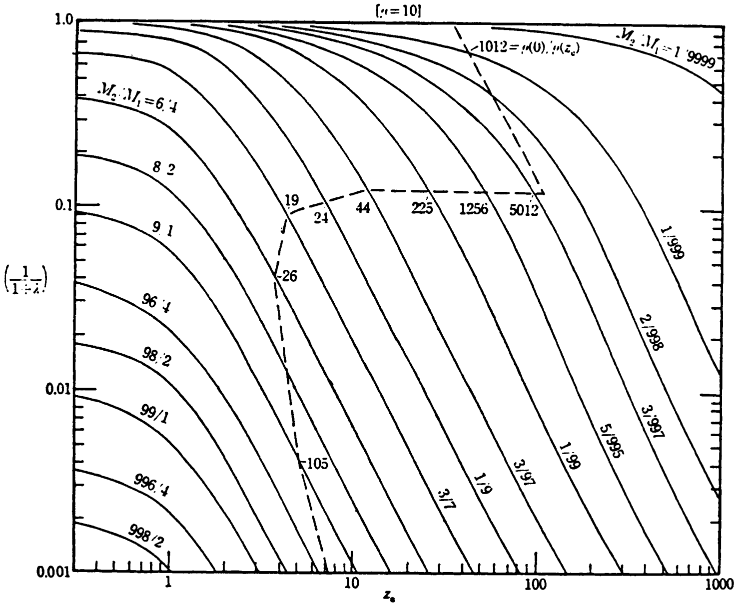

Fig. 7. Curves of fixed M_2/M_1 (solid curves) and of the marginal stability (dashed curve) in the $(1 + \lambda)^{-1}$-z_0 diagram for the case of μ = 10, where $\lambda = \rho_2(0)/\rho_1(0)$ and $\mu = m_2/m_1$. Note that not only each curve of a fixed M_2/M_1 but also each line of a constant λ crosses the curve of the marginal stability criterion only once. The left side of the marginal stability curve is the stable region. The density contrast $\rho(0)/\rho(z_0)$ between the center and the surface is monotonically increasing with increasing z_0 along the curve of a fixed M_2/M_1. The value of the critical density contrast at which the cluster becomes marginally stable is shown along the curve of the marginal stability.

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5. CONCLUSIONS AND FUTURE PROBLEMS

The basic processes in precollapse phase are:
1) Overflow from the tidal radius
2) Tidal shock
3) Gravothermal instability
4) Multi-mass effects, i.e., acceleration of evolution, mass-segregation instability, and lack of equipartition.

All these processes have now been investigated in detail. The primary future problem is to construct realistic models of globular clusters including the above mentioned effects and to see whether most clusters collapse in Hubble time and are currently in post-collapse phase.

REFERENCES

Antonov, V. A. 1962 Vestnik Lenigrad Univ. 7, 135.
Chernoff, D. F., Kochanek, C. S. and Shapiro, S. L. 1986 preprint.
Cohn, H. 1980 Astrophys. J. 242, 765.
Hachisu, I., Nakada, Y., Nomoto, K. and Sugimoto, D. 1978 Prog. Theor. Phys., Kyoto 60, 393.
Hachisu, I. and Sugimoto, D. 1978 Prog. Theor. Phys., Kyoto 60, 123.
Inagaki, S. 1985 in IAU Symposium No. 113, Dynamics of Star Clusters, J. Goodman and P. Hut, eds., Reidel, Dordrecht, p. 189.
Inagaki, S. and Hachisu, I. 1978 Publ. Astron. Soc. Japan 30, 39.
Inagaki, S. and Saslaw, W. C. 1985 Astrophys. J. 292, 339.
Inagaki, S. and Wiyanto, P. 1984 Publ. Astron. Soc. Japan 36, 391.
Katz, J. 1978 Monthly Notices Roy. Astron. Soc. 183, 765.
Katz, J. 1980 Monthly Notices Roy. Astron. Soc. 190, 497.
King, I. R. 1966 Astron. J. 71, 64.
Larson, R. B. 1970 Monthly Notices Roy. Astron. Soc. 147, 323.
Lynden-Bell, D. and Eggleton, P. P. 1980 Monthly Notices Roy. Astron. Soc. 191, 483.
Lynden-Bell, D. and Wood, R. 1968 Monthly Notices Roy. Astron. Soc. 138, 495.
Chernoff, D. F. and Shapiro, S. L. 1987 in IAU Symposium No. 126, Globular Cluster Systems in Galaxies, J. E. Grindlay and A. G. D. Philip, eds., Reidel, Dordrecht, p. XXX.
Spitzer, L., Jr. 1969 Astrophys. J. Letters 158, L139.
Spitzer, L., Jr. 1985 in IAU Symposium No. 113, Dynamics of Star Clusters, J. Goodman and P. Hut, eds., Reidel, Dordrecht, p. 119.
Vishniac, E. T. 1978 Astrophys. J. 223, 986.
Wiyanto, P., Kato, S. and Inagaki, S. 1985 Publ. Astron. Soc. Japan 37, 715.
Yosizawa, M., Inagaki, S., Nishida, M. T., Kato, S., Tanaka, Y. and Watanabe, Y. 1978 Publ. Astron. Soc. Japan 30, 279.

DISCUSSION

DJORGOVSKI: Since the mass contrast accelerates the collapse so much, it is possible to contemplate core collapse in dwarf ellipticals; for example, M 32 has the surface brightness profile which is a slope -1.2 power law for radii < 30". Do you think that it is possible that M 32 is collapsing now, or that it is a PCC galaxy?

INAGAKI: If the half-mass relaxation time of M 32 is smaller than several billion years, it is quite possible that M 32 is a PCC galaxy. However, a tentative calculation shows that the half-mass relaxtion time of M 32 is of the order of the Hubble time. Therefore it is unlikely that M 32 is a PCC galaxy. The power-law density profile of M 32 may be created by some other mechanism.

LEE, H. M.: How much mass is lost due to the high velocity tail of Maxwellian velocity distributions (estimated); and how much is lost due to the conductive flux through the tidal boundary?

INAGAKI: Since we did not take account of close encounters and put the value of distribution function zero at the energy corresponding to the tidal radius, no mass is lost due to high velocity tail of Maxwellian distribution.