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THE ROTATION OF THE EARTH AND ATOMIC TIME STANDARDS

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EDITED BY DIRK BROUWER
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The Symposium was organized by a committee consisting of G. M. Clemence, E. P. Fedorov, W. Markowitz and P. Melchior under the chairmanship of Professor A. A. Mikhailov.

Invited contributors presented papers in three different fields:

I. The Motion of the Pole
II. The Rotation of the Earth
III. Atomic Standards of Frequency.

The paper of B. van der Waerden was presented by D. Brouwer, that of B. Decaux by A. Danjon.

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PART I. THE MOTION OF THE POLE

NUTATION AS DERIVED FROM LATITUDE OBSERVATIONS

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Abstract. The results of several long series of latitude observations have been used for a separate determination of the coefficients of some nutational terms in obliquity and longitude.

The derived value of the constant of nutation is essentially smaller—and those of the coefficients of the semiannual and semimonthly terms larger—than the respective theoretical values, determined on the supposition that the earth as a whole is an elastic body. The theoretical value of the ratio of the axes of the nutational ellipse evidently does not need any correction.

On the basis of these data some conclusions of a qualitative character are made on the interaction between the core and the shell of the earth.

The object of the present paper is to show that some conclusions about the interaction between the earth's core and shell can be drawn from an investigation of nutation. For this purpose we must derive directly from observations more detailed information concerning nutation than was

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hitherto available. Latitude observations over long intervals are likely to give the most favorable material. The main, semiannual and fortnightly terms of nutation deserve special attention.

The effect of the main term on declinations may be expressed by the formula:

$$\Delta_0\delta = - N_0 (n_0 \cos \alpha \sin \Omega - \sin \alpha \cos \Omega), \quad (1)$$

where

N_0 = the adopted value of the constant of nutation,
n_0 = that of the ratio of the axes of the nutational ellipse,
α = the right ascension of a star,
Ω = the longitude of the ascending node of the moon's orbit.

Let us suppose that both N_0 and n_0 need corrections, and besides that there exists a lag of phase different for nutation in longitude and obliquity. Then equation (1) should be replaced by the following:

$$\Delta\delta = - (N_0 + \Delta N) \times [(n_0 + \Delta n) \cos \alpha \sin (\Omega - \beta_1) - \sin \alpha \cos (\Omega - \beta_2)]. \quad (2)$$

If the actual effect of nutation is expressed by formula (2) the analysis of latitude observations should reveal the difference

$$\Delta\varphi = \Delta_0\delta - \Delta\delta \quad (3)$$

since in the reduction of observations formula (1) was employed. This difference may be represented in the following form:

$$\Delta\varphi = A_1 \cos \alpha \cos \Omega + B_1 \sin \alpha \cos \Omega + A_2 \cos \alpha \sin \Omega + B_2 \sin \alpha \sin \Omega, \quad (4)$$

where

$$\left.\begin{array}{ll} A_1 = - N_0 n_0 \beta_1 & A_2 = N_0 \Delta n + \Delta N n_0 \\ B_1 = - \Delta N & B_2 = - N_0 \beta_2 \end{array}\right\}. \quad (5)$$

Thus, our problem is reduced to the determination of the coefficients A_1, A_2, B_1, B_2. I decided to use for this purpose observations of the international latitude stations. However, the results of these observations, as taken directly from publications of the Central Bureau, are unsuitable for an analysis for deriving the periodical variation of the form (4). It is necessary first to apply certain corrections. This was done by T. Hattori when he used the latitude values derived at the international stations for the determination of the constant of nutation (Hattori 1947, 1951). My preliminary calculations in some respects resemble those of Hattori but they differ substantially in some points.

Earlier I noticed that sometimes the scale values adopted by the Central Bureau of the I.L.S. had been subject to considerable errors. Contrary to Hattori, I made an attempt to free the observed latitudes from the effect of these errors, as well as that of a variation of the mean latitudes. Thanks to Uemae's work (1953), it became possible to exclude the errors made by the Central Bureau in applying corrections for the Ross terms of nutation.

I do not dwell here on the description of each phase of my calculation as all the necessary details are given in another paper (Fedorov 1958). I confine myself to giving the final result of this calculation. From the analysis of about 135,000 observations at Carloforte, Mizusawa and Ukiah the following expression has been obtained:

$$\Delta\varphi = - \underset{\pm 25}{0''.0081} \cos \alpha \cos \Omega - \underset{\pm 25}{0''.0128} \sin \alpha \cos \Omega + \underset{\pm 19}{0''.0120} \cos \alpha \sin \Omega - \underset{\pm 19}{0''.0004} \sin \alpha \sin \Omega. \quad (6)$$

The same material has been used for deriving the fortnightly term in latitude variation, but in addition I have availed myself of the result obtained by H. R. Morgan (1952) from observations with the Washington P.Z.T. from 1931 to 1951 and that obtained by A. J. Orlov (1952) from observations with the Pulkovo zenith-telescope from 1915 to 1928. Thus, the total number of observations used for deriving the fortnightly term is 230,000. Some details of this calculation are given in two other papers (Fedorov 1955, 1958). The final result is:

$$\Delta\varphi = + \underset{\pm 14}{0''.0086} \sin (2 ☾ - \alpha) - \underset{\pm 6}{0''.0019} \cos (2 ☾ - \alpha) + \underset{\pm 7}{0''.0021} \sin (2 ☾ + \alpha) + \underset{\pm 10}{0''.0001} \cos (2 ☾ + \alpha). \quad (7)$$

It will be of much interest to have also an expression for the semiannual term. However, in this case the analysis of routine latitude observations meets special difficulties. The only observations which are likely to provide favorable data for such an analysis are those of two bright zenith stars at Poltava. An attempt to derive the semiannual nutational term from these observations was made by N. A. Popov. He obtained the following result:

$$\Delta\varphi = 0''.027 \sin (2\odot - \alpha), \quad (8)$$
$$\pm 4$$

$\odot$ being the mean longitude of the sun.

Using the results of (6), (7) and (8), I have derived the expressions of the three above-mentioned nutational terms both in longitude (ψ_a) and obliquity ($\epsilon_a - \epsilon_0$). They are given in the Table together with the theoretical expressions of the same terms. The latter have been obtained assuming for the constant of nutation the value $9''.220$ which was found from the following relation between H, the mechanical ellipticity of the earth, μ the ratio of the moon's mass to that of the earth, and the constant of nutation:

$$N = 231981''.8 H \frac{\mu}{1 + \mu}. \quad (9)$$

Both this relation and the theoretical expressions for the nutational terms given in the Table were first deduced on the assumption that the earth is rigid, but they would be practically unaffected if allowance were made for its elasticity. If we compare these theoretical expressions of the nutational terms with the results of observations we shall notice at once some evident differences which cannot be ascribed solely to errors of observational data but may be explained as due to the dynamical effect of the earth's core.

We must keep in mind that equations of nutation govern the motion of the earth's angular momentum $\bar{G}$. Since the position of an observatory is a definite place on the earth's surface and an observer is thus always attached to the shell, so are the data obtained from astronomical observations relevant of the motion of the shell alone but not of the earth as a whole. It follows that for a comparison with observations we should take equations governing the motion of the earth's shell. We shall denote its angular momentum by $\bar{G}_s$.

Since both $\dot{\bar{G}}$ and $\dot{\bar{G}}_s$ are vectors lying in the equatorial plane XOY, they may be expressed by complex numbers, as follows:

$$\dot{\bar{G}} = G\,(\sin \epsilon \cdot \dot{\psi} + i\dot{\epsilon}) \quad (10)$$

$$\dot{\bar{G}}_s = G_s\,(\sin \epsilon \cdot \dot{\psi}_a + i\dot{\epsilon}_a). \quad (11)$$

Let $\bar{L}$ be the couple arising from the attraction of the moon and sun on the earth's shell. It is easy to show that

$$\bar{L} = hG_s\,(\sin \epsilon \cdot \dot{\psi} + i\dot{\epsilon}), \quad (12)$$

where h is the ratio of H_s, the mechanical ellipticity of the shell to that of the earth as a whole. Since we deal now with the shell, the effect of the core should be considered as an action of external forces. Denoting the moment of these forces by $\bar{M}$, we may write an equation for the motion of $\bar{G}_s$ in the following form:

$$\dot{\bar{G}}_s = \bar{L} + \bar{M}. \quad (13)$$

The couple $\bar{M}$ transfers the angular momentum between the shell and the core but does not affect the momentum of the earth as a whole.

Let us put

$$\bar{M} = X + iY. \quad (14)$$

Substituting (11), (12) and (14) in (13), we shall have

$$X + iY = G_s\,[\sin \epsilon\,(\dot{\psi}_a - h\dot{\psi}) + i\,(\dot{\epsilon}_a - h\dot{\epsilon})]. \quad (15)$$

Let us compare now this expression for $\bar{M}$ with that obtained on the assumption that the core is rigid or, in general, that no motion of the core relative to the shell is possible. For this special case we denote the couple arising from a mutual action between the core and shell by

$$\bar{M}' = X' + iY', \quad (16)$$

and the angular momentum of the shell by G_s'. Its direction is practically the same as that of the vector $\bar{G}$ and, consequently, its motion is governed by the equation

$$\dot{\bar{G}}_s' = G_s\,(\sin \epsilon \cdot \dot{\psi} + i\dot{\epsilon}), \quad (17)$$

in which ψ and ϵ are the same as in (10).

TABLE I

Nutation in longitude

Term	Theoretical, $\psi \sin\epsilon$	Derived from observations, $\psi_a \sin\epsilon$
Main	$-6''.869 \sin$ ☊	$-6''.853 \sin$ ☊
		$+0.008 \cos$ ☊
Fortnightly	$-0''.0812 \sin 2$☾	$-0''.0866 \sin 2$☾
		$+0.0019 \cos 2$☾
Semiannual	$-0''.507 \sin 2\odot$	$-0''.533 \sin 2\odot$

Nutation in obliquity

Term	Theoretical, $\epsilon - \epsilon_0$	Derived from observations, $\epsilon_a - \epsilon_0$
Main	$+9''.220 \cos$ ☊	$+9''.198 \cos$ ☊
		$-0.001 \sin$ ☊
Fortnightly	$+0''.0884 \sin 2$☾	$+0''.0894 \cos 2$☾
		$+0.0019 \sin 2$☾
Semiannual	$+0''.552 \cos 2\odot$	$+0''.578 \cos 2\odot$

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Substituting the values of $\dot{\bar{G}}_s'$ and $\bar{L}$ given by (17) and (12) in the following equation

$$\bar{M}' = \dot{\bar{G}}_s' - \bar{L}$$

we find

$$X' + iY' = G_s\,(1 - h)(\sin\epsilon\cdot\dot{\psi} + i\dot{\epsilon}). \quad (18)$$

Let

A_c = the equatorial moment of inertia of the core,
A = that of the earth as a whole,
H_c = the mechanical ellipticity of the core.

According to K. Bullen (1936)

$$A_c/A = 0.112, \quad H_c = 0.0026.$$

Then

$$h = 1.027.$$

If we denote the earth's angular velocity by n and put

$$\Omega = \omega_1 t, \quad 2\,☾ = \omega_2 t, \quad 2\odot = \omega_3 t$$

we shall have

$$\omega_1 = -\ 0.000146\ n,$$
$$\omega_2 = +\ 0.07300\ n,$$
$$\omega_3 = +\ 0.00547\ n.$$

The substitution of theoretical expressions for ψ and $\epsilon - \epsilon_0$ in (18) leads to the following equation:

$$X' + iY' = \bar{U}_1 + \bar{U}_2 + \bar{V}_1 + \bar{W}_1 \quad (19)$$

in which

$$\left.\begin{aligned} \bar{U}_1 &= +\ 0''.217\omega_1 G_s e^{+i\omega_1 t}, \\ \bar{U}_2 &= -\ 0''.032\omega_1 G_s e^{-i\omega_1 t} \\ \bar{V}_1 &= +\ 0''.0023\omega_2 G_s e^{+i\omega_2 t}, \\ \bar{W}_1 &= +\ 0''.014\omega_3 G_s e^{+i\omega_3 t} \end{aligned}\right\}. \quad (20)$$

Now let us take the values of ψ_a and $\epsilon_a - \epsilon_0$ from our Table. Being substituted in (15) they give:

$$\begin{aligned} X + iY = (1.09 + 0.02i)\,\bar{U}_1 & \\ + (1.09 + 0.13i)\,\bar{U}_2 & \\ + (-\ 0.43 + 0.83i)\,\bar{V}_1 - 0.86\bar{W}_1. & \quad (21) \end{aligned}$$

Having regard to the uncertainties of the observed values as well as the computed values of N and h, it is not easy to say to what extent this result is trustworthy. Nevertheless I should like to point out the following conclusions which, in my opinion, deserve some consideration:

(1) The actual magnitude of the vector $\bar{U}_1$, as inferred from observational data, is larger than that obtained theoretically for a rigid core;
(2) The actual directions of the vectors $\bar{V}_1$ and $\bar{W}_1$ are opposite to those for the rigid core.

At first glance these conclusions seem to contradict one another. However this contradiction vanishes under more close consideration. The observed changes of the vectors mentioned above agree in sign with that which would be expected on the theory of the dynamical effect of a liquid core. A quantitative comparison makes no sense because of the lack of accuracy of observational data.

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NUTATION AND THE VARIATION OF LATITUDE

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Abstract. A theoretical discussion by the author and R. O. Vicente uses geophysical estimates of the mechanical properties of the earth's shell and two extreme models for the core, chosen to make the mass and moment of inertia correct. The period found for the variation of latitude is in good agreement with observation. The 18.6-year nutation is in better agreement than has been found previously but is still not altogether satisfactory.

It is well known that the period of the 14-monthly variation of latitude is greatly affected by the elasticity of the earth. For a rigid earth the period would be about 305 days. The actual period is rather uncertain but can be taken as 440 days with an extreme uncertainty of 15 days. The difference was for a long time our best datum on the elasticity of the earth as a whole.

Seismology has shown the earth to have a central core, with a radius of about 0.55 of that of the outside; this does not transmit transverse waves and is presumably liquid. Seismology has

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